Approach of Adaptive Control for a Class of Nonlinear Systems Using Fuzzy Approximator

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Abstract:– Based on the Lyapunov synthesis approach and regarding the fuzzy system as approximator to approximate unknown functions in the system to be controlled, several adaptive control schemes have been developed during the last decade or more. Actually, (i) most of them just consider SISO systems (which can avoid the challenge of the coupling between control inputs); (ii) the system state have been involved in the fuzzy controller directly (in this way, there is no need to consider the problem of state observer). This paper develops a design methodology for a class of MIMO nonlinear systems with state observer. The overall adaptive scheme is shown to guarantee the tracking error, between the outputs of system and the desired values, to be asymptotical in decay, while maintaining all singles involved stable and forcing the estimated state to follow the real state rapidly.

Key-Words:– Stability, State observer, Fuzzy approximator, Adaptive law.

1 Introduction

Over the last decade or more, beside the traditional adaptive control and sliding model control techniques, neural control, fuzzy control have been appearing strongly capable in a large amount of research and industrial applications. The motivation is often that they provide an alternative to the traditional modeling and design of control systems where system knowledge and dynamic models in the traditional sense are uncertain and time varying. Actually as shown in [1][2], no matter either fuzzy control or neural control, the system, rather than "control", is used as the parameterized approximator that is finally expressed as a series of radial function (RBF) expansion due to its excellent approximation properties. A key element of this success has been the merger of adaptive system theory with approximation theory [3], where the unknown plants are approximated by parameterized approximators. This is why a large amount of research on adaptive control involving fuzzy approximator has appeared in this research field since the early 1990's.
On the other hand, most of adaptive control systems, where fuzzy approximator, is used proposed so far involve the system state directly, and which is sometimes unavailable especially in a nonlinear system. Although some encouraging challenges have been carried out using the concept of *fuzzy state observer* [7]-[8], there is still not a clear clue that shows the connection between the real control state, observer, fuzzy approximator, and the system controller. This greatly limits the flexibility of fuzzy control system to be applied to a practical system. Another concern over the proposed adaptive control systems is that they mostly pay attention on the single-input/single-out (SISO) systems, and the system design for which can not be extended to a mutiple-inputs/mutiple-outputs (MIMO) system straightforwardly. Therefore, the system design should be built based on MIMO system in terms of system applicability.
In this paper, our goal is to show an approach of adaptive control for a class of MIMO nonlinear systems with disturbance, in which state observer is proposed instead of using the unavailable system state. Considering the part closely related with the state observer as a subsystem, the whole system behavior, thus, becomes a standard singularly perturbed form [9]-[11], in which the gap between the real state and its corresponding value from the state observer decays to order $O(\epsilon)$ [1] in a very fast speed by choosing an arbitrarily small constant ϵ. Also until then, the gap is treated as part of system disturbance. To deal with the reconstruction error regarding the fuzzy approximator, we adopt a switching function with an alterable coefficient, which is tuned by an adaptive law based on the tracking error, in stead of the upper bound assumptions as well as treating the disturbance. The adaptive law to adjust all parameters will be developed based on the Lyapunov synthesis approach. It is shown that the proposed fuzzy controller guarantees the tracking error, between the output of the considered system and the desired value, to be shrunken to zero, while maintaining all signals involved in the system stable, and forcing the estimated state from the state observer to follow the real state rapidly.

[1]A vector function $f(t,\epsilon) \in \mathcal{R}^n$ is said to be $O(\epsilon)$ over an interval $[t_1,\ t_2]$ if there exist positive constants k and ϵ^* such that

$$||f(t,\ \epsilon)|| \leq k\epsilon, \quad \epsilon \in [0,\ \epsilon^*], \quad t \in [t_1,\ t_2]$$

where $||\cdot||$ is the Euclidean norm [12].

2 Problem Statement

Consider the following MIMO continuous-time nonlinear system:

$$\mathbf{x}^{(n)} = \mathscr{F}(\mathbb{X}) + \mathscr{B}(\mathbb{X})\mathbf{u} + \mathscr{D} \tag{1}$$

where $\mathbb{X} = [X_1^T, X_2^T, \ldots, X_m^T] \in R^{\sum_{i=1}^m n_i}$ with $X_i = [x_i, \dot{x}_i, \ldots, x_i^{(n_i-1)}]^T \in R^{n_i}$ being ith subsystem state vector is the global state vector, $\mathscr{F} = [f_1, f_2, \ldots, f_m]^T \in R^m$ is a vector function, $\mathscr{B} = [b_{i,j}] \in R^{m\times m}$ is the control gain matrix, both $\mathscr{F}$ and $\mathscr{B}$ are unknown nonlinear functions of the system state vector. $\mathbf{u} = [u_1, u_2, \cdots, u_m]^T$ is the control vector. $\mathbf{x}^{(n)} = [x_1^{(n_1)}, x_2^{(n_2)}, \ldots, x_m^{(n_m)}]^T \in R^m$. $\mathscr{D} = [d_1, d_2, \ldots, d_m]^T \in R^m$ is an uncertainty vector.

Let $\mathbf{x}_d = [x_{d_1}, x_{d2}, \ldots, x_{dm}]^T \in R^m$ be a desired trajectory vector and define the tracking error vector,

$$\tilde{\mathbf{x}} = \mathbf{x} - \mathbf{x}_d \tag{2}$$

where $\mathbf{x} = [x_1, x_2, \ldots, x_m]^T \in R^m$. The problem we consider in this paper is to design a controller vector $\mathbf{u}$ for (4) which ensures the tracking error vector (2) to be uniformly ultimately bounded, also the ultimate bound should be made arbitrarily small by choosing appropriately control parameters, while maintaining all signals in the system uniformly bounded.

