On the Generalized Solutions for Uncertain Systems with Applications to Optimal Control and Estimation Problems*

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Abstract: - The paper deals with the control and estimation problems for impulsive control system under uncertainty described by differential systems with measures. Uncertainty conditions are of a set–membership type, i.e. uncertain parameters and functional disturbances are taken to be unknown but bounded with given bounds (e.g., the model may contain unpredictable errors without their statistical description). Models of this kind arise in many applications ranging from space navigation to investment problems as well as ecological management. In this paper we consider solution concepts for such measure driven uncertain dynamical systems. The approaches are based on the techniques of approximation of the discontinuous generalized trajectory tubes by the solutions of usual differential systems without measure terms and provide convenient frameworks to derive optimality conditions in the form of either a Maximum Principle or Hamilton-Jacobi-Bellman equations. Another class of problems related to the addressed systems concerns the state estimation. The redesign of the approach for impulsive control systems in the framework of the new solution concept is presented through the characterization of the reachable set as a level set of the value function regarded as a solution of Hamilton-Jacobi-Bellman equations.

Keywords: - Impulsive control, dynamical system, trajectory tubes, uncertainty, estimation.

1 Introduction

In this paper the impulsive control problem for a dynamic systems with unknown but bounded initial states is studied. Such problems arise from mathematical models of dynamical and physical systems for which we have an incomplete description of time dependence of their generalized coordinates [1, 3, 4, 5, 6, 7, 8, 9].

We discuss the approaches to solution concepts for such uncertain dynamical systems based on ideas of well known discontinuous time substitution [10] and the techniques of differential inclusions theory [11, 12, 13, 3].

There is a long list of publications devoted to impulsive control optimisation problems, among them we mention here only the results related to the present investigation [14, 15, 7, 16, 17, 18, 19]. The question arises how the results of classical control theory established for uncertain dynamical systems driven by ordinary control functions can be extended to the case of impulsive systems containing measure (impulses). Our study combines both approaches mentioned above and presents results related to optimal control and state estimation for differential uncertain systems of impulsive structure.

In this paper we consider a dynamic control system described by a differential equation with

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a usual control function $u(\cdot)$ and a measure (or impulsive control component) $v(\cdot)$:

$$dx(t) = f(t, x(t), u(t))dt \ + \tag{1}$$

$$+ \ B(t, x(t), u(t))dv(t), \quad x \in R^n, \quad t_0 \le t \le T,$$

with unknown but bounded initial condition

$$x(t_0 - 0) = x^0, \quad x^0 \in X^0. \tag{2}$$

Here $u(t)$ is a usual (measurable) control with constraint

$$u(t) \in U, \quad U \subset R^m, \tag{3}$$

and $v(t)$ is an impulsive control function which is continuous from the right, with constrained variation

$$\mathrm{Var}_{t\in[t_0,T]} \ v(t) \le \mu, \tag{4}$$

where μ is a given positive number.

So we consider here the case when the system control variable consists of two parts $w = \{u, v\}$ with the first component u being of the ordinary type and the second one v being the measure (or the impulsive control). We assume also that $f(t, x, u)$ and $n \times k$-matrix $B(t, x, u)$ are continuous in their variables.

One of the principal points of interest of the theory of control under uncertainty conditions [1, 5, 6] is to study the set of all solutions

$$x[t] = x(t, t_0, x^0, u, v)$$

to (1) - (11). The guaranteed estimation problem consists in describing the set

$$X[\cdot] = X(\cdot\ , t_0, X^0) = \bigcup_{\{u(\cdot),\ v(\cdot)\}} \{ \ x[\cdot] \ \mid x[t] =$$

$$x(t, t_0, x^0, u, v), \ x^0 \in X^0 \ \ \} \tag{5}$$

of solutions to the system (1) - (11) under constraints (3) - (15) and the t − cross-section (t − cut) $X[t]$ of the $X[\cdot]$. The t − cut $X[t]$ is actually the attainability set (the reachable set) of the system at instant t from the initial set-valued "state" X^0. The set $X[t]$ may be treated also as the unimprovable set-valued estimate of the unknown state $x(t)$ of the system (1) - (11) under restrictions (3) - (15).

The mathematical background for investigations of set-valued estimates $X[t]$ ranging from theoretical schemes to numerical techniques may be found in [1, 5, 6, 20].

