An Adaptive Fuzzy Image Smoothing Filter for Gaussian Noise

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Abstract: - This paper presents a Gaussian noise filter that uses a sigmoid shaped membership function to model image information in the spatial domain. This function acts as a tunable smoothing intensification operator. With a proper choice of two sigmoid parameters 't' and 'a', the filter strength can be tuned for removal of Gaussian noise in intensity images. An image information measure, Total Compatibility is used to adaptively select these sigmoid parameters. A visible improvement in the smoothness of images is observed, and the output of filter is also compared with those of other standard smoothing methods.

Key-words: - image, noise, parameter, fuzzy logic, Gaussian, filter

1 Introduction

In the presence of noise, image smoothing is an important pre-processing step followed by other tasks such as edge detection, feature extraction and object recognition. Smoothing removes noise, but typically blurs edges as well. The conflicting need for both smoothing and preserving edges has given rise to the development of various filtering methods. Most of the effective approaches are nonlinear and adaptive in nature [3-4]. It is also desirable for the user to be able to control the amount of smoothing according to the application.

In a previous paper, we described the development of a tunable fuzzy filter for image smoothing [7]. We proposed a parameter tuned filter based on the sigmoid function, with good performances in Gaussian, Impulse as well as a combination of both noise environments. The level of smoothing required was controlled by setting the parameter value. In this paper, we present the same filter but with some modifications. To improve the ease of use, instead of the user having to determine the parameter value (or use the default), an image information measure, Total Compatibility [6] is used to adaptively select the parameter value. The effectiveness of this modified filter in removing Gaussian noise is highlighted through some experiments.

The paper is organized as follows. Section 2 describes the sigmoid function and the parameters that control its shape. Section 3 describes the filter design and how the sigmoid function is to be used to control the removal of noise. Section 4 explains how the Total Compatibility is used to adaptively tune the sigmoid parameters. Section 5 reports results of the filter on test images and the comparison with other filters. Section 6 concludes.

2 The Sigmoid Function

The sigmoid shaped function is no stranger to fuzzy processing. Zadeh operator, which is a non-parametric sigmoid function, has been used as the fuzzy intensification operator, INT [5]. A parametric form of the sigmoid function was proposed for graylevel image contrast intensification by Madasu [2]. Later, it was applied to color image contrast intensification as well [1]. This function termed as the new intensification operator, NINT has more flexibility in determining the exact shape of the sigmoid than INT.

We adopt the sigmoid function in [2] with 2 parameters, t and a for the present study, given by

$$\mu(k) = \frac{1}{1+e^{t(d-a)}} \tag{1}$$

where t controls the direction (and steepness) and a controls the position of the curve on the horizontal axis. d is the value calculated from image information, e.g., luminance differences. The shape of the sigmoid is determined by the choices for parameters a and t.

Using the above function, we can design a tunable fuzzy filter, which smoothes depending on the particular choice of parameters values.

3 Filter Design

Generally, an image I of size IxJ and intensity levels in the range (0, L-1) can be considered as a collection of fuzzy singletons in the fuzzy set notation,

$$I = \cup\{\mu_X(x_{i,j})\} = \{\mu_{i,j} / x_{i,j}\};\ i=1,2\ldots,I;\ j=1,2\ldots,J \quad (2)$$

where $\mu_X(x_{i,j})$ represents the membership of some property $\mu_{i,j}$ of $x_{i,j}$, where $x_{i,j} = 0,1,\ldots,L-1$ is the intensity at $(i,j)^{th}$ pixel.

For the transformation of the intensity $x_{i,j}$ in the range (0, L-1) to the fuzzy property plane in the interval (0,1), a membership function is used. The technique operates on a window. For example, let us consider the window of size 3x3.

As shown in Fig. 1, pixel luminance at location (i,j), is $x_{i,j}$.

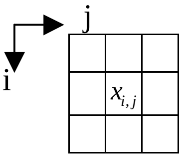

Fig. 1: A 3x3 pixel window

The window defines the central pixel, $x_{i,j}$ and its neighbours. The membership of the pixel $\mu_X(x_{i,j})$ will be used to calculate a noise estimate at each $x_{i,j}$

Noise can be removed from the central pixel by means of subtracting a noise estimate. So, the window moves over every pixel, where we calculate a noise estimate n. This n is to be subtracted from its original intensity, $x_{i,j}$ to get the output intensity, $y_{i,j}$.

$$y_{i,j} = x_{i,j} - n \quad (3)$$

The formula for the noise estimate at location (i,j) is obtained by

$$n = \frac{1}{N} \sum_{\substack{m,n=-k \\ m,n \neq 0}}^{k} \mu_3(x_{i+m,j+n})(x_{i,j} - x_{i+m,j+n}) \quad (4)$$

where n is the weighted average of the differences between the pixel of interest and its neighbours.