Here, we rewrite system (1) in a more general form as

$$x_i^{(n_i)} = f_i(\mathbb{X}) + \sum_{j=1}^{m} b_{i,j}(\mathbb{X})u_j + d_i \tag{3}$$

where $i = 1, 2, \ldots, m$. (3) is referred to as ith subsystem, which is corresponding to the independent coordinate x_i. Also, ith subsystem (3) can be rewritten in state-space representation,

$$\begin{cases} \dot{\mathcal{X}}_i = A_i\mathcal{X}_i + B_i\left(\sum_{j=1}^m b_{i,j}(\mathbb{X})u_j + d_i\right) \\ y_i = C_i^T\mathcal{X}_i \end{cases} \tag{4}$$

where,

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \in R^{n_i\times n_i}$$

$$B^T = [0\ 0\ \ldots\ 0\ 1] \in R^{n_i}$$

$$C^T = [1\ 0\ \ldots\ 0\ 0] \in R^{n_i}$$

and $\mathcal{X}_i^T = [x_{i,1},\ x_{i,2},\ \ldots,\ x_{i,n_i}] = \left[x_i,\ \dot{x}_i,\ \ldots,\ x_i^{(n_i-1)}\right]$ where not all $x_{i,l}$ ($l = 1, 2, \ldots, n_i$) are assumed to be available for measurement in this paper. In the remainder of this paper, the ranges for subscript i, and l are $1 \sim m$, and $1 \sim n_i$, respectively, unless it specifies.

Thus, for ith subsystem (3) the problem becomes developing u_i that ensures the tracking error,

$$\tilde{x}_i = y_i - x_{di} \tag{5}$$

to be uniformly ultimately bounded and the whole sytem's stability.

The nonlinear functions f_i and $b_{i,j}$ in (3) are unknown, so before developing our control system we have to solve the problem of approximating f_i and $b_{i,j}$. In the following section, it will be shown that using fuzzy IF-THEN rules, the unknown functions f_i and $b_{i,j}$ can be approximated by some parameterized fuzzy approximators.

To proceed with our development, we state our assumption on the system.

Assumption : The control gain $b_{i,j}$ is finite, nonzero, and of known sign for all $\mathbb{X}$; without loss of generality this sign can be taken as positive. The functions $\frac{d}{dt}(1/b_{i,i})$, and d_i are bounded.

It should be noted that here in this paper, we just suppose that the boundedness of $\frac{d}{dt}(1/b_{i,i})$, and d_i is existent, and its each real boundary does not need to be known in the development of control system.

3 Control System Approach

3.1 Fuzzy Approximator

The fuzzy model addresses the imprecision of the input and output variables directly by defining them with fuzzy sets in the form of membership functions. The basic configuration of the fuzzy model includes a fuzzy rules base, which consists of a collection of IF-THEN fuzzy rules. Now, we consider a fuzzy model with *singleton consequent, product inference*, *Gaussian membership function* in the antecedent, and *central average defuzzifier*, hence, such a fuzzy model can be written as

$$\mathcal{F}(Z) \quad = \quad W^T \cdot G(Z) \tag{6}$$

where $Z^T = [z_1, z_2, \ldots, z_n]$, $W^T = [w_1, w_2, \ldots, w_N]$ with N being the number of fuzzy rules; $G^T(Z) = [g_1(Z), g_2(Z), \ldots, g_N(Z)]$ with $g_j(Z) = \frac{\prod_{i=1}^n \mu_{A_j^i}(z_i)}{\sum_{j=1}^N \prod_{i=1}^n \mu_{A_j^i}(z_i)}$ where $\mu_{A_j^i}(z_i)$ is a Gaussian membership function, defined by

$$\mu_{A_j^i}(z_i) = \exp\left[-\left(\frac{z_i - \xi_j^i}{\sigma_j^i}\right)^2\right] \tag{7}$$

where ξ_j^i indicates the position, and σ_j^i indicates the variance of the membership function.

We now can show an important property of the fuzzy system above. As shown by Wang *et al* [1], the fuzzy system has the same pattern as a neural network. Exactly as a neural network, which has powerful abilities of learning and approximation, a fuzzy system with the Gaussian membership is capable of uniformly approximating any well-defined nonlinear function over a compact set U to any degree of accuracy. The following theorem [2] theoretically supports this claim.

Theorem 1 *For any given real continuous function f on the compact set $U \in \mathcal{R}^n$ and arbitrary ε^*, there* *exists an optimal fuzzy system expansion $\mathcal{F}^*(Z) = W^{*T} \cdot G(Z)$ such that*

$$\sup_{Z \in U} |f - \mathcal{F}^*(Z)| \leq \varepsilon^* \tag{8}$$

This theorem states that the fuzzy system (6) is a universal approximator on a compact set.