Thus, in this paper we actually apply the set-membership (bounding) approach combining with the techniques of approximation of the discontinuous generalized trajectory tubes by the solutions of usual differential systems (ordinary differential inclusions) without measure terms. These ideas provide convenient frameworks to derive optimality conditions in the form of either a Maximum Principle or Hamilton-Jacobi-Bellman equations. Another class of problems related to the addressed systems concerns the state estimation. The redesign of the approach for impulsive control systems in the framework of a convenient solution concept is presented through the characterization of the reachable set as a level set of the value function regarded as a solution of Hamilton-Jacobi-Bellman equations.

2 Optimal Control Problem

2.1 Problem Formulation

We study the following measure differential inclusion problem

$$(P) \quad \text{Minimize } h(x(1))$$

$$\begin{aligned} dx(t) &\in F(t, x(t))dt + \\ &\quad + \mathbf{G}(t, x(t))\mu(dt) \ \ \forall t \in [0, 1], \\ &x(0) = x_0, \quad \mu \in \mathcal{K} \end{aligned} \tag{6}$$

where

$$\begin{aligned} h : \mathbb{R}^n \mapsto \mathbb{R}, \quad F : [0, 1] \times \mathbb{R}^n \hookrightarrow \mathcal{P}(\mathbb{R}^n), \\ \mathbf{G} : [0, 1] \times \mathbb{R}^n \hookrightarrow \mathcal{P}(\mathbb{R}^{n\times q}), \\ \mathcal{K} := C^*([0, 1]; K) \end{aligned} \tag{7}$$

and K is a positive pointed convex cone in $\mathbb{R}^q$.

We consider mild assumptions on the data, i.e., we study the problem (P) under Lipschitz continuity dependence on the state variable and we do not assume the commutativity of the singular vector fields.

Therefore, the first question that arises in Problem (P) is how to define the solution $x(\cdot)$ to (6) or to its differential inclusion interpretation:

$$x(t) = x(0) + \int_0^t f(\tau, x(\tau))d\tau +$$

$$+\int_{[0,t]} G(\tau, x(\cdot))\mu(d\tau) \ \ \forall t \in [0,1],$$

where f and G are suitable selections of F and $\mathbf{G}$.

The main problem in this context is to define correctly the interaction between the evolving trajectory and the impulsive integrating measure. The approach presented here enables a definition of a solution concept which ensures the well posedness of the dynamic optimization control problem. The technique to derive optimality conditions is based on the reparameterization procedure that reduces the original problem to an auxiliary conventional one. Then, we apply existing conditions to this new problem and express them in terms of the data of the original problem.

2.2 The Concept of a Solution

Following [8], we introduce the solution concept **Definition 1**. We will call $x(\cdot) \in BV^+([0,1]; I\!R^n)$ as a *proper solution* to the measure differential inclusion (6) iff there exist $\mathcal{L}$-integrable f and $|\mu|$-integrable g, with $f(t) \in F(t, x(t))$ $\mathcal{L}-$ a.e. and $g(t) \in \tilde{G}(t, x(t^-); \mu(\{t\}))$ $|\mu|$-a.e., s.t. for all $t \in (0,1]$

$$x(t) = x(0) + \int_0^t f(\tau)d\tau + \int_{[0,t]} g(\tau)|\mu|(d\tau),$$

where $\tilde{G} : [0,1] \times I\!R^n \times K \hookrightarrow I\!R^n$ is given by

$$\tilde{G}(t,z;\alpha) := \begin{cases} \left\{ G(t,z)\frac{\mathrm{d}\mu}{\mathrm{d}|\mu|} \right\}, & \text{if } |\alpha| = 0 \\ \\ \left\{ \frac{\xi(\eta(t)) - \xi(\eta(t^-))}{|\alpha|} : \dot{\xi}(s) \in G(t, \xi(s))\dot{\gamma}(s) \right. \\ \bar{\eta}\text{-a.e., } \xi(\eta(t^-)) = z \text{ and } \gamma(\eta(t)) - \\ \left. \gamma(\eta(t^-)) = \alpha \right\}, & \text{if } |\alpha| > 0. \end{cases} \tag{8}$$

