New Simple Methods of Tuning Three-Term Controllers for Dead-Time Processes

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Abstract: - New simple methods are presented for tuning three-term controllers for dead-time processes. The methods are based on appropriate manipulations of the exponential dead-time term in the denominator of the closed-loop transfer function for a servo problem, and, subsequently, on appropriately matching the coefficients of corresponding powers of *s* in the numerators of the resulting transfer functions and those in their denominators. This technique gives simple algebraic systems of equations for the controller parameters. The proposed methods are universal and can readily be applied to a wide variety of linear process models with time delay. Simulation results show that our methods provide improved performances as compared to the most recent tuning methods. Moreover, the controller settings obtained by the proposed methods give a robust performance for uncertainty in the process model parameters.

Key-Words: - Pseudo-derivative feedback, on-line controller tuning, process control, dead-time processes.

1 Introduction

Time delays (also known as transport lags, dead times or time lags) may arise in physical, chemical, biological, electronic and economic processes and systems, as well as in the process of measurement and computation [1]. In [2], it has been pointed out that apparent time delays result when actual transportation or measurement delays are present, or when high order processes are approximated by means of lower order transfer functions. Note also that, a time delay may appear to be present due to the effect of the combination of a large number of time lags. Whatever the case is, time delays are present in most realistic processes and systems and must be carefully taken into account when model them.

Process modeling inherently involves a compromise between model accuracy and complexity on one hand, and the cost and effort required to develop the model, on the other hand. In the process control circles, there is much debate over how complex a model may reasonably be. The question as to whether the process is best modelled by a non-linear model with time delay or a linear model with time delay is also important, as it affects the complexity of the control problem. For the purpose of designing controllers, it appears that it is reasonable to suggest that, even if the process has no physical time delay, it may be possible to model such a (possibly high order) process by a linear low order model plus time delay. It appears also reasonable that either a first order plus dead time (FOPDT) model (for stable overdamped processes) or a second order plus dead time (SOPDT) model (for stable overdamped or underdamped processes), or finally an integrator plus dead time model (in case of processes with very large time constants) should be considered [2]-[5]. Either of these approximate process models would appear to be sufficiently accurate for many applications.

Of course, there are exceptions from this typical situation. Indeed, although most processes exhibit open loop stable behavior, several others exhibit multiple steady states due to system nonlinearities. Some of these steady states may be unstable. For several reasons, like safety, maximization of productivity and reduction of economic costs, it is often desirable to operate such processes around their unstable steady states. To approximate the open loop dynamics of such systems, many of these processes can be satisfactorily described by unstable transfer function models, the most popular being the unstable first order plus dead-time (UFOPDT) model and the unstable second order plus dead time (USOPDT) model (with either one or two unstable poles) [6]. On the other hand, whenever a process controlled variable encounters two (or more) competing dynamic effects, with different time constants, from the same manipulated variable, the resulting composite dynamic behaviour of this process can exhibit a troublesome inverse response or a large overshoot response (which corresponds to a positive or negative zero, respectively, in the open-loop transfer function) [7], [8].

Control methods for dead-time processes may be broadly classified into two main categories: The first category consists of methods based on structurally optimized controllers. The second category includes methods based on parameter optimized controllers. In the first class, the controller structure and parameters are adapted optimally to the structure and parameters of the process model. In the second class, the controller parameters are adapted to the controller structure.

The three-term Proportional-Integral-Derivative (PID) controller is the most common type of parameter optimized controller. In the process control, more than 95% of the control loops are of the PID type [5]. The main reason is its relatively simple structure, which can be easily understood and implemented in practice, and its ability to compensate many practical industrial processes. Several variations of the so-called “ideal” or “standard” PID controller have been proposed (for a review see [9]). Moreover, over the years, a variety of tuning methods for designing the parameters of PID controllers have been reported in the literature. The interested reader may refer to [9] for a detailed review of these methods. The available tuning methods may be broadly classified into six main categories: (a) Process reaction curve methods, (b) methods based on minimizing an appropriate performance criterion, (c) direct synthesis tuning methods, (d) ultimate cycle tuning methods, (e) robust tuning methods with an explicit robust stability and robust performance criterion, and (f) tuning methods based on several specific design criteria, like the quarter decay ratio specification, or the magnitude and frequency information at a particular phase lag. In many of these methods, one or two adjustable parameters are used to calculate the PID settings.

In many cases, the design of PID controllers for delayed processes is based on methods that were originally used for the controller design of delay-free processes. Moreover, most of the PID tuning methods reported in the literature are suitable only for stable dead-time processes, although in recent years some specific methods are proposed for tuning three-term controllers for integrating and unstable dead-time processes, as well as for inverse response or large overshoot processes with dead time. Few of these tuning methods are non-model specific, i.e. they are applicable regardless of the model of the process under control. Finally, in the majority of the available tuning methods, the design procedure is somewhat complicated. Therefore, there is a need for simple universal tuning methods with improved performances.

The focus of this paper is the control of dead-time processes using three-term controllers. The particular controller structure considered in the paper for the control of time delayed processes is the Pseudo-Derivative Feedback (PDF) controller structure, originally proposed in [10]. As it is explained in Section 2, the PDF controller is a variation of the standard PID controller. The reason for using PDF control instead of PID is mainly due to its advantages over conventional PID control. The main of them are: (a) The PDF controller can provide the closed-loop transfer function of the system with the absence of numerator dynamics. Therefore, it naturally ramps the controller effort, since it internalizes the set-point filter that one would apply to cancel the undesired zeros, introduced in the PID controller configuration, which produce excessive overshoot. (b) In the PDF controller, the D-action is moved from the direct control branch into the feedback, thus keeping the advantages of the differential action, but without difficulties caused by a differentiator placed in the direct branch (e.g., the “differential peak” in the control variable or the incorrect calculation of the derivative due to physical limitations of the differentiating device, etc.). (c) The PDF controller, with the integral action in the direct branch, is convenient for the reason of obeying the "one master principle" [11], i.e. the principle of one action in the direct branch. This prevents the closed-loop system from several deficiencies [12].

In this work, new simple methods are presented for tuning PDF controllers for dead-time processes. The methods are based on appropriate manipulations of the exponential dead-time term in the denominator of the closed-loop transfer function for a servo problem, and, subsequently, on matching the coefficient of corresponding powers of s in the numerators of the resulting transfer functions and that in their denominators. This technique gives simple algebraic systems of equations, the solution of which provides the PDF controller settings in terms of the parameters of the process model and on two adjustable parameters. The proposed methods are universal and can readily be applied to a wide variety of linear process models with time delay, from FOPDT models to unstable SOPDT models with an unstable zero. Although it is practically impossible to compare the proposed methods with the hundreds of methods reported in the literature [9], simulation results show that our methods provide improved performances as compared to the most recent tuning methods. Moreover, the controller settings obtained by the proposed methods give a robust performance for uncertainty in the process model parameters.

2 Three-Term Controller Structures for Dead-Time Processes

In the literature, several control techniques have been proposed for the control of dead-time systems. The most widely used are alternative schemes of the so-called PID or three-term controller. The continuous time “ideal” or “standard” PID controller for a single-input, single-output process model has the following transfer function.

$$G_C(s) = K_C\left(1 + \frac{1}{\tau_I s} + \tau_D s\right) \quad (1)$$

where K_C is the proportional gain, τ_I is the integral time constant and τ_D is the derivative time constant.

It is, however, uncommon to implement the PID controller structure provided above in practice. A number of alternative modified PID controller structures are used in the process control literature, and of course in industrial practice. The main of them are [1], [9]: (i) The ideal PID controller with first or second order filter, (ii) several types of the ideal PID controller with set-point weighting with or without first order filter, (iii) the so-called “classical” PID controller, (iv) several types of the non-interacting PID controller, (v) the industrial PID controller, (vi) the series PID controller with or without filtered derivative, (vii) the series PID controller with or without lead element, (viii) the PID controller with filtered derivative, (ix) the standard or ISA form PID controller, (x) several types of the two degree of freedom PID controller, and (xi) the I-PD controller (also called the Pseudo-

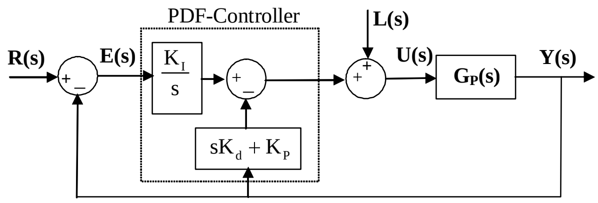

Figure 1. The Pseudo-Derivative Feedback control configuration.

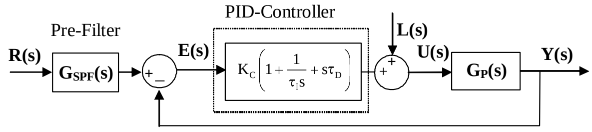

Figure 2. Ideal PID controller with set-point filter equivalent to the PDF control structure.

Derivative Feedback controller). Each particular variation of the PID controller has its own advantages and disadvantages, which will not be reviewed further here, because a comparison of the different PID controller types is beyond the scope of the present paper.

However, since the focus of this paper is on the derivation of new methods of tuning a particular class of three-term controllers, we will next briefly review this control structure, known as the Pseudo-Derivative Feedback (PDF) control. The PDF controller has originally been proposed in [10], and it is illustrated schematically in Figure 1. It is often called the I-PD controller, due to the fact that the three controller actions are separated. Integral action (which is dedicated to steady state error elimination) is located in the forward path of the loop, whereas proportional and derivative actions (which are mainly dedicated in assigning the desired closed-loop performance in terms of stability, responsiveness, disturbance attenuation, etc)) are located in the feedback path. This separation leads to a better understanding of the role of each particular controller action. Moreover, the PDF controller has some distinct advantages over the ideal PID controller of the form (1). The main of them are:

(a) A first advantage stems from the fact that PDF controllers can provide the closed-loop transfer function of the system with the absence of numerator dynamics. So, the speed response characterized by damping ratio and natural frequency can be easily predicted with adjusted control gains. In other analytic view, the PDF controller results in an equivalent transfer function to the PID controller of the form (1), having filtered input command, and the poles of the second-order set-point filter are cancelled out with the zeros introduced by the PID controller. Therefore, the set-point filter is of the form

$$G_{SPF}(s)=\frac{1}{\tau_D\tau_I s^2+\tau_I s+1} \quad (2)$$

The PDF controller naturally ramps the controller effort, since it internalizes the pre-filter that one would apply to cancel the undesired zeros introduced in the PID controller configuration, which produce excessive overshoot. The equivalence of the PDF and the ideal PID controller with set-point filter is shown in Figure 2 and it is based on the following relations between the parameters of the two alternatives control schemes

$$K_P=K_C \quad , \quad K_I=K_C/\tau_I \quad , \quad K_d=K_C\tau_D \quad (3)$$

It is worth noticing at this point that, in the case of regulatory control the two alternative control schemes presented above are identical in terms of performance, provided that relations (3) hold.

(b) Another fundamental advantage of the PDF controller over PID control lies in moving the D-action from the direct control branch into the feedback, thus keeping the advantages of the differential action, but without difficulties caused by a differentiator placed in the direct branch. Namely, it is known that the D-action can be used for increasing the speed of the system's response. The reason for moving it form the direct branch is that it causes the sudden change in the error signal, where the physiccal limitations of the differentiating device cause the incorrect calculation of the derivative, thus, accordingly, the real performance of the system will be smaller than the ideal performance set by the model. Also, the presence of the D-action and to the lesser extent also the P-action, in stepwise variation of the reference (input) signal, can cause the so called "differential peak" in the control variable, which cannot be handled physically by the majority of the executive organs. In the PDF control the D-action is moved into the feedback by the output, and since the output value represents the result of several integrations, it will vary slower than the other signals in the system, and thus the differentiator's response will be more realistic.