3.2 State Observer

To deal with the unknown functions such as $f_i(\mathbb{X})$, we will employ the fuzzy approximator above to estimate them. In the fuzzy approximator, as the input variables the system state is often used. However, as mentioned previously, not all $x_{i,l}$ in $\mathcal{X}_i$ are assumed to be available for measurement in this paper, therefore, first of all we have to design a state observer. We estimate the state $x_{i,l}$ using the observer

$$\begin{cases} \dot{\hat{x}}_{i,l} = \hat{x}_{i,l+1} + \frac{\alpha_{i,l}}{\epsilon^l}(y_i - \hat{x}_{i,1}), \quad l = 1, \dots, n_i - 1 \\ \dot{\hat{x}}_{i,n_i} = \frac{\alpha_{i,n_i}}{\epsilon^{n_i}}(y_i - \hat{x}_{i,1}) \end{cases} \tag{9}$$

where ϵ is a positive parameter to be specified. The positive constant $\alpha_{i,l}$ is chosen such that the roots of

$$s^{n_i} + \alpha_{i,1} s^{n_i - 1} + \cdots + \alpha_{i,n_i-1} s + \alpha_{i,n_i} = 0 \tag{10}$$

have negative real parts. Like (4), the state observer (9) can be rewritten in state-space representation,

$$\dot{\hat{\mathcal{X}}}_i = A_i \hat{\mathcal{X}}_i + D_i(\epsilon) L_i C_i^T (\mathcal{X}_i - \hat{\mathcal{X}}_i) \tag{11}$$

where,

$$D_i(\epsilon) = \begin{bmatrix} \frac{1}{\epsilon} & & & \\ & \frac{1}{\epsilon^2} & & \\ & & \ddots & \\ & & & \frac{1}{\epsilon^{n_i}} \end{bmatrix} \in R^{n_i \times n_i},$$

$$L_i^T = [\alpha_{i,1},\ \alpha_{i,2},\ \dots,\ \alpha_{i,n_i}] \in R^{n_i},$$

$$\hat{\mathcal{X}}_i^T = [\hat{x}_{i,1},\ \hat{x}_{i,2},\ \dots,\ \hat{x}_{i,n_i}] = \left[\hat{x}_i,\ \dot{\hat{x}}_i,\ \dots,\ \hat{x}_i^{(n_i-1)}\right].$$

Now, we define a matrix $N_i(\epsilon) \in R^{n_i \times n_i}$ as follows.

$$N_i(\epsilon) = \begin{bmatrix} \epsilon^{n_i-1} & & & \\ & \epsilon^{n_i-2} & & \\ & & \ddots & \\ & & & \epsilon^0 \end{bmatrix}.$$

Using the special structure of the matrices $N_i(\epsilon)$, $D_i(\epsilon)$, B_i, C_i, and L_i, it can be shown that

$$\begin{cases} N_i^{-1}(\epsilon) B_i = B_i \\ N_i^{-1}(\epsilon)\left[A_i - D_i(\epsilon) L_i C_i^T\right] N_i(\epsilon) = \frac{1}{\epsilon}\left[A_i - L_i C_i^T\right] \end{cases} \tag{12}$$

Let

$$e_{i,l} = \frac{1}{\epsilon^{n_i - l}}(x_{i,l} - \hat{x}_{i,l}) \tag{13}$$

and $E_i^T = [e_{i,1}, e_{i,2}, \dots, e_{i,n_i}] = \left[e_i, \dot{e}_i, \dots, e_i^{(n_i-1)}\right]$, which is the scaling estimation error. Thus, we have

$$E_i = N_i^{-1}(\epsilon)\left(\mathcal{X}_i - \hat{\mathcal{X}}_i\right) \tag{14}$$

From (4), (11), (12), and (14), it follows that

$$\dot{E}_i = \frac{1}{\epsilon}\left(A_i - L_i C_i^T\right) E_i + B_i \left(\sum_{j=1}^m b_{i,j}(\mathbb{X}) u_j + d_i\right) \tag{15}$$

where the characteristic equation of matrix $(A_i - L_i C_i^T)$ is (10). Including the state observer, at this stage the whole system behavior is dominated by (4), and (15). This is the standard singularly perturbed form. If attention is paid to term $\frac{1}{\epsilon}$ in (15), we can easily see that the evolutions of $\mathcal{X}_i$, and E_i do have a absolutely different transient speed, in which (15) is called the *fast model* whereas (4) is called the *slow model* [10]. In the following subsection, we will take advantage of the evolutions' different transient speeds to develop the fuzzy control system.

3.3 Structure of Controller

In this paper, we adopt the variable structure theory to construct our adaptive fuzzy control system. The sliding mode hyperplane is firstly defined as

$$s_i = \left(\frac{d}{dt} + \lambda\right)^{n_i - 1} \tilde{x}_i \quad \text{with } \lambda > 0 \tag{16}$$

where λ defines the bandwidth of the error dynamics of the system. The equation defines a time-varying hyperplane in $\mathcal{R}^{n_i}$ on which the tracking error $\tilde{x}_i$ decays exponentially to zero, so that perfect tracking can be asymptotically obtained by maintaining this condition. In this case the control objective becomes the design of a controller that ensures $s_i = 0$. The time derivative of the error metric can be written as

$$\begin{aligned} \dot{s}_i \quad &= \quad \sum_{j=1}^m b_{i,j} u_j + d_i + f_i \\ &\quad -x_{di}^{(n_i)} + \Lambda_i^T \tilde{\mathcal{X}}_i \end{aligned} \tag{17}$$