Here, $|\mu|$ is the total variation measure associated with μ, $\eta(\cdot)$ is a *time reparameterization* and $(\gamma(\cdot), \theta(\cdot))$ is a *h-graph completion.* We remind that the graph completion of a measure $\mu \in C^+(0,1)$ is a pair $(\theta, \gamma) : [0,1] \mapsto (I\!R^+)^{q+1}$ defined by

$$\gamma(s) := \begin{cases} M(\theta(s)), & \text{if } |\mu|(\{\theta(s)\}) = 0 \\ \\ M(t^-) + \int_{\eta(t^-)}^s v(\sigma)d\sigma, & \text{otherwise} \end{cases},$$

and $\theta : [0,1] \mapsto [0,1]$ is s.t. $\theta(s) = t$ for all $s \in \bar{\eta}(t)$ where

$$v(\cdot) \in V^t := \{v : \bar{\eta}(t) \to K | \sum_{i=1}^q v_i(s) = 1,$$

$$\int_{\bar{\eta}(t)} v(s)ds = \mu(\{t\})\},$$

$$M_i(0) := 0 \text{ and } M_i(t) := \int_{[0,t]} \mu_i(ds)$$

$$\text{for all } t > [0,1], \quad i = 1, ..., q,$$

$$\eta(t) := t + \sum_{i=1}^q M_i(t) \text{ and}$$

$$\bar{\eta}(t) := \begin{cases} \{\eta(t)\} & \text{if } |\mu|(\{t\}) = 0 \\ [\eta(t^-), \eta(t)] & \text{if } |\mu|(\{t\}) > 0. \end{cases}$$

The *reparameterized system* is

$$\dot{y}(s) \in F(\theta(s), y(s))\dot{\theta}(s) + G(\theta(s), y(s))\dot{\gamma}(s),$$

being $\dot{\gamma}(s)$ the variation rate of the control measure in the reparameterized time.

For a given μ and a pair of measurable selections (f, G) of $(F, \mathbf{G})$, we have a set of trajectories:

$$\mathcal{F}_{\mu,f,G} := \Big\{ y(\cdot) : \dot{y}(s) = f(\theta(s), y(s))\dot{\theta}(s) +$$

$$+ G(\theta(s), y(s))\dot{\gamma}(s), \quad \dot{\gamma}(s) \in K, \tag{9}$$

$$(\dot{\theta}(s), \dot{\gamma}(s)) \in \Omega, \ [0,1] \text{ a.e.} \Big\},$$

where $\Omega := \{w \in I\!R^+ \times K : \sum_{i=0}^q w_i = 1\}$, $\gamma(0) = 0$ and $\gamma(\eta(t)) = \mu([0,t])$, $\forall t \in [0,1]$.
Definition 2. The pair (θ, γ) is a *h-graph completion* associated to (μ, f, G) if

$$(\theta, \gamma) \in \text{argmin}\{h(y(1)) : \ y(\cdot) \in \mathcal{F}_{\mu,f,G}\}.$$

Let us introduce a notation

$$\sum\nolimits_h := \{(y, \theta, \gamma) : y \in \mathcal{F}_{\mu,f,G} \ ,$$

$$\forall \text{ h-graph completion } \ (\theta, \gamma)\}$$

for the set of all h-standard reparametrized control processes.

We mention here that under mild hypotheses a robust solution $x(\cdot)$ to the measure differential inclusion (6) exists if and only if there exists an absolutely continuous solution $y(\cdot)$ to the reparameterized differential inclusion such that

$$x(t) = y(\eta(t)) \ \ \forall t \in [0,1] \text{ and } \|x\|_{TV} \leq \|y\|_{TV}.$$

3 Applications to Control Problem

We require the following assumptions on the data:

(H1) h is Lipschitz continuous with constant K_h;

(H2) F is continuous, and for each t is Lipschitz continuous with respect to (w.r.t.) x with constant K_f;

(H3) F is a nonempty, convex and compact-valued function;

(H4) There are constants K_1 and K_2, such that,

$$\forall (t,x), \quad \forall v \in F(t,x) \quad |v| \le K_1 + K_2|x|;$$

(H5) G is bounded, convex-valued and Lipschitz continuous w.r.t. (t,x) with constant K_G;

(H6) F and G have closed graphs;

(H7) $\forall\ r>0$, $\exists\ k_0(r) \in I\!R$ s.t. $\exists$ a solution (x,μ) to (P) s.t. $\|\mu\| \le k_0(r)$, $\|x\| \le r$.

