Directional fractal dimension using parallel lines

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Abstract: - The concept of directional fractal dimension is introduced, based on the method of parallel lines which is applied to curves. The calculations are carried out on two perpendicular direction, considering the contours of the image. The results are applied to irregular structures and it is compared with the box-counting dimension in each case.

Key-Words: - Image processing, fractal dimension, box-counting, parallel lines, directional dimension, fractal anisotropy

1 Introduction

Several methods exist for the calculation of the dimension applied to fractal objects [1-5]. These methods can be theoretical or based in numerical implementations. Many authors have work on these aspects and Tricot carry out a compilation of the existent methods applied to fractal curves [6]. Also, the concept of generalized dimension was introduced through the work of Grassberger [7].

Among the numerical methods for the calculation of the fractal dimension, box-counting and parallel lines are two examples, but are the methods which here interest us. Using the parallel lines method, we consider lines along two perpendicular direction, which allows us to define a directional fractal dimension. This directionality is important when we wants to consider anisotropy in fractal structures [8,9] or the characterization of the same ones according to different directions. Our main interest has to do with the image processing.

2 Box-counting and parallel lines methods

The concept of fractal dimension is related to measurement of irregularities distribution in a complex structure. The concept “irregularities distribution” means that, for the case of structures generated through an iterative method (for example fractal objects), an initial pattern (or generator) can be observed at different scales superimposed (or included) successively with such initial pattern. Also, deterministic structures (such as polygons) can be included in such considerations, because only one iteration is necessary for their generation. Then, they are less irregular than other fractal structures. In this way, we can compare the irregularities distribution in each structure.

In the case of an image with different gray levels, a threshold can be established to binarize the image or to stand out the contours. Then, the selected method is applied on the contours of the obtained image. Box-counting dimension method can be used to calculate fractal dimension [1-5,10].

For a measure M_δ, with a number δ of divisions in the image F, we have the exponential law:

$$M_\delta(F) \sim c\,\delta^{-s} \tag{1}$$

being *c* y *s* constants, and s is the dimension of *F*, which is defined by:

$$s = -\lim_{\delta \to 0} \frac{\log(M_\delta(F))}{\log(\delta)} \tag{2}$$

If the images have irregularities in 3D, which represents a generalization of previous works, instead of two-dimensional boxes, then three-dimensional boxes must be considered over the surface.

In the parallel lines method, lines with a certain distance (*d*) between them are considered. Then it is computed the number of points (*M*) where such lines cut the irregular curve or the obtained contours of the original image. Finally, the values obtained *log(M) vs. log(d)*, in a similar form to the box-counting method (Eq. 2) are plotted, and the parallel lines fractal dimension is obtained.

3 Directional dimension

Here we will follow the umbralization processes between the values 30 and 255, establishing the contours for a certain image. Then, we apply the method of parallel lines in two perpendicular directions. Such process is shown in Figs. 1 and 2.

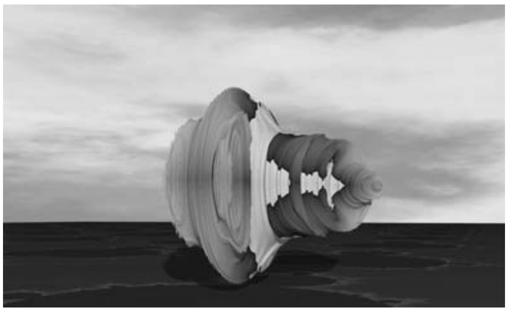


Figure 1 – Image of irregular structure

Fig. 1 is the original image with gray levels, and Fig. 2 shown the contours and the application procedure. The arrows with the numbers 1, 2, 3 represent the successive lines which are successively included in the process.