where $\Lambda_i^T = \left[0, \lambda^{n_i-1}, (n_i-1)\lambda^{n_i-2}, \dots, (n_i-1)\lambda\right]$, $\tilde{\mathcal{X}}_i = \mathcal{X}_i - X_{di}$, $X_{di}^T = \left[x_{di}, \dot{x}_{di}, \dots, x_{di}^{(n_i-1)}\right]$. Referring to system (3), it naturally suggests that when $b_{i,j}$, and f_i are known, a controller of the form

$$\begin{aligned} u_i \quad &= \quad b_{ii}^{-1}\Bigg[-k_d s_i - f_i + a_i \\ &\quad - \sum_{\substack{j=1 \\ j \neq i}}^m b_{i,j} u_j - d_i^* \cdot \mathrm{sgn}(s_i)\Bigg] \end{aligned} \tag{18}$$

where $k_d > 0$, $|d_i| \leq d_i^*$ with d_i^* being the boundary of d_i, $a_i = x_{di}^{(n_i)} - \Lambda_i^T \tilde{\mathcal{X}}_i$, leads to $\dot{s}_i \leq -k_d s_i$, and hence, $\tilde{x}_i \to 0$ as $t \to \infty$. However, the problem is how u_i can be determined when $b_{i,j}$, and f_i, as well as the upper boundary d_i^* for d_i, are unknown. What is more, the state X and the sliding mode s_i involving $\mathcal{X}_i$ can not be used in the controller directly due to the problem with $\mathcal{X}_i$'s measurement.

Using the estimated state $\hat{\mathcal{X}}_i$ instead of $\mathcal{X}_i$, we define a new sliding mode hyperplane

$$\hat{s}_i = \left(\frac{d}{dt} + \lambda\right)^{n_i - 1} (\hat{x}_i - x_{di}) \tag{19}$$

Using the relation $\hat{x}_i = x_i - \epsilon^{n_i-1} e_i$ from (14), $\hat{s}_i$ can be rewritten as

$$\hat{s}_i = s_i - \epsilon^{n_i-1}\Lambda_{1i}^T E_i \tag{20}$$

where $\Lambda_{1i}^T = \left[\lambda^{n_i-1}, (n_i-1)\lambda^{n_i-2}, \ldots, (n_i-1)\lambda, 1\right]$. Taking the time derivative of both sides of (20), it follows that

$$\dot{\hat{s}}_i = f_i + \sum_{j=1}^{m} b_{i,j} u_j - \hat{a}_i + d_{ei} \tag{21}$$

where $\hat{a}_i = x_{di}^{(n_i)} - \Lambda_i^T \left(\hat{\mathcal{X}}_i - X_{di}\right)$, and

$$d_{ei} = d_i + \Lambda_i^T \left(N_i(\epsilon) - \epsilon^{n_i-1} I\right) E_i. \tag{22}$$

in which the gap between $x_i^{(n_i)}$ and $\hat{x}_i^{(n_i)}$ can be covered by disturbance d_i. In other words, the gap can be viewed as a part of the disturbance that is unknown but dealt with by an adaptive law in this paper.
As shown latter on, the scaling estimation error E_i is bounded. Considering *Assumption* in section 2 stipulating the boundedness of disturbance d_i and E_i's boundedness, the inequality

$$|d_{ei}| \leq d_{ei}^* \tag{23}$$

is satisfied for some d_{ei}^*. As mentioned in the assumption, the value of d_{ei}^* is not involved in the system design, therefore, it is no necessary to know the value. To proceed with the system development, one task left is how to deal with the unknown functions $b_{i,j}$, and f_i. Here in this paper we employ the fuzzy approximator described previously to estimate them using the estimated state $\hat{\mathcal{X}}_i$. For the convenience of system development, we use the fuzzy approximator to estimate newly transformed functions $g_i = 1/b_{i,i}$, $h_i = f_i * g_i$, and $q_{i,j} = b_{i,j} * g_i$ $(j \neq i)$ instead of $b_{i,j}$, and f_i. Let us denote $g_i^*(\hat{\mathbb{X}}) = W_{gi}^{*T} G_{gi}(\hat{\mathbb{X}}), h_i^*(\hat{\mathbb{X}}) = W_{hi}^{*T} G_{hi}(\hat{\mathbb{X}})$, and $q_{i,j}^*(\hat{\mathbb{X}}) = W_{qij}^{*T} G_{qij}(\hat{\mathbb{X}})$ to be the optimal fuzzy approximators of the unknown functions g_i, h_i, and $q_{i,j}$, respectively. According to Theorem 1, there are some small positive values $\varepsilon_{gi}^*, \varepsilon_{hi}^*$ such that the errors,

$$\varepsilon_{gi} = g_i - g_i^* \tag{24}$$

$$\varepsilon_{hi} = h_i - h_i^* \tag{25}$$

which are referred to as *reconstruction errors*, satisfy the following inequalities,

$$|\varepsilon_{gi}| \leq \varepsilon_{gi}^* \tag{26}$$

$$|\varepsilon_{hi}| \leq \varepsilon_{hi}^* \tag{27}$$

Similarly, the reconstruction error for $q_{i,j}$ is defined as

$$\varepsilon_{qij} = \left(q_{i,j} - q_{i,j}^*\right) u_j \tag{28}$$

In the above expression, control input u_j is involved. In a control system, the boundedness for the control input, which can be confirmed in (31) and later development, is a basic requirement. Therefore, like (26) and (27), here it is reasonable to assume that ε_{bij} is bounded by a constant ε_{qij}^*:

$$|\varepsilon_{qij}| \leq \varepsilon_{qij}^* \tag{29}$$

And, one fact is that, according to *Assumption* in section 2, the time derivative of g_i, $\frac{d}{dt} g_i$, is supposed to be bounded in this paper. Therefore, there is a d_{gi}^* such that

$$|\frac{d}{dt} g_i| \leq d_{gi}^* \tag{30}$$

We also should note that the values of $\varepsilon_{gi}^*, \varepsilon_{hi}^*, \varepsilon_{qij}^*$, and d_{gi}^* do not need to be specified in this paper.