Theorem 1. Let (x,μ) solve the Problem (P) under assumptions (H1)-(H7). Then, there is $p \in BV([0,1]; I\!R^n)$ s.t.:

$$\begin{array}{rcl}
(-dp(t), dx(t)) & \in & \partial H_F(t,x(t),p(t))dt \ + \\
 & + & \partial H_G(t,x(t),p(t))\mu(dt) \\
 & & \qquad\qquad \mu \text{ and } \mathcal{L} \text{ a.e.}, \\
-p(1) & \in & L\partial h(x(1)), \\
0 & \ge & \sigma_K(H_G(t,x(t),p(t))), \\
 & & \qquad\qquad \forall t \in [0,1], \\
0 & \le & \sigma_K(H_G(t,x(t),p(t)))\ \mu \text{ a.e.};
\end{array}$$

$$\begin{array}{rcl}
(-\dot{\alpha}_t(s), \dot{\xi}_t(s)) & \in & \partial H_G(t,\xi_t(s),\alpha_t(s)) \cdot \bar{v}(s) \\
 & & \qquad\qquad \bar{\eta}(t) \text{ a.e.}, \\
0 & \le & \sigma_K(H_G(t,\xi_t(s),\alpha_t(s))) \\
 & & \qquad\qquad \bar{\eta}(t) \text{ a.e.}, \\
(x^*(t^-), p(t^-)) & = & (\xi_t(\eta(t^-)), \alpha_t(\eta(t^-))), \\
(x^*(t), p(t)) & = & (\xi_t(\eta(t)), \alpha_t(\eta(t))),
\end{array}$$

where $\bar{v}$ satisfies

$$\sigma_K(H_G(t,\xi_t(s),\alpha_t(s))) = H_G(t,\xi_t(s),\alpha_t(s)) \cdot \bar{v}(s),$$

$$\int_{\bar{\eta}(t)} \bar{v}(s)ds = \mu(\{t\}), \quad \sigma_K(k) := sup_{\varsigma \in K}\{k \cdot \varsigma\},$$

$$H_F(t,x^*(t),p(t)) := \sigma_{F(t,x(t))}(p(t)),$$

and

$$H_G(t,x^*(t),p(t)) := \left\{ \begin{array}{l} \{h_G(t) \cdot w^*(t)\} \text{ if } \mu(\{t\}) = 0 \\ \{h_G^s(t) : s \in \bar{\eta}(t)\} \text{ otherwise} \end{array} \right.$$

with

$$\begin{array}{l}
h_G(t) \cdot w^*(t) := \text{Sup}\left\{ p^T(t)G(t,x(t)) \cdot w : \right. \\
\qquad \left. w = \frac{d\mu}{d\bar{\mu}} \in K,\ G(t,x(t)) \in \bar{G}(t,x(t)) \right\} \\
h_G^s(t) \cdot \bar{v}^*(s) = \text{Sup}\left\{ \alpha_t(s)G(t,\xi_t(s)) \cdot v(s) : \right. \\
\qquad \left. v(\cdot) \in V^t,\ G(t,\xi_t(s)) \in \bar{G}(t,\xi_t(s)) \right\}.
\end{array}$$

Let us now consider the derivation of generalized Hamilton-Jacobi-Bellman equations in the context of this solution concept, [21, 24]. Let $X(\tau,\xi)$ be the set of feasible trajectories starting at (τ,ξ). Then,

- $R(\tau,\xi) := \{x(1) \in I\!R^n : \ x \in X(\tau,\xi)\}$ is the *reachable set* at time $t=1$ when $x(\tau) = \xi$,
- $V(\tau,\xi) := \min\{h(z) : \ z \in R(\tau,\xi)\}$ is the *value function.*

Theorem 2. Under assumptions (H1)-(H7) the value function V is locally Lipschitz continuous on $[0,1] \times I\!R^n$ and
1) for all $(t,x) \in (0,1) \times I\!R^n$

$$\max_{\substack{(w_0,w) \in K_1, \\ f \in F(t,x), \\ g \in \mathbf{G}(t,x)}} \{DV((t,x); -(w_0, fw_0 + gw))\} = 0,$$

2) for all $(t,x) \in [0,1] \times I\!R^n$

$$\max_{r \in K, g \in \mathbf{G}(t,x)} \{DV_t(x; -gr)\} \le 0, \tag{10}$$

3) for all $x \in I\!R^n$

$$\max\{\max_{r \in K, g \in \mathbf{G}(1,x)} \{DV_1(x; -gr)\},\ V(1,x) - \\ -h(x)\} = 0$$

(here $Df(x;v)$ is the lower Dini derivative of a l.s.c. function f w.r.t. x in the direction v and $V_t(x) = V(t,x)$).