(c) Finally, the application of the PDF control is also convenient from the reason of obeying the "one master principle" (according to terminology used in [11]), namely the principle of one action in the direct branch. The deficiency of the PID control laws family is also in that, since the controller is required to simultaneously respond on signals that can be conflicting. If for example the error signal (which is handled by the PID controller to produce the control input) is the sinusoidal signal, then its derivative and integral are moved for 90^o and -90^o with respect to the error signal, respectively, so the controller is forced to simultaneously process three different signals and generate the control input. Result of such an analysis, according to [10], is the conclusion that the most convenient is for the control algorithm not to contain more than one action in the direct branch. The application of this rule actually represents obeying "one master principle". Considering the previously presented analysis of the D-action, it is obvious that its application in the direct branch should be avoided. On the other hand, the P-action of the PDF controller can be placed in the direct branch, only in the case where there is no disturbing action, and where the fast response is not required. In the case of existence of disturbance and only of the P-action in the direct branch, the system would always present a certain error in the stationary state. Another deficiency of P-action application is also in that it is not realistic to consider that the control element instantaneously responds to the stepwise response. Thus, as the most convenient solution will be the placement of the integral action in the direct branch.

Due to its advantages over PID control, we shall next focus our attention to the PDF control structure and propose new simple methods for tuning its settings, in cases where it is applied to control a variety of linear time delayed process models.

3 The proposed tuning methods

Consider the general transfer function model of the form

$$G_P(s)=\frac{Kq(s)}{p(s)}\exp(-ds) \quad (4)$$

of a dead-time process, where $p(s)\in R^n[s]$ and $q(s)\in R^m[s]$ are polynomials of the indeterminate *s*, with $m\leq n$. Let us now apply to system (4) the PDF control structure depicted in Figure 1. It is not difficult to see that the closed-loop transfer function from reference *R(s)* to output *Y(s)* is given by

$$G_{CL}(s)=\frac{KK_I q(s)\exp(-ds)}{sp(s)+Kq(s)\left(K_d s^2+K_P s+K_I\right)\exp(-ds)} \quad (5)$$

We next propose four new methods for tuning the PDF controller settings. These methods are as follows:

3.1 Method I

Re-writing the exponential term in the denominator of (5) in the form

$$\exp(-ds) = \exp(-a_1 ds) / \exp[(1-a_1)ds]$$

for some $a_1 \in R^+$, relation (5) takes on the form

$$G_{CL}(s) = \frac{q_{cl,1}(s)}{p_{cl,1}(s)} \exp(-a_1 ds) \qquad (6)$$

where

$$q_{cl,1}(s) = KK_I q(s)$$

$$p_{cl,1}(s) = sp(s)\exp[(1-a_1)ds] + Kq(s)\left(K_d s^2 + K_P s + K_I\right)\exp(-a_1 ds)$$

Let us now consider the numerator and the denominator terms of equation (6) using the Taylor series expansion for $q_{cl,1}(s)$ and $p_{cl,1}(s)$. The coefficient of the constant term (coefficient of s^0) of the numerator is already equal to that of the denominator because of the presence of the integral action. Since the objective of the controller is to make $Y(s)/R(s)=1$, in order to obtain the controller settings, one should equate the coefficients of like powers of s of the numerator and the denominator of (6). However, since the design specifications for stable systems cannot usually be met by unstable systems or by systems with stable and unstable open-loop zero dynamics, one cannot generally force $Y(s)/R(s)$ as having the value 1. For this reason, in the sequel, we shall equate the coefficient of the corresponding powers of s of the numerator with that of the denominator multiplied by a factor $b_1 \in R^+$, except for the coefficient of s^0.

Thus, on equating the coefficients of s, we get

$$KK_I q^{(1)}(0) = b_1\left\{p(0) + KK_i\left[q^{(1)}(0) - a_1 dq(0)\right] + KK_P q(0)\right\}$$

or, equivalently

$$\mathbf{n}_1^T \begin{bmatrix} K_d \\ K_p \\ K_I \end{bmatrix} = b_1 p(0) \qquad (7a)$$

where

$$\mathbf{n}_1 = \begin{bmatrix} 0 \\ -b_1 Kq(0) \\ K\left[(1-b_1)q^{(1)}(0) + b_1 a_1 dq(0)\right] \end{bmatrix} \qquad (7b)$$

Similarly, on equating the coefficients of s^2, after some straightforward manipulations, we get

$$\mathbf{n}_2^T \begin{bmatrix} K_d \\ K_p \\ K_I \end{bmatrix} = 2b_1\left[p^{(1)}(0) + (1-a_1)dp(0)\right] \qquad (8a)$$

$$\mathbf{n}_2 = \begin{bmatrix} -2b_1 Kq(0) \\ 2Kb_1\left[-q^{(1)}(0) + a_1 dq(0)\right] \\ n_{2,3} \end{bmatrix} \qquad (8b)$$

$$n_{2,3} = K\left\{(1-b_1)q^{(2)}(0) + b_1\left[2a_1 dq^{(1)}(0) - a_1^2 d^2 q(0)\right]\right\} \qquad (8c)$$

Finally, on equating the coefficients of s^3, we get

$$\mathbf{n}_3^T \begin{bmatrix} K_d \\ K_p \\ K_I \end{bmatrix} = 3b_1\left[p^{(2)}(0) + 2(1-a_1)dp^{(1)}(0) + (1-a_1)^2 d^2 p(0)\right] \qquad (9a)$$

where

$$\mathbf{n}_3 = \begin{bmatrix} 6b_1 K\left[-q^{(1)}(0) + a_1 dq(0)\right] \\ n_{3,2} \\ n_{3,3} \end{bmatrix} \qquad (9b)$$

$$n_{3,2} = 3b_1 K\left[-q^{(2)}(0) + 2a_1 dq^{(1)}(0) - a_1^2 d^2 q(0)\right] \qquad (9c)$$

$$n_{3,3} = K\left\{(1-b_1)q^{(3)}(0) + b_1\left[3a_1 dq^{(2)}(0) - 3a_1^2 d^2 q^{(1)}(0) + a_1^3 d^3 q(0)\right]\right\} \qquad (9d)$$

Relations (7a), (8a) and (9a) constitute a set of linear algebraic equations with respect to K_d, K_p and K_I,, having the form

$$\mathbf{Nk} = \mathbf{z} \qquad (10)$$

where

$$\mathbf{N} = \begin{bmatrix} \mathbf{n}_1^T \\ \mathbf{n}_2^T \\ \mathbf{n}_3^T \end{bmatrix}, \quad \mathbf{k} = \begin{bmatrix} K_d \\ K_p \\ K_I \end{bmatrix}$$

$$\mathbf{z} = \begin{bmatrix} b_1 p(0) \\ 2b_1\left[p^{(1)}(0) + (1-a_1)dp(0)\right] \\ 3b_1\left[p^{(2)}(0) + 2(1-a_1)dp^{(1)}(0) + (1-a_1)^2 d^2 p(0)\right] \end{bmatrix}$$

The solution of (10) provides the PDF controller settings sought. It is worth noticing at this point that, in the case where the process model has no zeros, then $q^{(i)}(0)=0$, for $i=1,2,3$. Then, as it can be easily checked by relations (7)-(9), parameter b_1 can be cancelled-out in both sides of (10), and does not play any role. In this case, only the adjustable parameter a_1 is used in the method. However, as it will be shown later on in section 4, the adjustable parameter b_1 plays an important role in cases where the process model contains at least one zero. Since, in this subsection, we present a general form of Method I applicable to systems with or without zeros, we next use a typical value $b_1=1$ for this adjustable parameter in the case of systems without zeros, although any other value of it is also acceptable.

3.2 Improved Method I

In this method, we still equate the coefficient of power of s in the numerator of (6) to b_1 times that of the denominator whereas the coefficients of s^2 and s^3 is set to c ($=\beta b_1$) times that of the denominator. Now, there are three tuning parameters: a_1, b_1 and β.

However, to make the tuning procedure simple we can keep β to a standard value (say $\beta=10$). Hence, there are once again only two tuning parameters a_1 and b_1.

Similar to the steps given in the previous sub-section for Method I, we get the following linear algebraic equations for the PDF settings as:

$$\overline{\mathbf{N}}\mathbf{k} = \overline{\mathbf{z}}$$

where

$$\overline{\mathbf{N}} = \begin{bmatrix} \mathbf{n}_1^T \\ \overline{\mathbf{n}}_2^T \\ \overline{\mathbf{n}}_3^T \end{bmatrix}, \quad \mathbf{k} = \begin{bmatrix} K_d \\ K_p \\ K_I \end{bmatrix}$$

$$\overline{\mathbf{n}}_2 = \begin{bmatrix} -2cKq(0) \\ 2Kc\left[-q^{(1)}(0) + a_1 dq(0)\right] \\ \overline{n}_{2,3} \end{bmatrix}$$

$$\overline{n}_{2,3} = K\left\{(1-c)q^{(2)}(0) + c\left[2a_1 dq^{(1)}(0) - a_1^2 d^2 q(0)\right]\right\}$$

$$\overline{\mathbf{n}}_3 = \begin{bmatrix} 6cK\left[-q^{(1)}(0) + a_1 dq(0)\right] \\ \overline{n}_{3,2} \\ \overline{n}_{3,3} \end{bmatrix}$$

$$\overline{n}_{3,2} = 3cK\left[-q^{(2)}(0) + 2a_1 dq^{(1)}(0) - a_1^2 d^2 q(0)\right]$$

$$n_{3,3} = K\left\{(1-c)q^{(3)}(0) + c\left[3a_1 dq^{(2)}(0) - 3a_1^2 d^2 q^{(1)}(0) + a_1^3 d^3 q(0)\right]\right\}$$

$$\overline{\mathbf{z}} = \begin{bmatrix} b_1 p(0) \\ 2c\left[p^{(1)}(0) + (1-a_1)dp(0)\right] \\ 3c\left[p^{(2)}(0) + 2(1-a_1)dp^{(1)}(0) + (1-a_1)^2 d^2 p(0)\right] \end{bmatrix}$$

Note that, in the case of systems without zeros, the improved Method I gives exactly the same controller settings as those obtained by Method I. This is due to the fact that in this case, both b_1 and c can be cancelled out in both sides of the controller design equation, and hence, once again, it remains only one adjustable parameter: a_1. However, as it will be verified in the next section, in the case of systems with zeros, the proposed improved method gives an enhanced performance as compared to Method I, since then, both b_1 and c are present and play a very important role.

3.3 Method II

Re-writing the exponential term in the denominator of (5) in the form

$$\exp(-ds) = \exp(-a_2 ds) / \exp\left[(1-a_2)ds\right]$$

for some $a_2 \in R^+$, relation (5) takes on the form

$$G_{CL}(s) = \frac{q_{cl,2}(s)}{p_{cl,2}(s)} \exp(-ds) \quad (11)$$

where

$$q_{cl,2}(s) = KK_I q(s) \exp\left[(1-a_2)ds\right]$$

$$p_{cl,2}(s) = sp(s)\exp\left[(1-a_2)ds\right] + Kq(s)\left(K_d s^2 + K_P s + K_I\right)\exp(-a_2 ds)$$

Using an argument analogous to that of Method I, we next equate the coefficients of the correspond-ding powers of s of the numerator of (11) with that of its denominator multiplied by a factor $b_2 \in R^+$ (except for the coefficient of s^0).