μ_3 is the membership function for the transformation of the intensity $x_{i,j}$ in the range (0, L-1) to the fuzzy property plane in the interval (0,1).

$$\mu_3(x_{i+m,j+n}) = \frac{1}{1+e^{\frac{t(|x_{m,n} - x_{m+i,n+j}| - a)}{L-1}}} \quad (5)$$

where $m,n \neq 0$, -k<m,n<k, k = 1,2,3… (depends on window size, e.g. if window size is 3x3, k=1)

Membership function μ_3 is used for assigning the weight of the contribution from a particular neighbouring pixel towards the output. If the μ_3 for a particular neighbour pixel is higher, it contributes more to the output intensity. In this membership function are 2 tunable parameters, t and a and one variable d (see section 2) derived from the image. To filter Gaussian noise, t is set to be positive. d is the absolute intensity difference between a particular neighbour and the central pixel. Parameter a sets the cutoff point for this difference, to result either in membership more or less than 0.5.

If the central pixel $x_{i,j}$ is a Gaussian tail-end noise pixel, then a should be set larger, so that the noisy central pixel can be neutralized by the neighbours' contribution, but not too large to cause detail blurring. Conversely, if the central pixel $x_{i,j}$ is part of a uniform area or Gaussian noise, then a should be small to give high membership to only neighboring pixels similar in intensity to the central.

In the membership function, the value of a determines the selectivity. It should be observed that by varying the value of a $(0 < a \leq L-1)$, different nonlinear behavior could be obtained. For removing Gaussian noise, a bigger value of a will smooth more, but also result in more blurring. Therefore a needs to be chosen carefully.

4 Parameter Tuning

4.1 Parameter *a*

We propose to improve the performance of the filter by tuning the a parameter according to another membership function, which reflects the local image characteristics.

4.1.1 Compatibility

The fuzzy membership function of total compatibility was defined by Choi and Krishnapuram [6] as a means to quantify the local area characteristics in an image.

If a given central pixel is an impulse noise pixel or Gaussian tail-end noise pixel, then the gray level of this pixel will be significantly different from its neighbors. This means the degree that the neighboring pixels is compatible with this noisy central pixel will be small. However, before we can start measuring compatibility, we also need to make sure that the neighboring pixels are not also impulse noise pixels themselves. If they were, then the compatibility measured would not be accurate. So first, we have to establish the reliability of the neighboring pixels.

To gauge the reliability of a pixel, we need to take into account the gray level differences between it and its neighbors.

Reliability can be measured using variable $\beta_{x_{i,j}}$, where

$$\beta_{x_{i,j}} = \frac{1}{N} \sum_{x_{m,n} \in A} (x_{i,j} - x_{m,n})^2 \quad (6)$$

reflects the variance of the intensity differences between the central pixel and its neighboring pixels. If $\beta_{x_{i,j}}$ is small, then it is likely to be reliable. Conversely, if $\beta_{x_{i,j}}$ is large, then it could possibly be a Gaussian tail-end noise pixel.

Let $\mu_{x_{i,j}}$ represents the degree of compatibility of a neighboring pixel $x_{m,n}$ with respect to central pixel $x_{i,j}$. The fuzzy membership function is defined by:

$$\mu_{x_{i,j}} = \exp\left(-\frac{(x_{i,j} - x_{m,n})^2}{\beta_{x_{i,j}}}\right) \quad (7)$$

The variable $\beta_{x_{i,j}}$ is large when the neighbouring pixel is unreliable, making the compatibility correctly low even though the difference in intensity levels may be small.

In order to find out whether a particular central pixel is an Gaussian tail-end noise pixel, we have to consider the compatibilities of *all* the neighboring pixels $x_{m,n}$ with respect to the central pixel $x_{i,j}$. To evaluate this property, we can simply take the mean of $\mu_{x_{i,j}}$, as described by the function: Total compatibility, $\mu_C = \frac{1}{N}\sum_{\substack{m,n=-k \\ m.n\neq 0}}^{k} \mu_{x_{i,j}} \quad (0 \le \mu_C \le 1)$ (8)

If the central pixel $x_{i,j}$ is part of an edge or Gaussian tail-end noise pixel, the compatibilities $\mu_{x_{i,j}}$ will be low, resulting in a low μ_C. Conversely, if the central pixel $x_{i,j}$ is part of a uniform area or Gaussian noise, μ_C would be high.