4 Estimation Problem

The approach presented above may also be used to solve the state estimation problem for impulsive dynamical system. The estimation problem in the deterministic setting is studied under uncertainty conditions with set–membership description of uncertain variables considered to be unknown but bounded with known bounds, [2, 6, 5, 3]. Such problems arise from mathematical models of dynamical and physical systems for which we have either an incomplete description or a loose mode of time dependence of their generalized coordinates. The techniques to describe the trajectory tubes for impulsive differential inclusion are studied by using dynamic programming results articulated with the described concept of proper solution.

One of the main points of interest in control theory under uncertainty, [6], concerns the study of the set of all solutions $x[t] = x(t, t_0, x_0)$ to (6) with unknown but bounded initial state x_0:

$$x_0 \in X_0, \tag{11}$$

where X_0 is a compact subset of $I\!R^n$. The "guaranteed" estimation problem consists in describing the set

$$X[\cdot] = \cup\{x[\cdot] = x(\cdot, t_0, x_0) \mid x_0 \in X_0\}$$

of solutions (6) and (11), being the t − cross section of this set, $X[t]$, the reachable set (information set) at time t of the system from X_0 at time t_0.

The information set can be treated as a *level set* of a generalized solution $V(t, x)$ to the HJB equation (10), where $V(t, x)$ is a value function given by

$$V(t,x) = \inf_{x[\cdot]} \{\phi(t_0, x[t_0]) \mid x[\cdot] = x(\cdot, t_0, x_0),$$
$$x[\cdot] \text{ solves (6) s. t. } x[t] = x\}\,, \tag{12}$$

being ϕ an appropriately chosen function (e.g., $\phi(t_0, x) = d^2(x, X_0)$ with X_0 as in (11) where $d(x, M)$ is the *distance function* from x to $M \subset I\!R^n$). Then, according to Theorem 2, techniques of proper solutions can be used in finding $V(t, x)$ and, thus, in constructing trajectory tubes and their cross sections as level sets of V [2].

In the estimation problems the so called measurement equation is considered also

$$y(t) = g(t, x, \xi(t)) \tag{13}$$

with $\xi(t)$ being the unknown but bounded "noise" or disturbance. The latter equation may be expressed as the state ("viability" [12]) constraint:

$$0 \in G(t, x), \tag{14}$$

where G is a given set-valued map.

In this case we have to modify the value function (12)

$$\tilde{V}(t,x) = \min\{d^2(x(t_0), X_0) + \int_{t_0}^{t} d^2(0, G(t, x[t]))dt$$
$$\mid x[\cdot] = x(\cdot, t_0, x_0), x[\cdot] \text{ solves (6)s.t.} x[t] = x\}. \tag{15}$$

It should be mentioned that the value function $\tilde{V}$ in optimization problem (12) (or (15)) can be found through the techniques of viscosity ([2]) or minimax ([4]) solutions of the corresponding H-J-B equations. The above approach gives us a possibility to produce also other estimates for the information sets $X[t]$ through the comparison principle that allows to connect the given approach to the techniques of ellipsoidal or box-valued calculus developed for systems with linear structure ([6], [20]).

5 Conclusions

The approaches based on the techniques of approximation of the discontinuous generalized trajectory tubes by the solutions of usual differential systems without measure terms and on the results of differential inclusion theory were considered. The techniques presented in the paper provide convenient frameworks to derive optimality conditions in optimal control theory and to solve the state estimation problems for dynamical systems under uncertainty conditions.

References:

[1] A.B. Kurzhanski, *Control and Observation under Conditions of Uncertainty*, Nauka, Moscow, 1977.

[2] J.S. Baras and A.B. Kurzhanski, Nonlinear Filtering: The Set-Membership (Bounding) and the H_∞ Approaches, *Proc. of the IFAC NOLCOS Conference*, Tahoe, CA, Plenum Press, 1995.