Thus, on equating the coefficients of s, we get

$$\mathbf{m}_1^T \begin{bmatrix} K_d \\ K_p \\ K_I \end{bmatrix} = b_2 p(0) \quad (12a)$$

with

$$\mathbf{m}_1 = \begin{bmatrix} 0 \\ -b_2 Kq(0) \\ K\left\{(1-b_2)q^{(1)}(0) + \left[d + (b_2-1)a_2 d\right]q(0)\right\} \end{bmatrix} \quad (12b)$$

Similarly, on equating the coefficients of s^2, after some straightforward manipulations, we get

$$\mathbf{m}_2^T \begin{bmatrix} K_d \\ K_p \\ K_I \end{bmatrix} = 2b_2\left[p^{(1)}(0) + (1-a_2)dp(0)\right] \quad (13a)$$

where

$$\mathbf{m}_2 = \begin{bmatrix} -2b_2 Kq(0) \\ 2b_2 K\left[-q^{(1)}(0) + a_2 dq(0)\right] \\ m_{2,3} \end{bmatrix} \quad (13b)$$

$$m_{2,3} = K\left\{(1-b_2)q^{(2)}(0) + 2\left[(1-a_2) + 2b_2 a_2\right]dq^{(1)}(0) + \left[(1-a_2)^2 - b_2 a_2^2\right]d^2 q(0)\right\} \quad (13c)$$

Finally, on equating the coefficients of s^3, we get

$$\mathbf{m}_3^T \begin{bmatrix} K_d \\ K_p \\ K_I \end{bmatrix} = 3b_2\left[p^{(2)}(0) + 2(1-a_2)dp^{(1)}(0) + (1-a_2)^2 d^2 p(0)\right] \quad (14a)$$

where

$$\mathbf{m}_3 = \begin{bmatrix} 6b_2 K\left[-q^{(1)}(0) + a_2 dq(0)\right] \\ m_{3,2} \\ m_{3,3} \end{bmatrix} \quad (14b)$$

$$m_{3,2} = 3b_2 K\left[-q^{(2)}(0) + 2a_2 dq^{(1)}(0) - a_2^2 d^2 q(0)\right] \quad (14c)$$

$$m_{3,3} = K\left\{(1-b_2)q^{(3)}(0) + \left[3(1-a_2) + 3b_2 a_2\right]dq^{(2)}(0) + \left[3(1-a_2)^2 - 3b_2 a_2^2\right]d^2 q^{(1)}(0) + \left[(1-a_2)^3 + b_2 a_2^3\right]d^3 q(0)\right\} \quad (14d)$$

Relations (12a), (13a) and (14a) constitute a set of linear algebraic equations with respect to K_d, K_p

and K_I,, having the form

$$\mathbf{M}\mathbf{k} = \mathbf{h} \tag{15}$$

where

$$\mathbf{M} = \begin{bmatrix} \mathbf{m}_1^T \\ \mathbf{m}_2^T \\ \mathbf{m}_3^T \end{bmatrix} \quad , \quad \mathbf{k} = \begin{bmatrix} K_d \\ K_p \\ K_I \end{bmatrix}$$

$$\mathbf{h} = \begin{bmatrix} b_2 p(0) \\ 2b_2\left[p^{(1)}(0) + (1-a_2)dp(0)\right] \\ 3b_2\left[p^{(2)}(0) + 2(1-a_2)dp^{(1)}(0) + (1-a_2)^2 d^2 p(0)\right] \end{bmatrix}$$

The solution of (15) provides the PDF controller settings sought. Note that, as it can be easily checked by a simple inspection of relations (12)-(14), in the case of Method II, the adjustable parameter b_2 is involved in the controller design equation (15), even in cases where the process model has no zeros.

3.4 Improved Method II

In this method, we still equate the coefficient of power of s in the numerator of (11) to b_2 times that of the denominator whereas the coefficients of s^2 and s^3 is set to b_3 ($=\gamma b_2$) times that of the denominator. Now, there are three tuning parameters: a_2, b_2 and γ. However, to make the tuning procedure simple we can keep γ to a standard value (say $\gamma=5.55$). Hence, there are once again only two tuning parameters a_2 and b_2.

Similar to the steps given in the previous sub-section for Method II, we get the following linear algebraic equations for the PDF settings as:

$$\overline{\mathbf{M}}\mathbf{k} = \overline{\mathbf{h}}$$

where

$$\overline{\mathbf{M}} = \begin{bmatrix} \mathbf{m}_1^T \\ \overline{\mathbf{m}}_2^T \\ \overline{\mathbf{m}}_3^T \end{bmatrix} \quad , \quad \mathbf{k} = \begin{bmatrix} K_d \\ K_p \\ K_I \end{bmatrix}$$

with

$$\overline{\mathbf{m}}_2 = \begin{bmatrix} -2b_3 K q(0) \\ 2b_3 K\left[-q^{(1)}(0) + a_2 d q(0)\right] \\ \overline{m}_{2,3} \end{bmatrix}$$

$$\overline{m}_{2,3} = K\left\{(1-b_3)q^{(2)}(0) + 2\left[(1-a_2) + 2b_3 a_2\right]dq^{(1)}(0) + \left[(1-a_2)^2 - b_3 a_2^2\right]d^2 q(0)\right\}$$

$$\overline{\mathbf{m}}_3 = \begin{bmatrix} 6b_3 K\left[-q^{(1)}(0) + a_2 d q(0)\right] \\ \overline{m}_{3,2} \\ \overline{m}_{3,3} \end{bmatrix}$$

$$\overline{m}_{3,2} = 3b_3 K\left[-q^{(2)}(0) + 2a_2 d q^{(1)}(0) - a_2^2 d^2 q(0)\right]$$

$$\overline{m}_{3,3} = K\left\{(1-b_3)q^{(3)}(0) + \left[3(1-a_2) + 3b_3 a_2\right]dq^{(2)}(0) + \left[3(1-a_2)^2 - 3b_3 a_2^2\right]d^2 q^{(1)}(0) + \left[(1-a_2)^3 + b_3 a_2^3\right]d^3 q(0)\right\}$$

$$\overline{\mathbf{h}} = \begin{bmatrix} b_2 p(0) \\ 2b_3\left[p^{(1)}(0) + (1-a_2)dp(0)\right] \\ 3b_3\left[p^{(2)}(0) + 2(1-a_2)dp^{(1)}(0) + (1-a_2)^2 d^2 p(0)\right] \end{bmatrix}$$

As it will be verified in the next section, the proposed method gives an improved performance as compared to Method II.

4 Simulation Results

We next apply the tuning methods presented in the previous section to a variety of dead-time processes, in order to illustrate its wide applicability in facing diverse process characteristics.

4.1 Example 1

Let us first consider a stable first order plus dead time (FOPDT) model of the form

$$G_P(s) = K\exp(-ds)/(Ts+1)$$

with $K=1$, $T=1$ and $d=0.5$, which is a typical example for testing tuning methods for three-term controllers [13]. The proposed Method I is applied here by selecting $a_1=2.2$ and $b_1=1$. With this selection, the PDF controller settings obtained by Method I are $K_P=2.0992$, $K_I=2.8174$, $K_d=0.2045$. The PDF controller settings obtained when applying Method II with $a_2=2.2$ and $b_2=15$ are $K_P=2.1785$, $K_I=2.9986$, $K_d=0.2182$. The settings of a PID controller tuned according to the method reported in [13] are $K_C=2.5$, $\tau_i=1.25$ and $\tau_D=0.2167$. Note that the method proposed in [13] has favorably been compared to other well known tuning methods for FOPDT models (e.g. the open-loop Ziegler-Nichols and the IMC tuning methods). Figure 3 shows the comparison of the servo-response of the proposed methods with that of the method reported in [13], as well as the performance comparison of the above

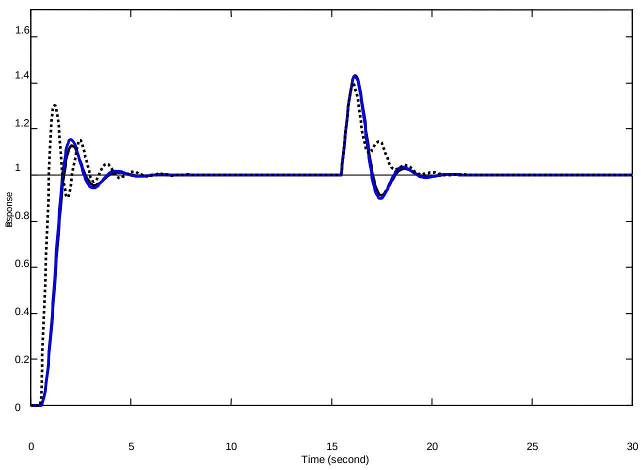

Figure 3. Comparison for stable FOPDT systems. Solid blue line: Method I, solid black line: Method II, dotted line: Method of [13].

Table 1. ISE, IAE and ITAE values for Example 1.

Criterion	Method I	Method II	Method of [13]
ISE-servo	1.0123	0.9987	0.6816
IAE-servo	1.2908	1.2962	0.9993
ITAE-servo	1.0625	1.1124	0.8933
ISE-load	0.1364	0.1324	0.1104
IAE-load	0.4888	0.4915	0.5002
ITAE-load	0.7677	0.7972	0.8799

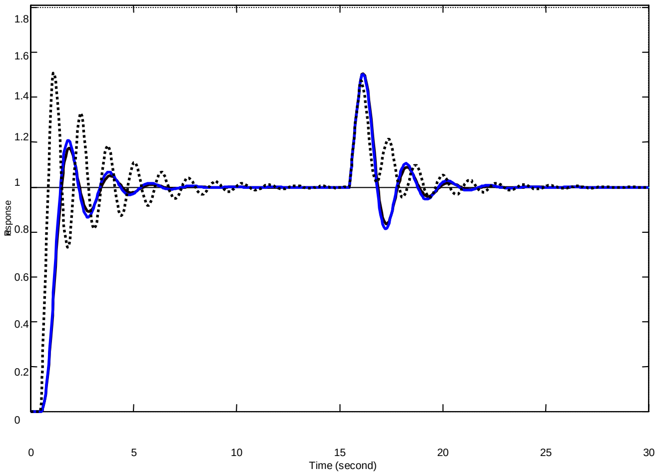

Figure 4. Closed-loop responses under parametric uncertainty in *K*. Other legend as in Fig. 1.

methods for the regulatory control problem. The servo performance obtained by our methods is better than that obtained by the method in [13], in terms of overshoot and settling time, while all three methods are similar in terms of regulatory control performance. Moreover, Table 1 gives the ISE, IAE and ITAE values for all three methods. Obviously, the ISE, IAE and ITAE values for the method in [13] are smaller than that obtained by the present methods in the case of the servo-problem. The same holds for the ISE value in the case of the regulatory problem, whereas, in the regulatory problem case, the values of the IAE and ITAE obtained by our methods are smaller than those obtained by the method in [13]. Finally, the robustness of the proposed methods is evaluated by allowing a 20% uncertainty in the process gain. Figure 4 shows the responses in this case. Similar responses are obtained in the case of a 20% uncertainty in *T* or *d*. The robust performances obtained by the proposed methods are better than that obtained by the method in [13]. In particular, it is easy for the interested reader to check that our methods can tolerate more than 30% decreasing uncertainty in *T*, whereas the method in [13] does not.

4.2 Example 2

Let us now consider a stable second order plus dead time (SOPDT) model of the form

$$G_P(s) = K\exp(-ds)/(T_1 s+1)(T_2 s+1)$$

with $K=1$, $d=0.5$, $T_1=2$, $T_2=1$. The proposed Method I is applied here by selecting $a_1=4.5$ and $b_1=1$. With this selection, the PDF controller settings obtained by Method I are $K_P=4.4815$, $K_I=2.4362$, $K_d=0.6667$. The PDF controller settings obtained when applying Method II with $a_2=4.5$ and $b_2=15$ are $K_P=4.4561$, $K_I=2.6082$, $K_d=0.6836$. The settings of a PID controller tuned according to the method reported in [14] are $K_C=3.125$, $\tau_i=3.125$ and $\tau_D=1.125$, with the tuning parameter in [14] having the value $\lambda=0.5$, in order to obtain a desired closed-loop response of the form $exp(-ds)/(\lambda s+1)$. Figure 5 shows the comparison of the servo-response of the proposed methods with that of the method reported in [14], as well as the performance comparison of the above methods for the regulatory control problem. The performance obtained by our methods, in the case of the servo problem, is better than that obtained by the method in [14], in terms of overshoot and settling time. Our methods provide a considerably better regulatory control performance. Table 2 gives the ISE, IAE and ITAE values for all three methods. Obviously, the ISE, IAE and ITAE values for our methods are smaller than those obtained by the method in [14], for both the servo-problem and the regulatory problem, except for the case of the ISE-servo criterion. Note that in [14], an alternative tuning method is reported, which provides a closed-loop performance of the form $exp(-ds)/(\lambda s+1)^2$. The PID setting obtained by this alternative method are $K_C=1.9444$, $\tau_i=2.9167$ and $\tau_D=0.6024$, with the tuning parameter $\lambda=0.5$. Figure

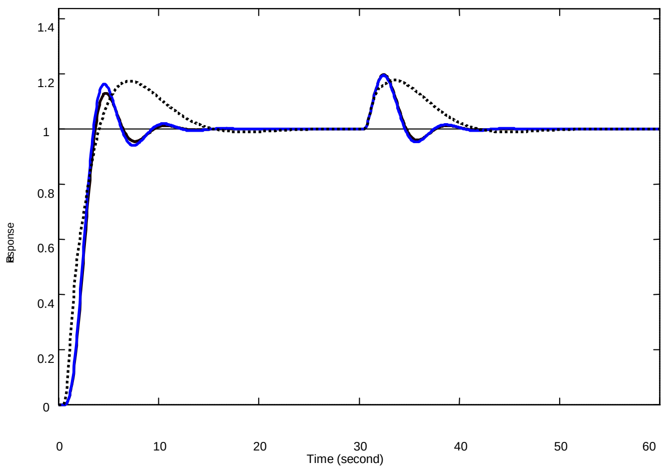

Figure 5. Comparison for stable SOPDT systems. Solid blue line: Method I, solid black line: Method II, dotted line: Method of [14].