Therefore it is useful to let parameter *a* vary according to μ_C. When μ_C is small it is desirable to set a higher *a* so that higher differences due to tail-end noise can be corrected. However we do not want to blur the edges, so there will be a upper limit to the value of a.

If μ_C is high, we want to assign parameter *a* a small value so that the Gaussian noise pixel will be replaced by the correction term contributed only by the compatible neighbours. It is analogous to averaging between similar pixels. By experiment, it has been found that the filter usually performs well when *a* is within the range 40-80 for intensity images of 256 levels.

Thus, we set *a* = -40 * μ_C + 80 (9)

4.2 Parameter *t*

Parameter *t* determines the direction of the sigmoid curve. For Gaussian denoising purposes, t must be a positive number. Through experiments, it has been found that values of t above 20 work quite well and can be fixed for the whole image as there was no significant improvement found by varying the value of t from pixel to pixel.

5 Results

In our experiments, we used the 256x256 Lena (Fig. 2(a)) and 512x512 Baboon (Fig. 3(a)) images. The images were digitized into 256 gray levels. The window size used in our proposed filters was of size 7x7.

Both images were corrupted by Gaussian noise with mean = 0 and variance = 0.005 as shown in Fig.s 2(b) and 3(b). The noise matrices were generated using a MATLAB subroutine. In addition to our proposed filters, the noisy images were filtered with the 5x5 Wiener filter and also with an image enhancement technique combining sharpening and noise reduction [8]. The reason for using the 5x5 window in the Wiener filter is, because a 3x3 window tends not to smooth the flat areas enough whereas the 7x7 window tends to over smooth with most details washed out. The results of all the filters are presented in Fig. 3 and 4 below. The result of the application of the 5x5 Wiener filter on Lena is shown in Fig. 3(c). The filter manages to smooth the noise, but the edges are quite soft and the flat areas have a slight mottled appearance.

The result of the combined sharpening and noise reduction filter [8] is shown in Fig. 3(d). It was applied with parameter alpha = 50, as the parameter is user defined and at this value it appears to balance sharpness and reduced noise by visual inspection. This filter yields a sharper result but with increased unevenness in the flat areas.

The results of the proposed filter with parameters fixed [7] is show in Fig. 3(e). Values were fixed at a = 50 and t = 20. These values were chosen by experiment. Most areas are smoothed correctly and edges and details are preserved. However, some noise remains.

Finally, the result of the application of our proposed filter with tuned parameters is shown in Fig. 3(f). The homogenous areas are smooth and the edges are still sharp.

The next set of pictures involves a slice of the 512x512 baboon image. The result of the application of the 5x5 Wiener filter on Lena is shown in Fig. 4(c). The result is smooth but blurred, whereas the flat areas around the nose have a slight mottled appearance.

The result of the combined sharpening and noise reduction filter [8] is shown in Fig. 4(d). This filter yields a sharper result but with increased noise at high contrast areas and unevenness in the flat areas.

The results of the proposed filter with parameters fixed [7] is show in Fig. 3(e). Values were fixed at a = 50 and t = 20. Most areas are smoothed correctly and edges and details are preserved. However, some noise remains.

Finally, the result of the application of our proposed filter with tuned parameters is shown in Fig. 3(f). The homogenous areas are smooth while the edges are still sharp.

The root-mean-square error (RMSE) and Signal-to-noise ratios (SNR) of the processed images with respect to the original uncorrupted images are reported in Table 1. Noisy SNR denotes the SNR of the noisy images with respect to the original uncorrupted images. The results show the effectiveness of the proposed tuned parameter filter in removing Gaussian noise.

Table 1: Comparison of performances of different filters on Lena Image

Filter	RMSE	Noisy SNR	Final SNR
5x5 Wiener	9.8762	0.2162	18.0287
Combined Sharpening and Noise Reduction	14.6990		14.5799
Proposed Filter with fixed values, a=50,t=20	14.0744		15.0056
Proposed Filter with parameter tuning method	10.8552		17.1689

Table 2; Comparison of performances of different filters on 512x512 Baboon Image

Filter	RMSE	Noisy SNR	Final SNR
5x5 Wiener	16.1826	0.2029	12.7554
Combined Sharpening and Noise Reduction	18.5601		11.4336
Proposed Filter with fixed values, a=50,t=20	16.5050		12.6949
Proposed Filter with parameter tuning method	14.4996		13.7575

6 Conclusions

In a previous paper, we described the development of a tunable fuzzy filter for image smoothing [7]. It was a parameter tuned filter based on the sigmoid function, with good performances in Gaussian, impulse as well as a combination of both noise environments. In this paper, an improvement on that filter was made, where the filter automatically selects the optimum parameter values based on image information. The image information used is Total Compatibility which measures the similarity and compatibility among pixels. Experimental results have shown that the proposed filter performs better than some other adaptive techniques in denoising images corrupted with Gaussian noise. Presently, work is on to extend the work for impulse noise and different levels of noise.