However, the optimal vectors W_{gi}^*, W_{hi}^*, and W_{qij}^* in the optimal fuzzy approximators are also unknown, so their estimates, denoted $\hat{g}_i(\hat{\mathbb{X}}) = \hat{W}_{gi}^T G_{gi}(\hat{\mathbb{X}}), \hat{h}_i(\hat{\mathbb{X}}) = \hat{W}_{hi}^T G_{hi}(\hat{\mathbb{X}})$, and $\hat{q}_{i,j}(\hat{\mathbb{X}}) = \hat{W}_{qij}^T G_{qij}(\hat{\mathbb{X}})$ are adopted.

Inspired by the control structure in (18), our fuzzy controller is now described as

$$u_i = u_{fdi} + u_{fzi} + u_{sdi} \tag{31}$$

where u_{fdi}, u_{fzi}, and u_{sdi} are an error's feedback component, fuzzy component and sliding component, respectively. The error's feedback component u_{fdi}, concretely expressed by,

$$u_{fdi} = -k_d \hat{s}_i - \frac{1}{2}\hat{d}_{gi}\hat{s}_i, \qquad k_d > 0 \tag{32}$$

is a kind of feedback of tracking error $(\hat{x}_i - x_{di})$, in which the coefficient $\hat{d}_{gi}$ is the estimate of d_{gi}^* in (30), and is tuned by,

$$\dot{\hat{d}}_{gi} = \frac{1}{2}\gamma_{d_g}\hat{s}_i^2, \qquad \gamma_{d_g} > 0 \tag{33}$$

The fuzzy component u_{fzi}, expressed by,

$$\begin{aligned} u_{fzi} \;=\; & -\hat{h}_i + \hat{g}_i\hat{a}_i - \sum_{\substack{j=1 \\ j\neq i}}^{m} \hat{q}_{i,j} u_j \\ & - \left(\hat{\varepsilon}_{hi} + \hat{\varepsilon}_{gi}|\hat{a}_i| + \sum_{\substack{j=1 \\ j\neq i}}^{m} \hat{\varepsilon}_{qij}|u_j|\right) \mathrm{sgn}(\hat{s}_i) \end{aligned} \tag{34}$$

will cover the unknown functions $g_i, h_i, q_{i,j}$, and attempt to compensate the estimating errors. At the same time, the adaptive laws are synthesized by

$$\dot{\hat{W}}_{hi} = \Gamma_{hi} G_{hi}(\hat{\mathbb{X}})\hat{s}_i \tag{35}$$

$$\dot{\hat{W}}_{gi} = -\Gamma_g G_{gi}(\hat{\mathbb{X}})\hat{a}_i\hat{s}_i \tag{36}$$

$$\dot{\hat{W}}_{qij} = \Gamma_q G_{qij}(\hat{\mathbb{X}}) u_j \hat{s}_i \tag{37}$$

$$\dot{\hat{\varepsilon}}_{hi} = \gamma_h |\hat{s}_i| \tag{38}$$

$$\dot{\hat{\varepsilon}}_{gi} = \gamma_g |\hat{a}_i \hat{s}_i| \tag{39}$$

$$\dot{\hat{\varepsilon}}_{qii} = \gamma_q |u_j \hat{s}_i| \tag{40}$$

where $\hat{W}.$, and $\hat{\varepsilon}.$ are the estimates of $W_{\cdot}^*$, and $\varepsilon_{\cdot}^*$, respectively; $\Gamma.$, and $\gamma.$ are some appropriate symmetric positive definite matrices, or positive constants which determine the rates of adaptation.

The sliding component u_{sd}, expressed by,

$$u_{sdi} = -\hat{d}_{egi}\text{sgn}(\hat{s}_i) \tag{41}$$

copes with the disturbance d_i in (1). And the coefficient $\hat{d}_{eg}$, which is the estimate of $(d^*_{ei} \cdot g^*_i)$, is tuned by an adaptive law as follows:

$$\dot{\hat{d}}_{egi} = \gamma_{d_{eg}}|\hat{s}_i| \tag{42}$$

where $\gamma_{d_{eg}}$ is the rate of adaptation as well.