Table 2. ISE, IAE and ITAE values for Example 2.

Criterion	Method I	Method II	Method of [14]
ISE-servo	2.0294	1.9952	1.6000
IAE-servo	2.7480	2.7779	3.1140
ITAE-servo	5.3101	5.7523	12.5240
ISE-load	0.0720	0.0698	0.1410
IAE-load	0.5692	0.5754	1.1264
ITAE-load	1.9762	2.0892	6.1810

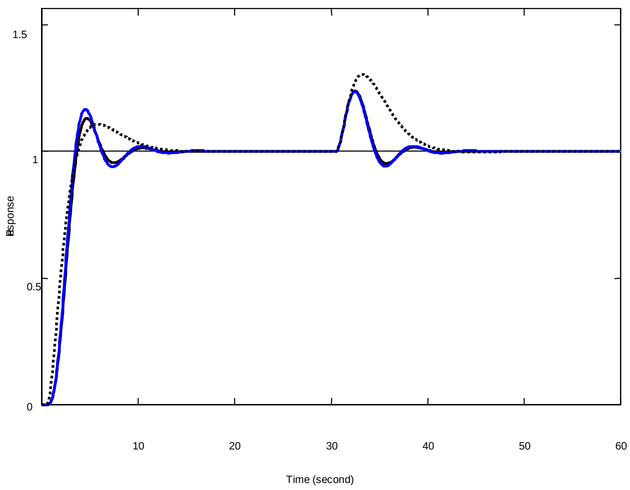

Figure 6. Comparison for stable SOPDT systems. Solid blue line: Method I, solid black line: Method II, dotted line: Alternative method of [14].

6 gives the comparison of the obtained responses. Obviously, our methods are favorably compared to the method given in [14], especially in the case of the regulatory problem.

4.3 Example 3

Let us next consider an integrating plus dead time (IPDT) model of the form

$$G_P(s) = K_P \exp(-ds)/s$$

with $K=1$, $d=1$ (a typical example for testing tuning methods for such process models). The proposed Method I is applied here by selecting $a_1=2.3$ and $b_1=1$. With this selection, the PDF controller settings obtained by Method I are $K_P=1.1342$, $K_I=0.4931$, $K_d=0.3043$. The PDF controller settings obtained when applying Method II with $a_2=2.3$ and $b_2=15$ are $K_P=1.1626$, $K_I=0.5253$, $K_d=0.3142$. The settings of a PID controller tuned according to the method reported in [15] are $K_C=1.2346$, $\tau_i=4.5$ and $\tau_D=0.45$. Figure 7 shows the comparison of the servo-response of the proposed methods with that of the method reported in [15], as well as the performance comparison of the above methods for the regulatory control problem. The servo performance obtained by our methods is better than that obtained

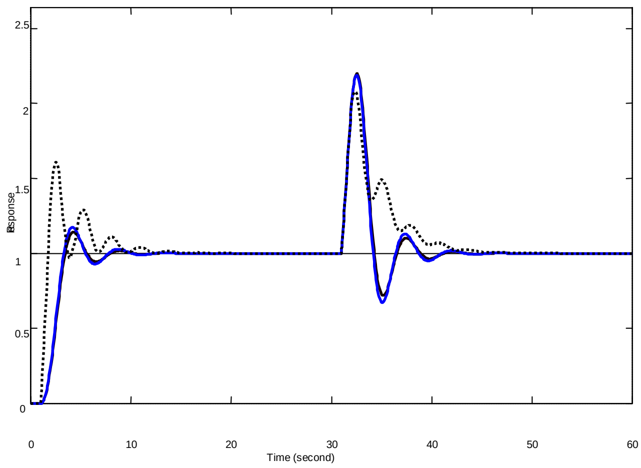

Figure 7. Comparison for IPDT systems. Solid blue line: Method I, solid black line: Method II, dotted line: Method of [15].

Table 3. ISE, IAE and ITAE values for Example 3.

Criterion	Method I	Method II	Method of [15]
ISE-servo	2.1287	2.1038	2.7861
IAE-servo	2.7456	2.7774	2.7800
ITAE-servo	4.9144	5.2822	7.7016
ISE-load	2.2169	2.1702	2.0869
IAE-load	2.9361	2.9938	3.6449
ITAE-load	10.0737	10.7485	16.4382

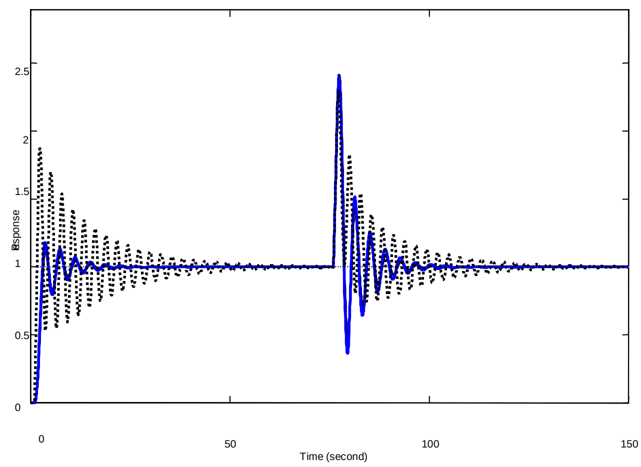

Figure 8. Closed-loop responses for IPDT models under uncertainty in *K*. Other legend as in Fig. 6.

by the method in [15], in terms of overshoot and settling time, while all three methods are similar in terms of regulatory control performance. Table 3 gives the ISE, IAE and ITAE values obtained by the three methods in comparison, for both the servo and the regulatory control problem. The IAE and ITAE values obtained by our methods are smaller than those obtained by the method in [15], which is better in the case of ISE. Finally, the robustness of the proposed methods is evaluated by allowing a 25% uncertainty in the process gain. Figure 8 shows the responses in this case. The robust performances obtained by the proposed methods are better than that obtained by the method in [15]. However, for a 25% uncertainty in *d* the proposed methods seem to be less robust than the method reported in [15].

4.4 Example 4

Let us next consider a double integrating plus dead time (I^2PDT) model of the form

$$G_P(s) = K_P \exp(-ds)/s^2$$

Tuning methods of PID controllers for such processes are very limited. In [16], a method is proposed in order to tune a controller of the form

$$G_C(s) = K_c(1 + 1/\tau_i s)E(s) + K_c(b-1)R(s) - K_c \tau_d sY(s)$$

but the controller and the design method is quite complicated. Several methods in order to tune an ideal PID controller for I^2PDT processes have been presented only in [17]. We next consider the example treated in [17] in order to evaluate the proposed methods. In this example, $K=2.3574$ and $d=0.5017$. The proposed Method I is applied here by selecting $a_1=4.5$ and $b_1=1$. With this selection, the PDF controller settings obtained by Method I are $K_P=0.4993$, $K_I=0.2212$, $K_d=0.5637$. The PDF controller settings obtained when applying Method II with $a_2=4.5$ and $b_2=15$ are $K_P=0.5069$, $K_I=0.2368$, $K_d=0.5652$. The settings of a PID controller tuned according to the method reported in [17] are $K_C= 0.5588$, $\tau_i=10$ and $\tau_D=1.6456$. Figure 9 shows the comparison of the servo-response of the proposed methods with that of the method reported in [17], as well as the performance comparison of the above methods for the regulatory control problem. All

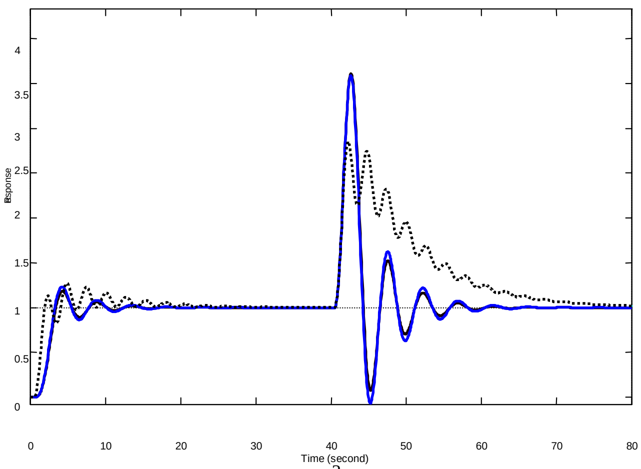

Figure 9. Comparison for I^2PDT systems. Solid blue line: Method I, solid black line: Method II, dotted line: Method of [17].

Table 4. ISE, IAE and ITAE values for Example 4.

Criterion	Method I	Method II	Method of [17]
ISE-servo	2.1174	2.1104	1.3633
IAE-servo	3.0116	3.1602	3.0006
ITAE-servo	7.5910	9.1882	19.4424
ISE-load	11.8591	12.0646	17.5451
IAE-load	8.3752	8.9754	17.8955
ITAE-load	39.1491	46.0640	179.1174

three methods are similar in terms of overshoot, while our methods are better in terms of settling time. However, in case of regulatory control a smaller error is obtained when the method in [17] is applied. Table 4 gives the ISE, IAE and ITAE values obtained by the three methods in comparison, for both the servo and the regulatory control problem. In general, considerably smaller values of the above criteria are obtained when applying the proposed methods. Finally, note that, as it can be easily checked by the interested reader, our methods can tolerate a 20% uncertainty in the process gain or in the process delay, whereas the method in [17] does not, giving an unstable response in both cases.

4.5 Example 5

Let us next consider a first order lag plus integral plus dead time (FOLIPDT) model of the form

$$G_P(s) = K\exp(-ds)/[s(Ts+1)]$$

with $K=1$, $T=1$, $d=2$. The proposed Method I is applied here by selecting $a_1=4$ and $b_1=1$. With this selection, the PDF controller settings obtained by Method I are $K_P=0.2813$, $K_I=0.0352$, $K_d=0.1250$. The PDF controller settings obtained when applying Method II with $a_2=4$ and $b_2=15$ are $K_P=0.2860$, $K_I=0.0376$, $K_d=0.1288$. The settings of a PID controller with derivative filtering of the form

$$G_c(s) = K_c(1 + (1/\tau_i)s + \tau_D s)(1/(\tau_f s + 1))$$

tuned according to the method reported in [18] are K_C=0.2449, τ_i=12, τ_D=0.9167, τ_f=0.551. Figure 10 shows the comparison of the servo-responses of the proposed methods with that of the method reported in [18], as well as the performance comparison of the above methods for the regulatory control problem. The servo performance obtained by our methods is better than that obtained by the method in [18], in terms of overshoot. Our methods also give a smaller maximum error in case of regulatory control. Table 5 gives the ISE, IAE and ITAE values obtained by the three methods in comparison, for both the servo and the regulatory control problem. The values obtained by our methods are

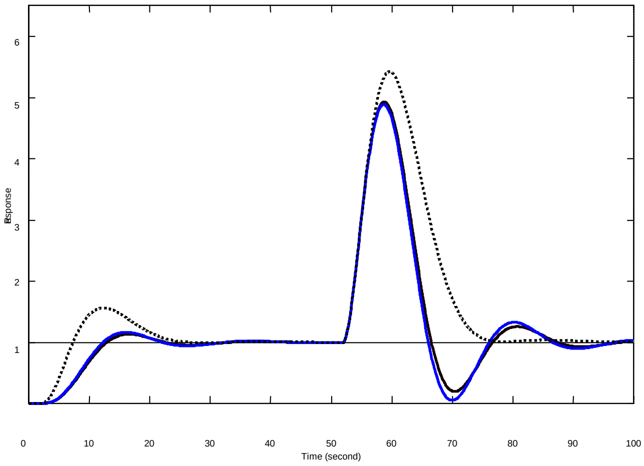

Figure 10. Comparison for FOLIPDT systems. Solid blue line: Method I, solid black line: Method II, dotted line: Method of [18].