[3] A.B. Kurzhanski and V.M. Veliov (Eds), *Set-valued Analysis and Differential Inclusions*, Progress in Systems and Control Theory, Birkhauser, Boston, 1990.

[4] N.N. Krasovskii and A.I. Subbotin, *Positional Differential games*, Nauka, Moscow, 1974.

[5] A.B. Kurzhanski and T.F. Filippova, On the Theory of Trajectory Tubes — a Mathematical Formalism for Uncertain Dynamics, Viability and Control, *Advances in Nonlinear Dynamics and Control: a Report from Russia, Progress in Systems and Control Theory, (A.B. Kurzhanski, (Ed))*, Vol.17, Birkhauser, Boston, 1993, pp.22–188.

[6] A.B. Kurzhanski and I. Valyi, *Ellipsoidal Calculus for Estimation and Control.* Birkhauser, Boston, 1997.

[7] T.F. Filippova, State Estimation Problem for Impulsive Control Systems, *Proc. 10th Mediterranean Conference on Automation and Control*, Lisbon, Portugal, 2002.

[8] F.L. Pereira and T.F. Filippova, On a Solution Concept to Impulsive Differential Systems, *Proc. of the 4th MathTools Conference*, S.-Petersburg, Russia, 2003.

[9] L.E. Ramos-Velasco, S. Celikovsky, V. Lopez-Morales and and V. Kucera, Generalized Output Regulation for a Class of Nonlinear Systems via the Robust Control Approach, *Proceedings of the WSEAS Multiconference MATH2004*, Maiami, Florida, April 21-23, 2004.

[10] R. Rishel, An Extended Pontryagin Principle for Control System whose Control Laws Contain Measures, *SIAM J. Control*, 1965.

[11] A.F. Filippov, *Differential Equations with Discontinuous Right-hand Side*, Nauka, Moscow, 1985.

[12] J.-P. Aubin and H. Frankowska, *Set-Valued Analysis*, Birkhauser, Boston, 1990.

[13] J.-P. Aubin and I. Ekeland, *Applied Nonlinear Analysis*, N.-Y., John Wiley & Sons, 1984.

[14] A. Bressan and F. Rampazzo, Impulsive Control Systems without Commutativity Assumptions, *J. of Optimization Theory and Applications.* Vol.81,1994.

[15] T.F. Filippova, On the State Estimation Problem for Impulsive Differential Inclusions with State Constraints, *Nonlinear Control Systems, NOLCOS'2001, Preprints of the 5th IFAC Symposium*, S.Petersturg, Russia, 2001.

[16] B. Miller, Optimization of Dynamic Systems with Generalized Control. *Aut. i Telemekhanika*, No.6, 1989, pp. 22–34.

[17] F.L. Pereira and G.N. Silva, Necessary Conditions of Optimality for Vector-Valued Impulsive Control Problems, *Systems and Control Letters*, Vol. 40, 2000, pp.205–215.

[18] M. Motta and F. Rampazzo, Dynamic programming for nonlinear systems driven by ordinary impulsive controls. *SIAM J. Control Optim.*, 1996.

[19] S.T. Zavalischin and A.N. Sesekin, *Impulsive Processes. Models and Applications*, Nauka, Moscow, 1991.

[20] E.K. Kostousova and A.B. Kurzhanski, Theoretical Framework and Approximation Techniques for Parallel Computation in Set-membership State Estimation, *Proc. of the Symposium on Modelling Analysis and Simulation, Lille, France, July 9-12, 1996*, Vol. 2, 1996, pp.849–854.

[21] A. C. Matos and F. L. Pereira, Hamilton-Jacobi Conditions for a Class of Impulsive Control Problems. *10th Mediterranean Conference on Control and Automation*, Lisboa, Portugal, July 13-17, 2002.

[22] R.B. Vinter and F.M.F.L. Pereira, A maximum principle for optimal processes with discontinuous trajectories , *SIAM J. Contr. and Optimization*, Vol.26, No.1, 1988, pp. 155–167.

[23] V.A. Dykhta and O.N. Sumsonuk, A maximum principle for optimal impulsive processes and it's applications, *Proceedings of 4th European Control Conference*, Brussels, FR-A-D3, 1997.

[24] G.N. Silva and R.B. Vinter, Measure differential inclusions, *J. Math. Anal. Appl.*, 202, pp. 727–746, 1996.