3.4 Analysis of Stability

Now, consider the following Lyapunov function candidate,

$$\begin{aligned} V_1 &= \frac{1}{2}\Bigg(g_i\hat{s}_i^2 + \frac{1}{\gamma_{d_g}}\tilde{d}^2_{gi} + \frac{1}{\gamma_h}\tilde{\varepsilon}^2_{hi} + \frac{1}{\gamma_g}\tilde{\varepsilon}^2_{gi} + \frac{1}{\gamma_{d_{eg}}}\tilde{d}^2_{egi} \\ &\quad + \sum_{\substack{j=1\\ j\neq i}}^{m} \tilde{W}^T_{qij}\Gamma_q^{-1}\tilde{W}_{qij} + \sum_{\substack{j=1\\ j\neq i}}^{m}\frac{1}{\gamma_q}\tilde{\varepsilon}^2_{qij} \\ &\quad + \tilde{W}^T_{hi}\Gamma_h^{-1}\tilde{W}_{hi} + \tilde{W}^T_{gi}\Gamma_g^{-1}\tilde{W}_{gi}\Bigg) \end{aligned} \tag{43}$$

where,

$$\tilde{d}_{gi} = d^*_{gi} - \hat{d}_{gi} \tag{44}$$
$$\tilde{\varepsilon}_{gi} = \varepsilon^*_{gi} - \hat{\varepsilon}_{gi} \tag{45}$$
$$\tilde{\varepsilon}_{hi} = \varepsilon^*_{hi} - \hat{\varepsilon}_{hi} \tag{46}$$
$$\tilde{d}_{egi} = d^*_{ei}g^*_i - \hat{d}_{egi} \tag{47}$$
$$\tilde{W}_{qij} = W^*_{qij} - \hat{W}_{qij} \tag{48}$$
$$\tilde{\varepsilon}_{qij} = \varepsilon^*_{qij} - \hat{\varepsilon}_{qij} \tag{49}$$
$$\tilde{W}_{hi} = W^*_{hi} - \hat{W}_{hi} \tag{50}$$
$$\tilde{W}_{gi} = W^*_{gi} - \hat{W}_{gi} \tag{51}$$

Using expressions (44-51), and adaptive law (33), (35-40), and (42) into the time derivative of the Lyapunove function candidate follows,

$$\dot{V}_1 < -k_d\hat{s}_i^2 \tag{52}$$

Therefore, all signals in (43), which also are signals involved in the system, are bounded, and $\hat{s}_i \to 0$, as $t \to \infty$, which also means $(\hat{x}_i - x_{di}) \to 0$, as $t \to \infty$. However, our goal is to drive, not the estimated state $\hat{x}_i$, but the real state x_i to follow the desired value. To this end, one way, the direct way, is to show x_i surely is following x_{di}, and another way, the indirect way, is to show that $\hat{x}_i$ is equal to x_i, the error between them decays to almost zero at a very fast speed. Here, we take the indirect approach. Let us pay attention to the fast model (15). Consider the following Lyapunov function candidate,

$$V_2 = E_i^T P_i E_i \tag{53}$$

where $P_i^T = P_i > 0$ is the solution of the Lyapnove equation,

$$P_i\left(A_i - L_iC_i^T\right) + \left(A_i - L_iC_i^T\right)_i^T P_i = -I_i \tag{54}$$

Based on the analysis above, the boundedness of controller (31) is guaranteed. Therefore, inequality

$$|f_i + b_iu_i + d_i| \le k_i \tag{55}$$

is satisfied for some $k_i \ge 0$ subject to *Assumption* stipulated in section 2. Substituting (54) into the time derivative of V_2, it follows that

$$\begin{aligned} \dot{V}_2 &\le -\frac{1}{\epsilon}||E_i||^2 + 2k_i||P_iB_i||||E_i|| \\ &\le -\frac{\gamma_i}{\epsilon}V_2, \qquad \text{if } V_2 \ge \epsilon^2\beta_i \end{aligned} \tag{56}$$

where $\gamma_i = \frac{1}{2\lambda_{max}(P_i)}$, and $\beta_i = 16||P_iB_i||^2k_i^2\lambda_{max}(P_i)$. This implies,

$$V_2 \le V_2(0)e^{-\frac{\gamma_i}{\epsilon}t} \tag{57}$$

We can get that there exists

$$\mathcal{T}_i = \frac{\epsilon}{\gamma_i}\ln\left(\frac{V_2(0)}{\epsilon^2\beta_i}\right) \tag{58}$$

such that for $t \ge \mathcal{T}_i$, V_2 satisfies

$$V_2 \le \epsilon^2\beta_i \tag{59}$$

From (53), we have $||E_i||^2 \le \frac{1}{\lambda_{min}(P_i)}V_2$. Therefore, it follows that

$$||E_i|| \le \mu_i\epsilon, \qquad \text{if } t \ge \mathcal{T}_i \tag{60}$$

where $\mu_i = \sqrt{\frac{\beta_i}{\lambda_{min}(P_i)}}$. Therefore, the scaling estimation error decays to the order $O(\epsilon)$. Since ϵ can be chosen arbitrarily small, we can make $\mathcal{T}_i$ in (58) arbitrarily small as well. Consequently, considering the relation in (14), we conclude the estimated state $\hat{\mathcal{X}}_i$ closely follows its real state $\mathcal{X}_i$ at a very fast speed, *i.e.,* $\hat{\mathcal{X}}_i \to \mathcal{X}_i$. Furthermore, we have $x_i \to x_{di}$.

4 Simulation Example

In order to verify the proposed design procedure, we apply the approach developed in previous section to the Duffing forced-oscillation system:

$$\ddot{x}(t) = -a\dot{x}(t) - bx^3(t) + c\cos(t) + u. \tag{61}$$

Its behavior is chaotic in unforced case, *i.e.,* $u = 0$. The unforced trajectory of the system is shown in Fig.1 in phase plane $(x, \dot{x})$ for $x(0) = \dot{x}(0) = 2, a = 0.11, b = 1, c = 12$, and time period $[0, 60]$. Now, we use the control approach proposed in this paper to force the state $x(t)$ to follow a desired trajectory $x_d(t) = \sin(t)$. In the phase plane, the desired trajectory is a unit circle: $x_d^2(t) + \dot{x}_d^2(t) = 1$. In this simulation, we choose the initial membership functions as shown in Fig.2 for both $x(t)$ and $\dot{x}(t)$.