Table 5. ISE, IAE and ITAE values for Example 5.

Criterion	Method I	Method II	Method of [18]
ISE-servo	7.2377	7.1188	6.7191
IAE-servo	9.7388	9.8378	10.5663
ITAE-servo	66.0693	71.3031	90.6408
ISE-load	99.7272	96.6300	156.7971
IAE-load	39.5177	39.9090	48.9996
ITAE-load	484.7655	511.2033	561.0848

smaller than those obtained by the method reported in [18] (except for the ISE-servo case). Finally, all methods are similar in terms of robustness. As it can be easily checked by the interested reader, all three methods can readily tolerate a simultaneous 20% uncertainty in all process parameters.

4.6 Example 6

Consider next an unstable first order plus dead time (UFOPDT) model of the form

$$G_P(s) = K\exp(-ds)/(Ts-1)$$

with $K=1$, $T=1$ and $d=0.5$. This is a typical example for testing tuning methods for such process models. Method I is applied here by selecting $a_1=3$ and $b_1=1$. With this selection, the PDF controller settings obtained by Method I are $K_P=2$, $K_I=0.6667$, $K_d=0.25$. The PDF controller settings obtained when applying Method II with $a_2=3$ and $b_2=15$ are $K_P=2.0211$, $K_I=0.7124$, $K_d=0.254$. The settings of a PID controller tuned according to the method reported in [13] are $K_C=2.19345$, $\tau_i=2.7792$ and $\tau_D=0.2561$. Figure 11 shows the comparison of the servo-response of the proposed methods with that of the method reported in [13], as well as the performance comparison of the above methods for the regulatory control problem. Obviously, the servo-response obtained by our method is considerably better. The regulatory control performance obtained by our method is similar to that obtained by the method in [13], in terms of settling time. However, our method produces a more oscillatory response with greater error. To settle this concern, we next apply Method I with $a_1=2.5$ and $b_1=1$. With this selection, the PDF controller settings obtained by Method I are $K_P=2.44$, $K_I=1.152$, $K_d=0.4$. The PDF controller settings obtained when applying Method II with $a_2=2.5$ and $b_2=15$ are $K_P=2.4744$, $K_I=1.2287$, $K_d=0.4061$. Figure 12 shows the obtained responses. The error in the regulatory control case is now almost equal to that obtained by the enhanced method in [13], although the response is still slightly more oscillatory.

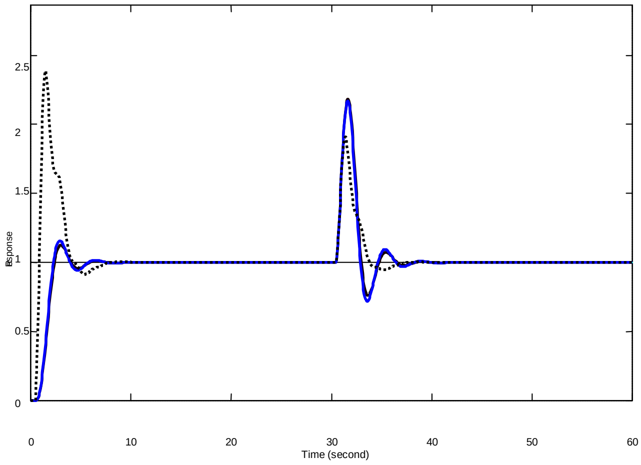

Figure 11. Comparison for UFOPDT systems for $a_1=a_2=3$. Solid blue line: Method I, solid black line: Method II, dotted line: Method of [13].

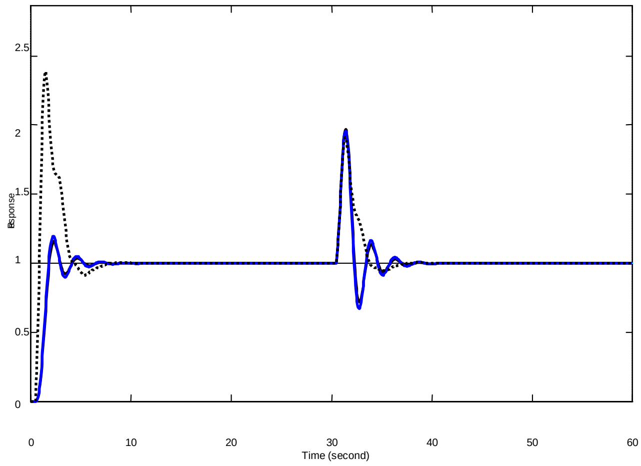

Figure 12. Comparison for UFOPDT systems for $a_1=a_2=2.5$. Solid blue line: Method I, solid black line: Method II, dotted line: Method of [13].

A considerably better response can be obtained using the improved Method II. Setting γ=5.55 and applying the improved Method II, with $a_2=4.518$

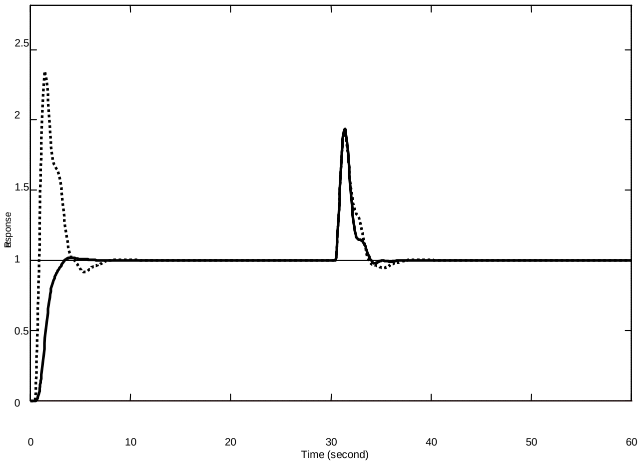

Figure 13. Comparison for UFOPDT systems. Solid black line: Improved Method II, dotted line: Method of [13].

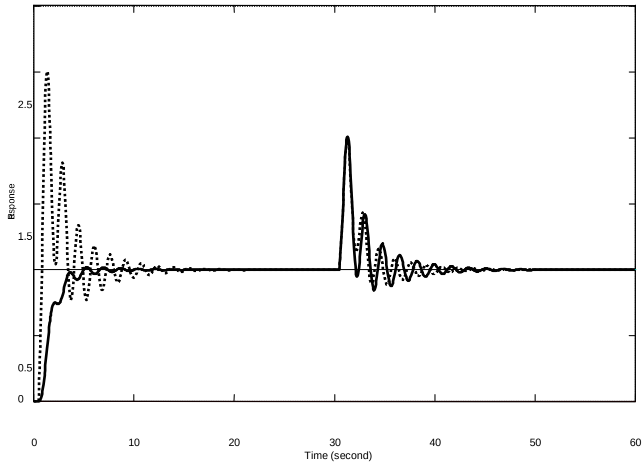

Figure 14. Closed-loop responses for UFOPDT models under uncertainty in *K*. Other legend as in Fig. 12.

Table 6. ISE, IAE and ITAE values for Example 6.

Criterion	Improved Method II	Method of [13]
ISE-servo	1.3116	205.0302
IAE-servo	1.6788	201.2670
ITAE-servo	1.6908	2.0003
ISE-load	0.7544	0.8121
IAE-load	1.2652	1.4797
ITAE-load	2.2275	3.0759

and $b_2=2.747$, we obtain $b_3=15.2458$, and the controller settings $K_P=2.3276$, $K_I=0.8202$, $K_d=0.4896$. Figure 13 shows the obtained responses. Obviously, the servo-performance obtained by our improved Method II is the best and produce no overshoot. The regulatory control performance is better than that obtained by the method in [13]. Table 6 summarizes the ISE, IAE and ITAE values obtained by the two methods in comparison. The robustness of the proposed methods is evaluated by allowing a 20% uncertainty in the process gain. Figure 14 shows the responses in this case. The methods in comparison have similar robust performances. Similar results can be obtained for a 20% uncertainty in *T* or *d*. Finally, it is worth noticing that the method presented in [13] cannot work for values of $d/T>1.5$. In contrast, both Method I and II (and of course the improved Method II) is applicable for values of d/T up to *2*. The interested reader can verify that for $d=1.75$, Method I applied for $a_1=15$ and $b_1=1$, provides the PDF controller settings $K_P=1.0019$, $K_I=7.2562\times10^{-5}$, $K_d=0.775$. The PDF controller settings obtained when applying Method II with $a_2=15$ and $b_2=15$ are $K_P=1.0019$, $K_I=7.7744\times10^{-5}$, $K_d=0.775$. The improved Method II, applied for $a_2=15$, $b_2=2.747$, $b_3=15.2458$ ($\gamma=5.55$), gives, in this case, the controller settings $K_P=1.0127$, $K_I=7.345\times10^{-4}$, $K_d=0.8456$. All these settings give stable closed-loop responses, with the improved Method II providing the best response. The PID controller settings obtained by the method in [13], with the tuning parameters involved having the values *24* and *14.4*, are $K_C=1.0671$, $\tau_i=42.9665$ and $\tau_D=0.8708$. However, these setting lead to an unstable closed-loop response.

4.7 Example 7

In this example, we will examine the efficiency of our methods in tuning PDF controllers for second order models with dead time, having an unstable pole (U^1SOPDT). These models have the form

$$G_P(s)=K\exp(-ds)/[(T_1s-1)(T_2s+1)]$$

To this end, consider the process model reported in [19], with $K=1;T_1=5$, $T_2=2.07$, $d=0.939$. Method I is applied here by selecting $a_1=4.2$ and $b_1=1$. With this selection, the PDF controller settings obtained by Method I are $K_P=5.8839$, $K_I=1.2384$, $K_d=7.6396$. The PDF controller settings obtained when applying Method II with $a_2=4.2$ and $b_2=15$ are $K_P=5.9622$, $K_I=1.3256$, $K_d=7.6693$. The settings of a PID controller tuned according to the method reported in [19] are $K_C=7.144$, $\tau_i=6.684$ and $\tau_D=1.655$. Figure 15 shows the comparison of the responses obtained by the proposed methods and the method in [19]. Obviously, the servo-performance obtained by our method is considerably better. However, the regulatory control performance obtained by the method reported in [19] is better in terms of maximum error and settling time. A better response can be obtained if we apply the improved method II for $a_2=7$, $b_2=3.1$, $b_3=50$. In this case, the PDF controller settings obtained by the improved Method II are $K_P=7.2976$, $K_I=1.3243$, $K_d=11.2167$. Figure

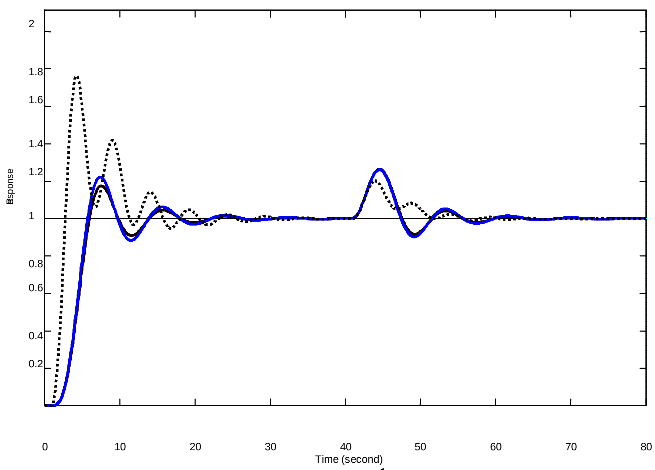

Figure 15. Comparison for U^1SOPDT systems with a_1=a_2=4.2, b_1=1, b_2=15. Solid blue line: Method I, solid black line: Method II, dotted line: Method of [19].

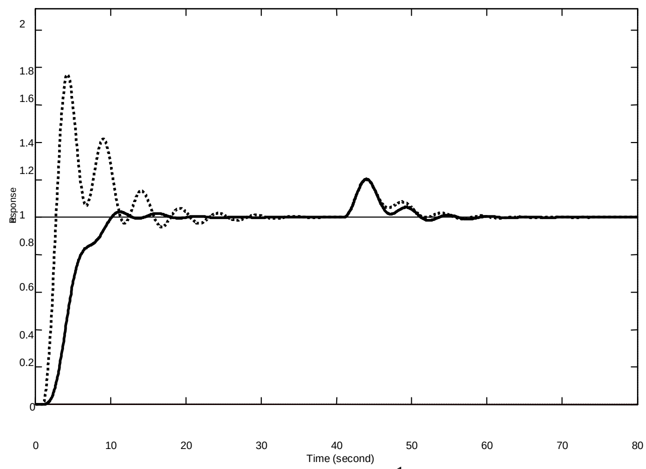

Figure 16. Comparison for U^1SOPDT systems. Solid line: Improved Method II, dotted line: Method of [19].