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(a) Original

(b) Noisy

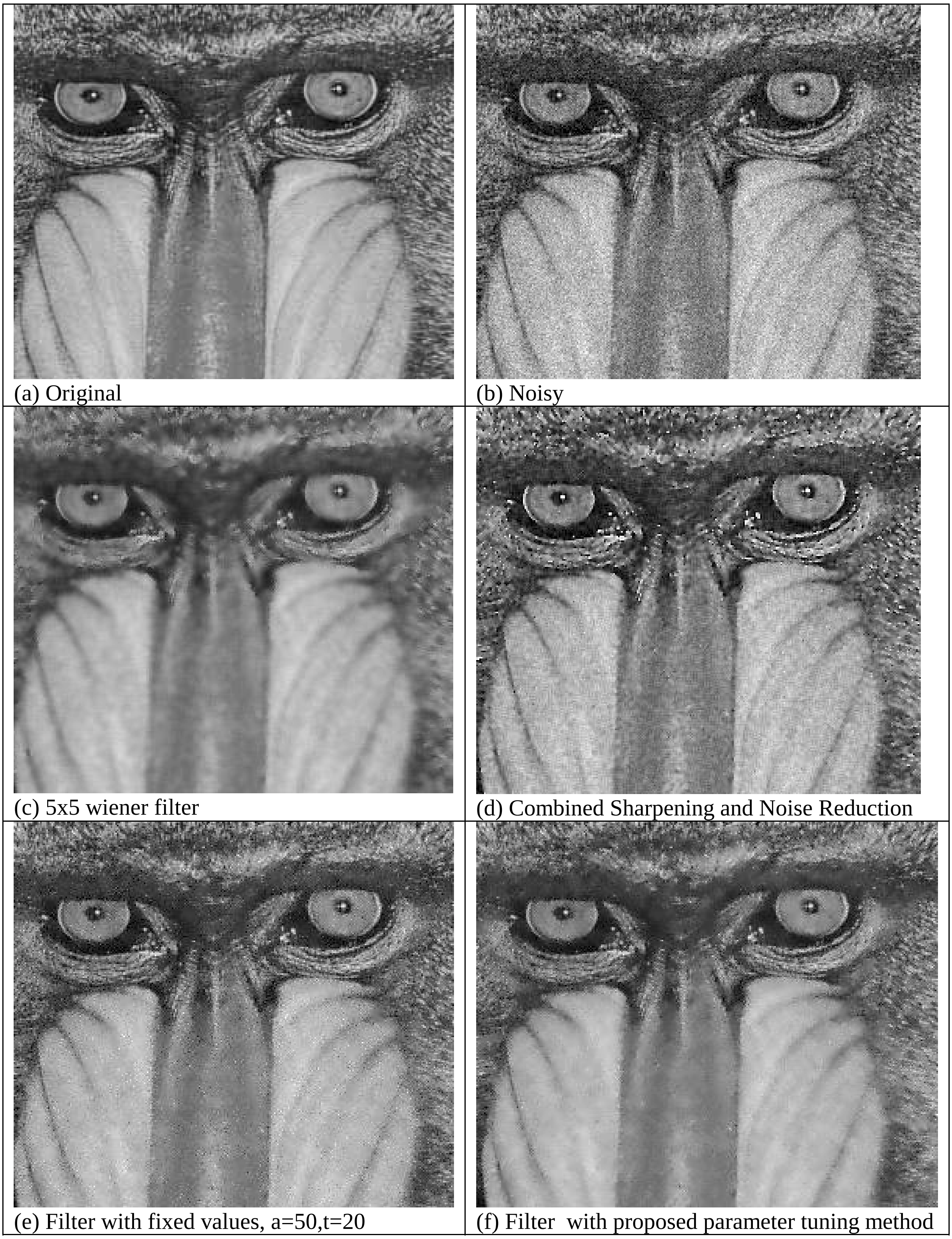
(c) 5x5 Wiener filter

(d) Combined Sharpening and Noise Reduction

(e) Filter with fixed values, a=50,t=20

(f) Filter with proposed parameter tuning method

Fig. 2: Lena: Original, Noisy Images and Results of Filtering



(a) Original

(b) Noisy

(c) 5x5 wiener filter

(d) Combined Sharpening and Noise Reduction

(e) Filter with fixed values, a=50,t=20

(f) Filter with proposed parameter tuning method

Fig. 3: Cropped section of Baboon: Original, Noisy Images and Results of Filtering