Clearly, the two input variables lead to $7 \times 7 = 49$ fuzzy rules at most as follows:

$$R_j : IF\ x\ is\ A^1_j,\ \dot{x}\ is\ A^2_j\ THEN\ f\ is\ w_j$$

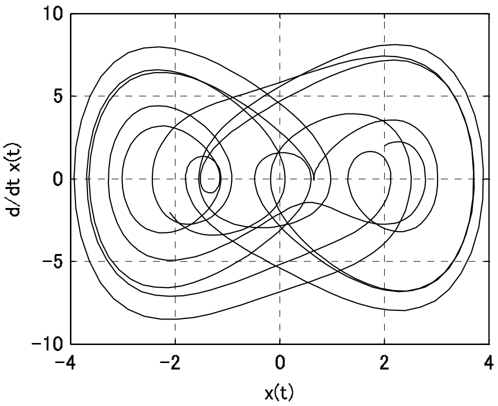

Figure 1: Unforced trajectories $(x, \dot{x})$

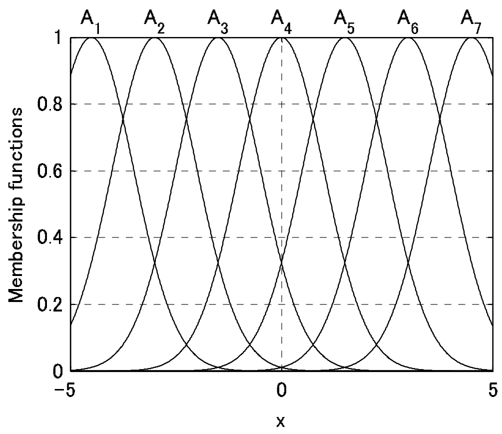

Figure 2: Membership functions in precedent

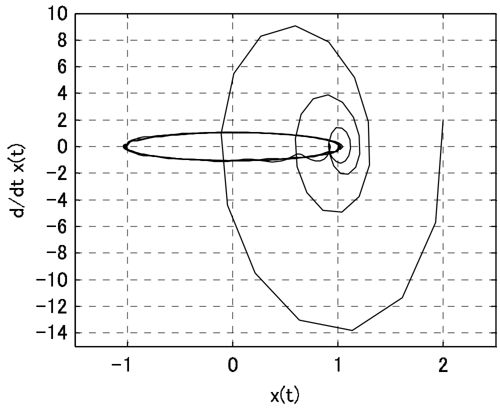

Figure 3: Controlled trajectories $(x, \dot{x})$

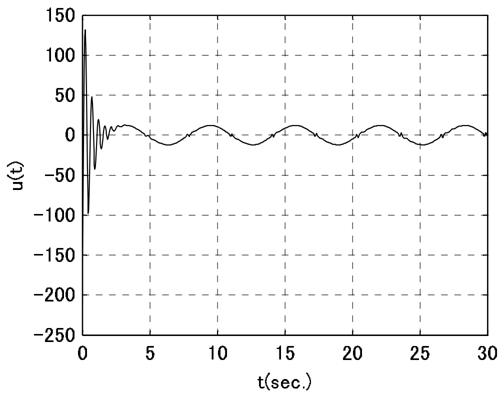

Figure 4: Amount of control law $u(t)$

To verify the control scheme, suppose that we have no knowledge regarding the function $f = -a\dot{x}(t) - bx^3(t) + c\cos(t)$, so the initial consequents w_j are selected randomly. Control law u in (31) was used. The error's feedback component $u_{fd}(t)$ is synthesized by (32), and (33) where $k_d = 1, \gamma_{d_g} = 0.2$. The fuzzy component $u_{fz}(t)$ is synthesized by (34), (35), and (38) where $\Gamma_h = 0.1I$ with I being an appropriate identity matrix, $\gamma_h = 0.2$. Further, the sliding component is determined by (41), and (42) where $\gamma_{d_{eg}} = 0.2$. In addition, we take the values that $\lambda = \alpha_1 = 1$, $\alpha_2 = 0.1$, $\varepsilon = 0.01$, $\Gamma_g = 0.1I$ and $\gamma_g = 0.2$ in this simulation.

Simulation result is shown in Figs.3-6. The closed-loop trajectories are depicted with the initial conditions $x(0) = \dot{x}(0) = 2$ in Fig.3. The control input $u(t)$ is shown in Fig.4. Further, the estimated errors of $(\hat{x}(t) - x(t))$, and $(\dot{\hat{x}}(t) - \dot{x}(t))$ are displayed in Fig.5, and Fig.6, respectively. We see that our control approach can handle well a system with some unknown time-variable facts such as $\cos(t)$ to track a time-varying desired trajectory.