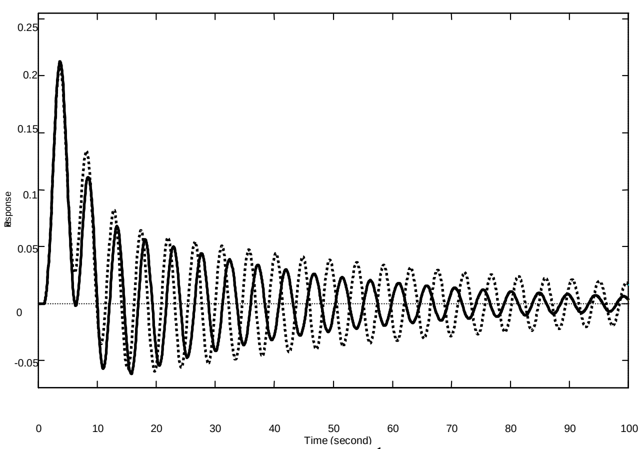

Figure 17. Comparison for U^1SOPDT systems, in terms of the regulatory control performance under 20%uncertianty in K. Solid line: Improved Method II, dotted line: Method of [19].

16 shows a comparison of the responses obtained by our method and that reported in [19]. A considerably better servo response is obtained by our method. Moreover, in this case, a better regulatory control performance is also obtained by our method. Finally, Figure 17 shows the regulatory control performances obtained by the two methods in comparison, in the case of a 20% uncertainty in *K*. Obviously, the closed-loop system with the PDF controller designed according to the improved Method II can better tolerate the assumed uncertainty.

4.8 Example 8

In this example, we consider a second order plus dead time model with two unstable poles (U^2SOPDT model) of the form

$$G_P(s) = K\exp(-ds)/[(T_1 s-1)(T_2 s-1)]$$

with $K=2$, $T_1=3$, $T_2=1$ and $d=0.3$ [19]. Method I is applied here with $a_1=6$ and $b_1=1$. With this selection, the PDF controller settings obtained by Method I are $K_P=1.2083$, $K_I=0.9491$, $K_d=3.3875$. The PDF controller settings obtained when applying Method II with $a_2=6$ and $b_2=15$ are $K_P=1.2281$, $K_I=1.0165$, $K_d=3.39$. The settings of a PID controller tuned according to the method reported in [19] are $K_C=2.3153$, $\tau_i=1.7843$ and $\tau_D=1.8859$. Figure 18 shows the comparison of the responses obtained by the proposed methods and the method in [19]. Obviously, the servo-responses obtained by the methods in comparison are similar (the method in [19] gives a greater overshoot). However, the response obtained by the method of [19], in the case of the regulatory control problem is considerable better than those obtained by the proposed methods.

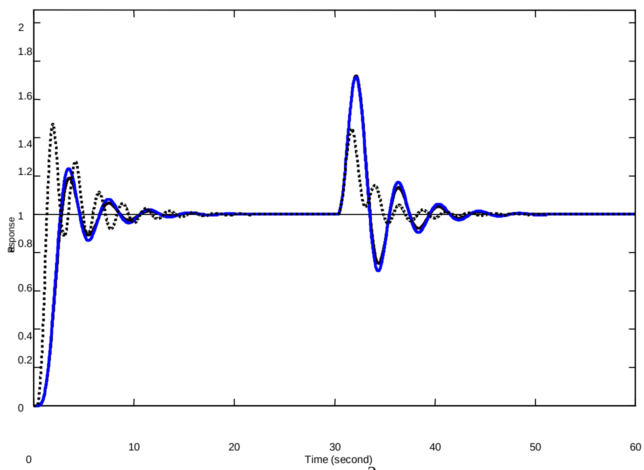

Figure 18. Comparison for U^2SOPDT systems with a_1=a_2=6, b_1=1, b_2=15. Solid blue line: Method I, solid black line: Method II, dotted line: Method of [19].

To obtain a better response, we apply the improved method II for $a_2=12$, $b_2=3$, $b_3=50$. In this case, the PDF controller settings obtained by the improved Method II are $K_P=2.0061$, $K_I=1.0024$, $K_d=4.4853$. Figure 19 shows a comparison of the responses obtained by our method and that reported in [19]. A considerably better servo response is obtained by our method, while a better regulatory control performance is also achieved. Note that, in this case, the value of the ISE-load criterion obtained by the method of [19] is *3.1972*, while our method provides a respective value of *3.0273*. Finally, Figure 20 shows the regulatory control performances obtained by the two methods in comparison, in the case of a 20% uncertainty in *K*. Obviously, the closed-loop system with the PDF controller designed according to the improved Method II can better tolerate the assumed uncertainty.

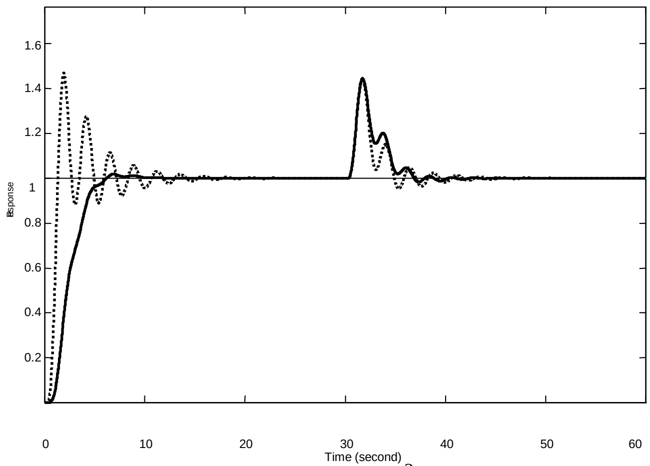

Figure 19. Comparison for U^2SOPDT systems. Solid line: Improved Method II, dotted line: Method of [19].

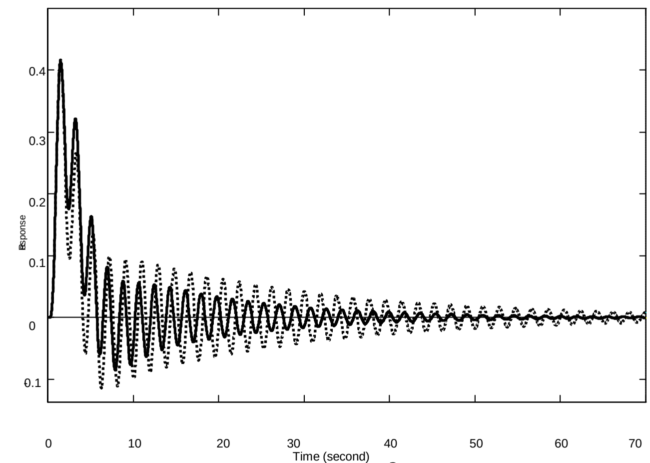

Figure 20. Comparison for U^2SOPDT systems, in terms of the regulatory control performance under 20%uncertianty in K. Solid line: Improved Method II, dotted line: Method of [19].

4.9 Example 9

In this example, we consider an integrating and unstable with dead time (I+UFOPDT) process model of the form

$$G_P(s) = K\exp(-ds)/\left[s(Ts-1)\right]$$

with $K=1$, $T=1$ and $d=0.2$ [19]. Method I is applied here with $a_1=6$ and $b_1=1$. With this selection, the PDF controller settings obtained by Method I are $K_P=3.3333$, $K_I=2.7778$, $K_d=3$. The PDF controller settings obtained when applying Method II with $a_2=6$ and $b_2=15$ are $K_P=3.3719$, $K_I=2.9752$, $K_d=3.0033$. The settings of a PID controller tuned according to the method reported in [19] are $K_C=0.8412$, $\tau_i=3.3066$ and $\tau_D=2.8113$. Figure 21 shows the comparison of the responses obtained by the proposed methods and the method in [19]. Obviously, the proposed methods have a better performance, particularly in the case of regulatory control.

4.10 Example 10

In this example, we consider an overdamped second order plus dead time process model with a stable zero (overdamped SOPDT+SZ), of the form

$$G_p(s) = K(T_z s+1)\exp(-ds)/\left[(T_1 s+1)(T_2 s+1)\right] \quad (16)$$

Process models of the form (16), which are also known as “large overshoot” systems because their open-loop dynamic behavior can have an initial large overshoot response, are much more difficult to control than the usual first-order, second-order or integrating plus dead-time systems. Very few discussions on proper controller tuning for such models have been reported in the literature, the most recent being that reported in [20], where a formal method is presented in order to obtain the settings of PID

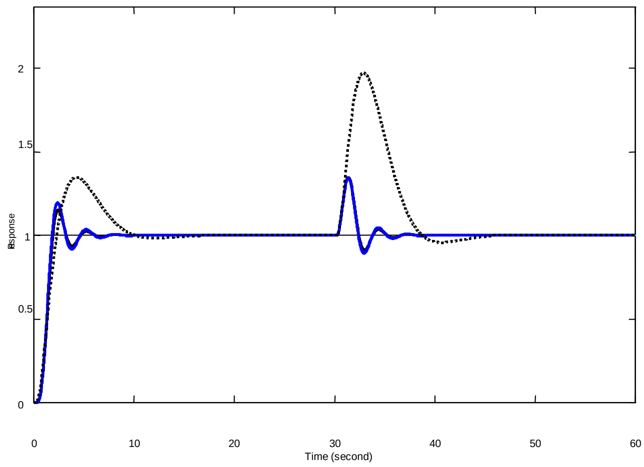

Figure 21. Comparison for I+UFOPDT systems with $a_1=a_2=6$, $b_1=1$, $b_2=15$. Solid blue line: Method I, solid black line: Method II, dotted line: Method of [19].

controllers of the form

$$G_c(s) = K_c\left(1+\frac{1}{\tau_I s}\right)\left(\frac{\tau_D s+1}{\tau_F s+1}\right)$$

In the present example, $K=1$, $T_1=3$, $T_2=2$, $T_z=1.5$, $d=0.5$. With these system parameters, the settings of a PID controller tuned according to the method in [20] are $K_C=2.4853$, $\tau_i=3$, $\tau_D=2$ and $\tau_F=1.5$. Method I, with $a_1=1$, $b_1=1$, gives the PDF controller parameters as $K_P=7.6420$, $K_I=6.4015$, $K_d=1.8333$. The PDF controller settings obtained by the application of Method II, with a_2=1.96, b_2=0.6, are $K_P=6.2721$, $K_I= 6.1628$, $K_d=1.1656$. Figure 22 shows the comparison of the responses obtained by the proposed methods and the method in [20]. The servo-responses obtained by our methods give some small overshoot, but the responses in the case of the regulatory control problem are considerably better than that obtained by the method of [20]. A better

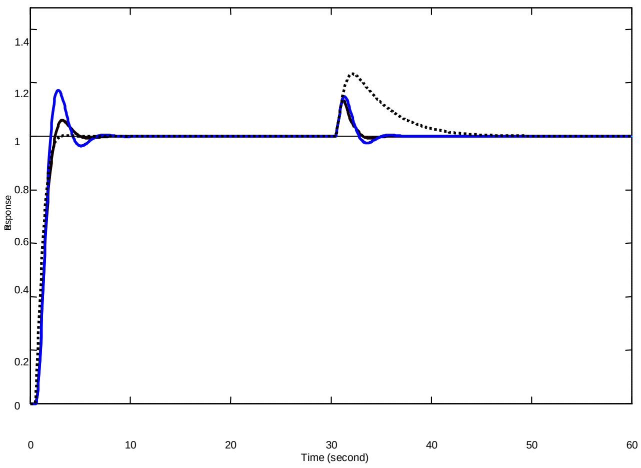

Figure 22. Comparison for overdamped SOPDT+SZ systems. Solid blue line: Method I, solid black line: Method II, dotted line: Method of [20].

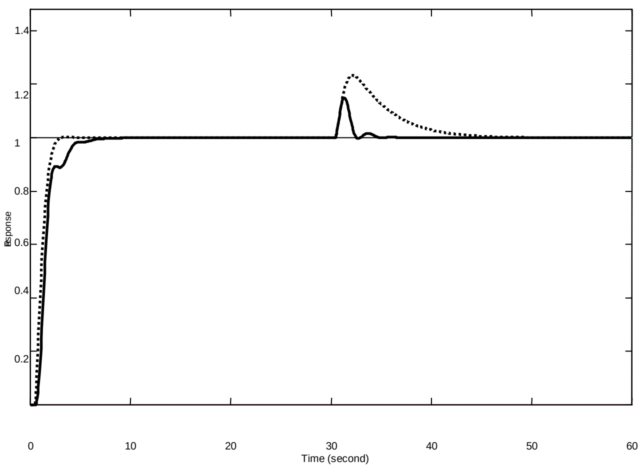

Figure 23. Comparison for overdamped SOPDT+SZ systems. Solid line: Improved Method II, dotted line: Method of [20].

servo response can be obtained when applying the improved method II with a_2=1.6, b_2=0.5, b_3=50 are $K_P=8.196$, $K_I=5.4094$, $K_d=0.7706$. Figure 23 shows the comparison of the obtained responses. Obviously our method gives the better response.

4.11 Example 11

In this example, we consider an underdamped second order plus dead time process model with a stable zero (underdamped SOPDT+SZ), of the form

$$G_P(s) = K(T_z s+1)\exp(-ds)/(\omega^2 s^2 + 2\zeta\omega s + 1)$$

with $\omega=1$, $\zeta=0.5$, $K=1$, $T_z=1.5$, $d=0.5$. Method I is applied here with $a_1=2.2$ and $b_1=0.6$. With this selection, the PDF controller settings obtained by Method I are $K_P=1.3917$, $K_I=1.1389$, $K_d=0.2335$. The PDF controller settings obtained when applying Method II with $a_2=1.69$ and $b_2=0.5$ are $K_P=1.4009$, $K_I=1.4507$, $K_d=0.2582$. The settings of a PID controller tuned according to the method reported in [20] are $K_C=0.8284$, $\tau_i=1$, $\tau_D=1$ and $\tau_F=1.5$. Figure 24 shows the comparison of the responses obtained by the proposed methods and the method in [20]. Obviously, the proposed methods have a better performance, particularly in the case of regulatory control. A similar performance can be obtained in the case where the PDF controller is designed according to the improved method II with $a_2=1.5$, $b_2=0.4$, $b_3=50$. In this case the obtained PDF controller settings are $K_P=1.5428$, $K_I=1.0707$, $K_d=0.1934$.

4.12 Example 12

In this example, we consider a second order plus dead time process model with two unstable poles and a stable zero (U^2SOPDT+SZ). The process model considered has the form

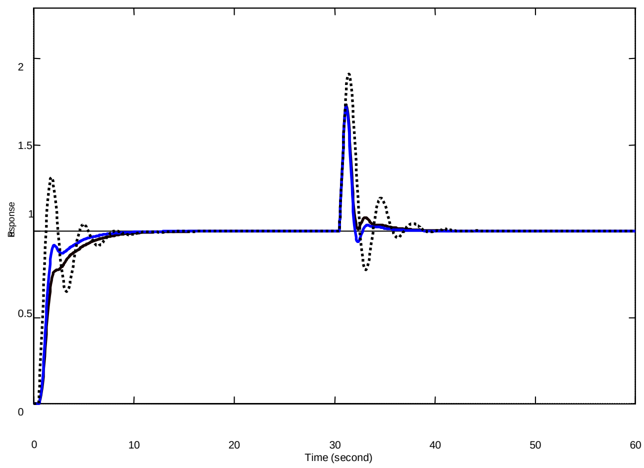

Figure 24. Comparison for underdamped SOPDT+SZ systems. Solid blue line: Method I, solid black line: Method II, dotted line: Method of [20].

$$G_P(s) = K(T_z s + 1)\exp(-ds)/[(T_1 s - 1)(T_2 s - 1)]$$

with $K=2$, $T_z=5$, $T_1=3$, $T_2=1$ and $d=0.3$ [19]. For this particular example, the method reported in [19] fails to provide settings for a PID (or, in general, for a simple three-term controller), since, as stated in [19], the strong lead term *(5s+1)* causes tuning parameters τ_I and τ_D of the PID controller to be negative values. For this reason, in [19], a four-term PID lag controller is necessary to control the process. However, if we apply the proposed Method I with $a_1=2.5$, $b_1=0.8$, we obtain the PDF controller settings $K_P=1.1804$, $K_I=0.8402$, $K_d=0.1227$. Moreover, the PDF controller settings obtained when applying Method II with $a_2=3$ and $b_2=0.6$ are $K_P=0.9329$, $K_I=0.4432$, $K_d=0.2012$. Figure 25 shows the closed-loop responses obtained when applying the above tree-term controllers to the process model considered. Obviously, the proposed methods are capable to give acceptable three-term controller settings even in the case where other methods fail.

4.13 Example 13

In this example, we consider an overdamped second order plus dead time process model with an unstable zero (overdamped SOPDT+UZ). The process model considered has the form

$$G_P(s) = K(-T_z s + 1)\exp(-ds)/[(T_1 s + 1)(T_2 s + 1)] \quad (17)$$

Process models of the form (17), which are also known as "inverse response" systems because their open-loop dynamic behavior can have an initial "wrong way" response, are also difficult to control. The interested reader can refer to [20], [21] for a review of methods for proper controller tuning for such models. In particular, in [20], a simple method is presented in order to obtain the settings of PID controllers of the form

$$U(s) = K_C\left(1 + \frac{1}{\tau_I s}\right)\left(R(s) - \frac{\tau_D s + 1}{0.1\tau_D s + 1}Y(s)\right)$$

In the present example, $K=1$, $T_1=1$, $T_2=1$, $T_z=0.6$, $d=1$. With these system parameters, the settings of a PID controller tuned according to the method in [20] are $K_C=0.2813$, $\tau_i=1$, $\tau_D=1$. Method I, with $a_1=4.1$, $b_1=1$, gives the PDF controller parameters as $K_P=1.0202$, $K_I=0.4927$, $K_d=0.5415$. The PDF controller settings obtained by the application of Method II, with $a_2=6$, $b_2=50$, are $K_P=0.5028$, $K_I=0.2316$, $K_d=0.5533$. Figure 26 shows the comparison of the servo responses obtainned by the proposed methods and the method in [20]. The servo-responses obtained by our methods give smaller closed-loop jump due to the inverse response, while Method I gives a small overshoot. Figure 27 shows the responses obtained in the case of regulatory control. In this case, all three methods give the same closed-loop jump while our methods give a smaller maximum error. Moreover, the ISE-load values in this case are obtained as *2.1018* for the Method of [20], *1.3965* for Method I and *2.1776* for Method II. Therefore, Method I gives the better performance. A similar good regulatory control performance can be obtained by applying the improved Method II with $a_2=6$, $b_2=20$ and $b_3=50$. Controller parameters in this case are obtained as $K_P=0.7242$, $K_I=0.2728$, $K_d=0.9893$. The ISE-load value for improved Method II is *1.6416*. Figure 28 shows the obtained responses.

4.14 Example 14

In this example, we consider an underdamped second order plus dead time process model with an unstable zero (underdamped SOPDT+UZ). The pro-

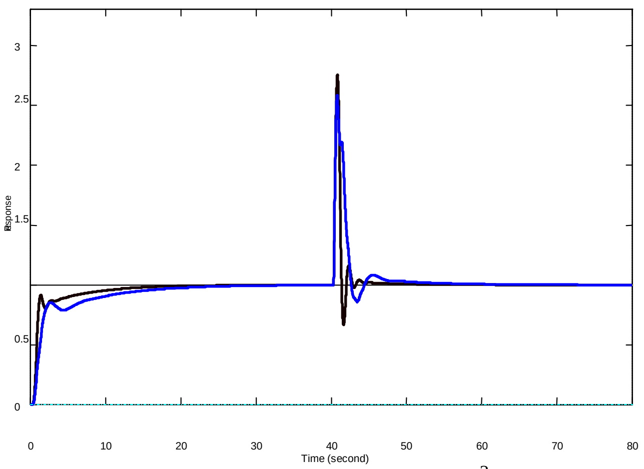

Figure 25. Closed-loop responses for U^2SOPDT+SZ systems. Black line: Method I, blue line: Method II.

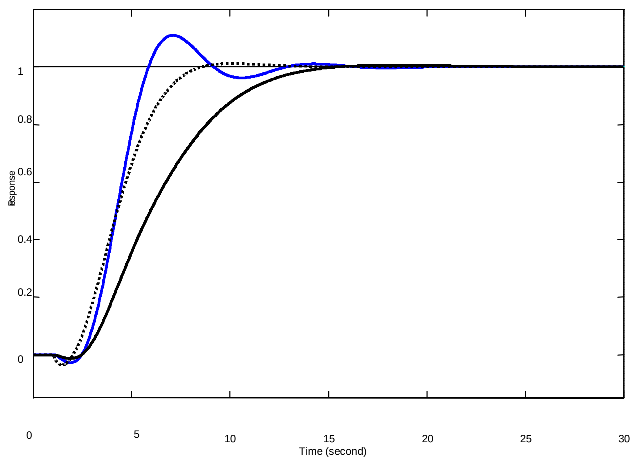

Figure 26. Comparison of servo-responses obtained for SOPDT+UZ systems: Solid blue: Method I, solid black: Method II, dotted: Method of [20].

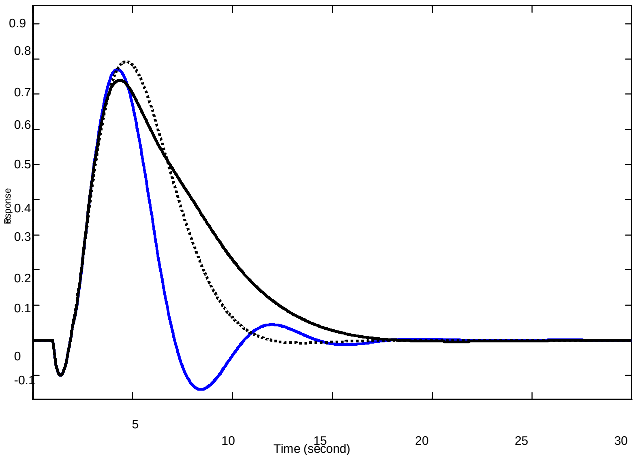

Figure 27. Comparison of the regulatory control responses obtained for SOPDT+UZ systems: Solid blue line: Method I, solid black line: Method II, dotted line: Method of [20].

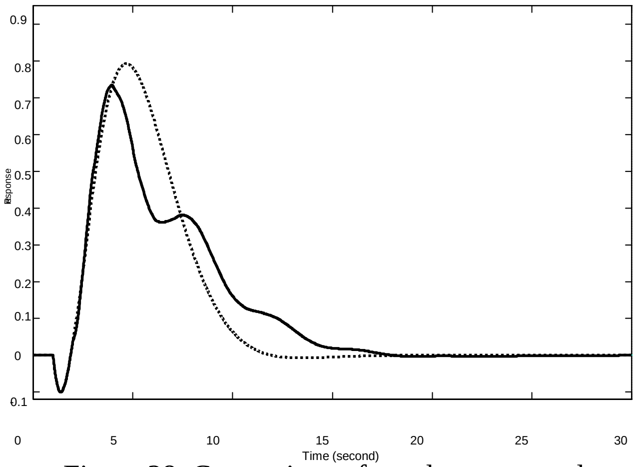

Figure 28. Comparison of regulatory control responses obtained for SOPDT+UZ systems: Solid: Improved Method II, dotted: Method of [20].

cess model considered has the form

$$G_P(s) = K(-T_z s + 1)\exp(-ds)/\left(\omega^2 s^2 + 2\zeta\omega s + 1\right) \quad (18)$$

For the present example $K=1$, $T_z=0.6$, $d=1$, $\omega=1$, $\zeta=0.5$. For models of the form (18), a method has been presented in [20] for tuning a special class of PID controllers having the form

$$U(s) = K_C\left(\left(1 + \frac{1}{\tau_I s}\right)R(s) - \left(1 + \frac{1}{\tau_I s} + \frac{\tau_D s}{0.1\tau_D s + 1}\right)Y(s)\right)$$

In the sequel, we will see that it is not necessary to resort to such a peculiar and complicated PID controller structure, as in [20]. In particular, we will see that more conventional three-term controllers are sufficient to adequately control a process model of the form (18).