We also should note that, when the tracking error $\tilde{x}(t)$ enters around the sliding surface, sign function $\mathrm{sgn}(\hat{s})$ begins working frequently so that such a control law (31) leads to control chattering. Chattering is undesirable in practice because it involves high control activity, and further may excite unmodeled high frequency plant dynamics. This problem can be eliminated by adopting a saturation function,

$$\mathrm{sat}\left(\frac{s}{\phi}\right) = \begin{cases} 1 & \frac{s}{\phi} \geq 1, \\ -1 & \frac{s}{\phi} \leq 1, \\ \frac{s}{\phi} & \text{otherwise.} \end{cases}$$

where ϕ is a little constant instead of $\mathrm{sgn}(s)$, and a smoothed sliding mode $s_\phi = s - \phi \cdot \mathrm{sat}(s/\phi)$ instead of $s(t)$ [6]. What's more, the system stability analysis is almost same with what appeared in this paper. Actually, the results given above were performed with s_ϕ, and $\mathrm{sat}(s/\phi)$ where $\phi = 0.01$.

5 CONCLUSION

In this paper, we proposed an adaptive controller for a class of nonlinear systems with state observer. The results achieved in this paper can be summarized in a theorem as follows.

Theorem 2 *If the plant (1), subject to Assumption in section 2, is controlled by (31-32), (34), and (41) with the adaptive law (33), (35-40), (42), and the state observer (9), then all signals involved in the control system will remain bounded, and the tracking error will asymptotically shrink to zero, at same time, the estimated state follows its real state.*

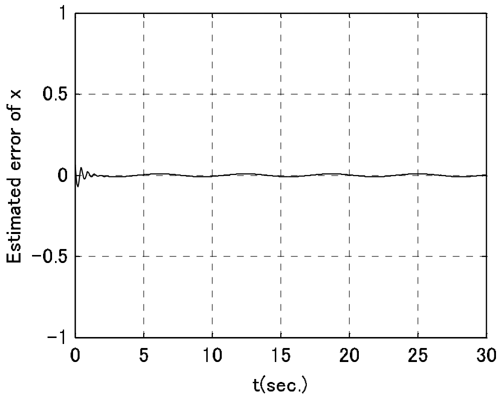

Figure 5: Estimated error $(\hat{x}(t) - x(t))$.

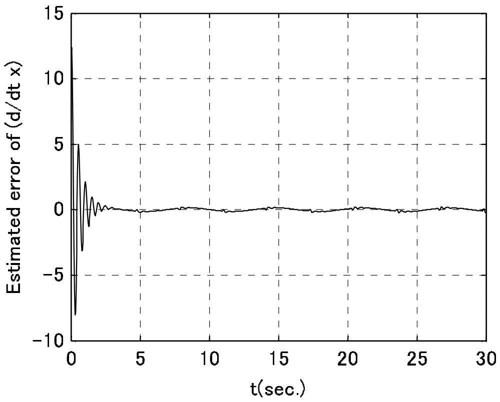

Figure 6: Estimated error $(\dot{\hat{x}}(t) - \dot{x}(t))$.

References:

[1] C.-H. Wang, H.-L. Liu, and T.-C. Lin, "Direct adaptive fuzzy-neural control with state observer and supervisory controller for unknown nonlinear dynamical systems," *IEEE Trans. on Fuzzy Systems,* vol. 10, no. 1, pp. 39-49, 2002.

[2] L.-X. Wang, and J. M. Mendel, "Fuzzy basis functions, universal approximation, and orthogonal least-squares learning," *IEEE Trans. on Neural Networks*, vol. 3, pp. 807-813, 1992.

[3] L.-X. Wang, "Stable adaptive fuzzy control of nonlinear systems," *IEEE trans. Fuzzy Systems,* vol. 1, no. 2, pp. 146-155, 1993.

[4] C.-Y. Su, and Y. Stepanenko, "Adaptive control of a class of nonlinear systems with fuzzy logic," *IEEE trans. Fuzzy Systems,* vol. 2, no.4, pp. 285-294, 1994.

[5] M. M. Polycarpou, and M. J. Mears, "Stable adaptive tracking of uncertain systems using nonlinear parameterized on-line approximators," *Int. J. Control*, vol. 73, no. 3, pp. 363-384, 1998.

[6] H. Han, C.-Y. Su, and Y. Stepanenko, "Adaptive control of a class of nonlinear systems with nonlinearly parameterized fuzzy approximators," *IEEE Trans. on Fuzzy Systems,* vol. 9, no. 2, pp. 315-323, 2001.

[7] X.-J. Ma, Z.-Q. Sun, and Y.-Y. He, "Analysis and design of fuzzy controller and fuzzy observer," *IEEE Trans. on Fuzzy Systems,* vol. 6, no. 1, pp. 41-51, 1998.

[8] S. Tong, and H.-X. Li, "Fuzzy adaptive sliding-mode control for MIMO nonlinear systems," *IEEE Trans. on Fuzzy Systems,* vol. 11, no. 3, pp. 354-360, 2003.

[9] Hassan K. Khalil, "Robust servomechanism output feedback controllers for feedback linearizable sytems," *Automatica,* vol. 30, no. 10, pp. 1587-1599, 1994.

[10] F. Esfandiari, and Hassan K. Khalil, "Output feedback stabilization of fully linearizable systems," *Int. J. Control,* vol. 56, no. 5, pp. 1007-1037, 1992.

[11] Hassan K. Khalil, "Adaptive output feedback control of nonlinear systems represented by input-output models," *IEEE Trans. on Automatic Control,* vol. 41, no. 2, pp. 177-188, 1996.

[12] Peter V. Kokotovic, Hassan K. Khalil, and John O'Reilly, *Singular Perturbation Methods in Control: Analysis and Design*, Academic Press, 1986.