To this end, we next apply Method I with $a_1=4.4$, $b_1=0.8$. The obtained PDF controller settings are $K_P=0.4034$, $K_I=0.3302$, $K_d=0.3487$. Based on the equivalence between the PDF controller and the ideal PID controller, the equivalent PID controller settings are $K_c=0.4034$, $\tau_I=1.2216$, $\tau_D=0.8646$, while the set-point filter transfer function (2) is given by $G_{SPF}(s)=1/(1.0561s^2+1.2216s+1)$. Moreover, application of the proposed Method II, with $a_2=6$, $b_2=8.5$, gives the PDF controller settings $K_P=0.2275$, $K_I=0.2066$, $K_d=0.0.6718$. The settings of the equivalent PID controller are $K_c=0.2275$, $\tau_I=1.1012$, $\tau_D=2.9525$, while the set-point filter has the form $G_{SPF}(s)=1/(3.2514s^2+1.1012s+1)$. Finally, the parameters of the modified PID controller designed according to the method reported in [20] are $K_c=0.2525$, $\tau_I=0.8989$, $\tau_D=1.0114$. Figure 29 shows the obtained responses for both the servo and the regulatory control problem. Method II gives less overshoot, while Method I is comparable to the method of [20] in terms of overshoot and settling time. Our methods give less closed-loop jump, and in the case of regulatory control, less maximum error. Moreover, the ISE-load values in this case are obtained as *3.3637* for the Method of [20], *2.9925* for Method I and *3.2668* for Method II. Overall, our methods give a better performance.

4.15 Example 15

In this example, we consider a second order plus dead-time model with an unstable pole and an unstable zero (U[1]SOPDT+UZ model). The process model has the form

$$G_P(s) = K(-T_z s + 1)\exp(-ds)/\left[(T_1 s - 1)(T_2 s + 1)\right]$$

To the authors' best knowledge there is no method in the literature for tuning three term controller for such processes. Here, we test the proposed me-

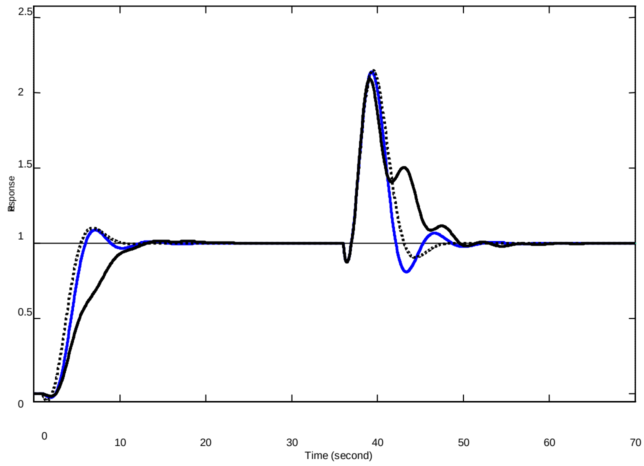

Figure 29. Comparison for underdamped SOPDT+UZ systems: Solid blue line: Method I, solid black line: Method II, dotted line: Method of [20].

thods as possible candidates. To this end, let $K=0.1$, $T_Z=0.6$, $d=0.3$, $T_1=T_2=1$. Application of Method I for $a_1=20$, $b_1=50$ gives the PDF controller settings $K_P=10.559$, $K_I=0.0848$, $K_d= 10.8565$. Method II applied for $a_2=25$, $b_2=50$ gives $K_P=10.6358$, $K_I= 0.08$, $K_d= 11.2271$. The improved Method II for $a_2= 25$, $b_2=15$, $b_3=50$ gives $K_P= 10.7895$, $K_I=0.1042$, $K_d =11.5911$. Figures 30-32 show the responses obtainned by the application of the above controllers. The improved Method II gives the smaller maximum error. Clearly, the proposed methods are very efficient in designing three term controllers for these peculiar dead-time processes.

4.16 Example 16

In this final example, we consider an unstable first order plus dead time process with an unstable zero. The process model has the form

$$G_P(s) = K(-T_z s + 1)\exp(-ds)/(Ts - 1) \qquad (19)$$

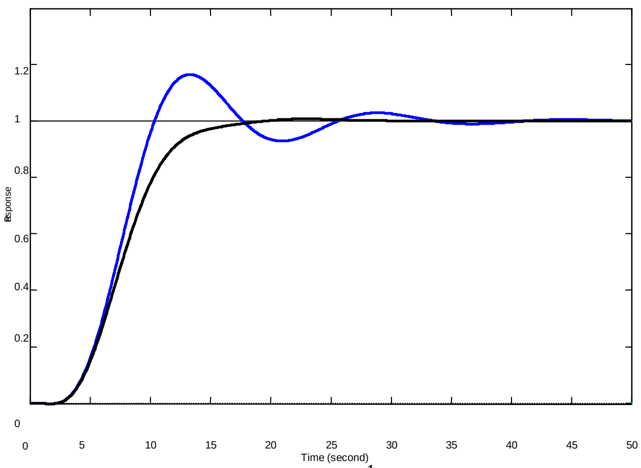

Figure 30. Comparison for U[1]SOPDT+UZ systems in case of set-point tracking: Blue line: Method I, black line: Method II.

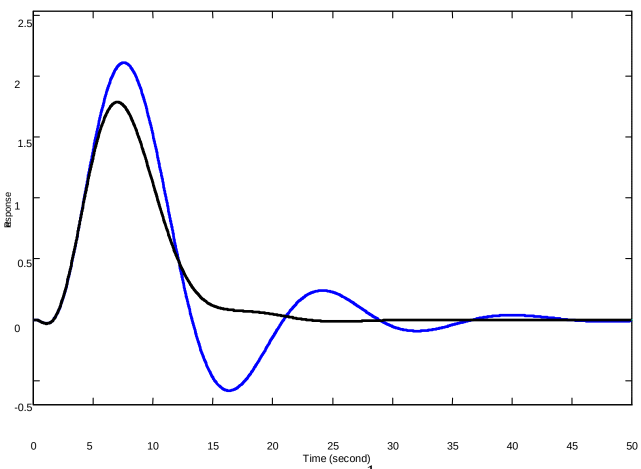

Figure 31. Comparison for U[1]SOPDT+UZ systems in case of regulatory control: Blue line: Method I, black line: Method II.

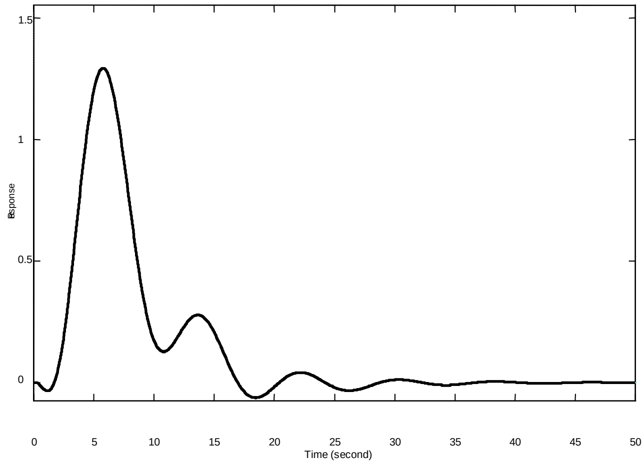

Figure 32. Regulatory control performance of improved Method II when applied to U[1]SOPDT+UZ systems.

Here $K=1$, $T=1$, $d=0.25$, $T_z=0.25$. Applying Method I with $a_1=7$, $b_1=15$, yields the PDF controller parameters as $K_P=1.4987$, $K_I=0.2514$, $K_d=0.0023$. Application of Method II with $a_2=11$, $b_2=15$ yields $K_P=1.4342$, $K_I=0.1541$, $K_d=0.0463$. For the process model (19) with parameters as above, typical values of the settings of a PI controller designed according the method reported in [21], [22], are $K_C=1.7$, $\tau_I=10$. Figure 33 shows the comparison of the performances of the above three controllers. The proposed method outperform the method reported in [21] in terms of overshoot, closed-loop jump and settling time. However, the regulatory control performance obtained by the PI controller that is designed according to the method of [21], [22], is better in terms of maximum error. To settle this concern, we next apply the improved Methods I and II with $a_1=5$, $b_1=5$, $c=50$, and $a_2=10$, $b_2=15$, $b_3=50$, respectively.

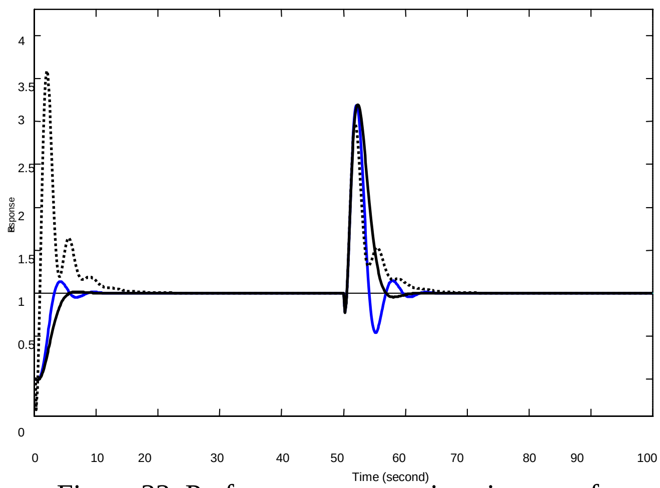

Figure 33. Performance comparison in case of UFOPDT+UZ systems: Solid blue line: Method I, solid black line: Method II, dotted line: Method [21]

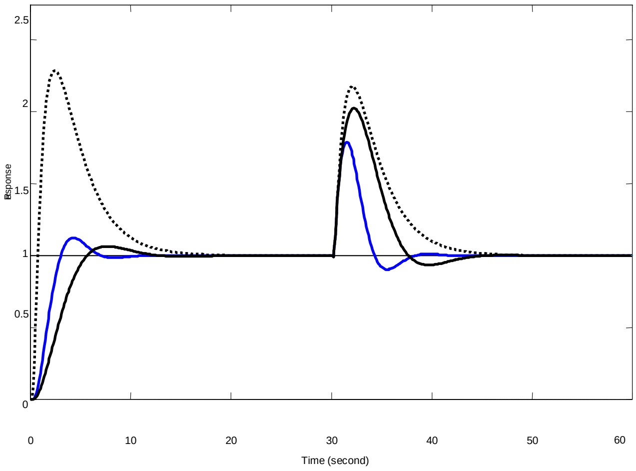

Figure 34. Performance comparison in case of UFOPDT+UZ systems: Solid blue line: Improved Method I, solid black line: Improved Method II, dotted line: Method of [21]

Then, the obtained PDF controllers settings are K_P= *1.8952*, K_I=*0.6174*, K_d=*0.1676*, and K_P=*1.6714*, K_I =*0.2599*, K_d=*0.2254*, respectively. The closed-loop performances obtained by the improved Methods I and II and the method of [21], [22] are illustrated in Figure 34. Clearly, in addition to a considerably better servo-response, our method provides a much better regulatory control performance, with the improved Method I giving a smaller maximum error.

5 Conclusions

Simple methods are proposed for three-term controller settings for dead time processes. The proposed methods are universal and can be applied to a wide variety of dead-time process models, from stable to unstable and from models without zeros to those representing inverse response and large overshoot. The proposed methods give simple sets of equations for the controller settings, and they are robust for uncertainty in the model parameters. A series of simulation examples verify that, in most cases, the proposed methods give the best performance when compared with the most recent tuning methods reported in the literature. In any case, the proposed methods give at least a performance similar to that obtained by other methods. On the other hand, in some cases the methods reported here is the only available tool in order to design three-term controllers for dead-time processes. Inevitably this paper presents some preliminary results of a work in progress. Important issues, like e.g. the theoretical analysis of stability and robustness of the proposed methods, are currently under investigation.

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