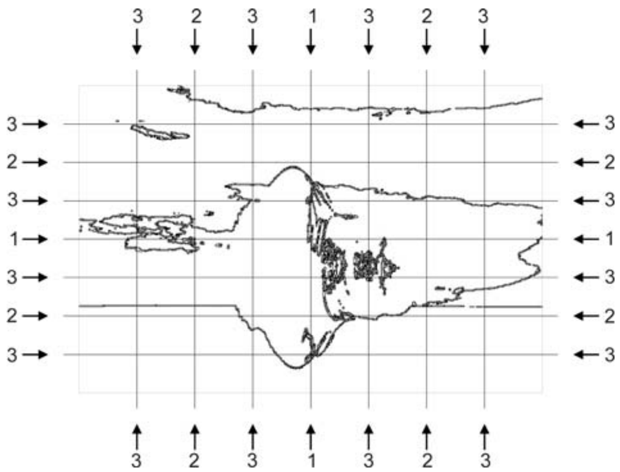


Figure 2 – The method of parallel lines in each direction.

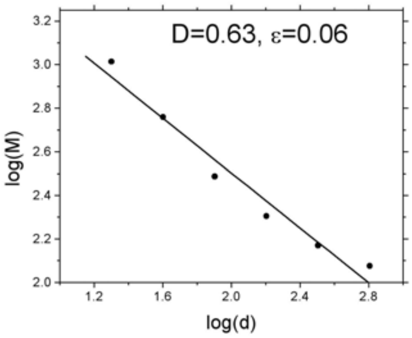
For the first pair of points to plot in Figs. 2a and 2b, the numbers of points M corresponds to the intersection of the line marked with 1 and the contours of the figure. The second point corresponds to the line marked with 1 and 2 and its intersection with the contours, and so on.

We also exemplify with the results obtained for the case of the Mandelbrot set. This fractal is obtained by means of iterative formula in the complex plane:

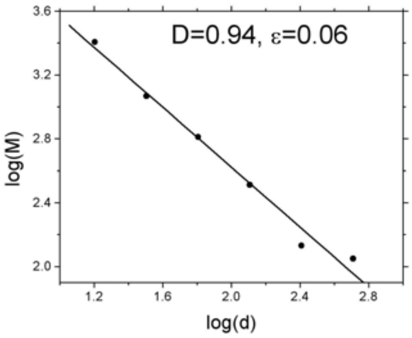
$$z_{n+1} = z_n^2 + C \tag{3}$$

being C a constant. The Mandelbrot set contains small copies of the *"main cardioid"* shown in Fig. 4. These small copies are connected with the main cardioid by filaments which are formed by other tiny cardioids. These structures are called the "Mandelbrot hair" or filaments [11,12].

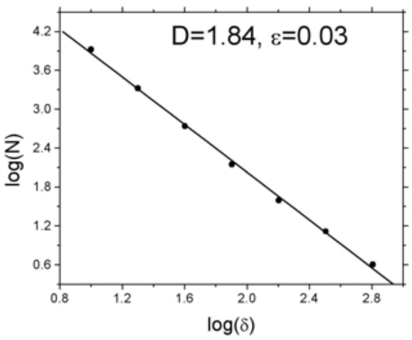
Starting from the results obtained for the case of an arbitrary irregular structure and for the Mandelbrot set, we can affirm that our method of dimensional characterization in two different directions is useful when there are anisotropies, and it can be important the directional property of the irregularities in the image. Although new studies are required, we can see that in both cases the sum of both directional dimensions is smaller than the box-counting dimension, which is clear if we think that the slope of the straight line will be smaller when it is evaluated the intersection between a line and the contours, since they tend to similar values when the boxes become very small or the lines are very close.



(a)



(b)



(c)

Figure 3 – Calculation of dimension for the image in Fig. 1: (a) vertical lines, (b) horizontal lines, (c) box-counting.

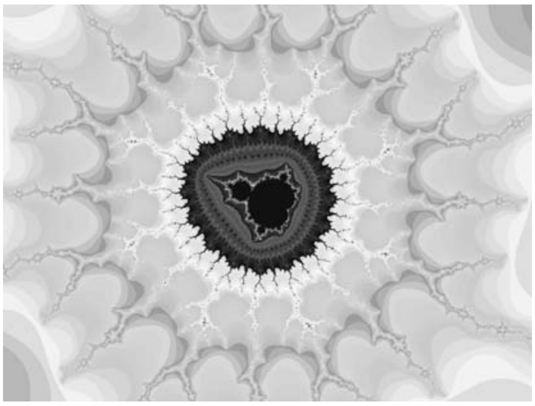
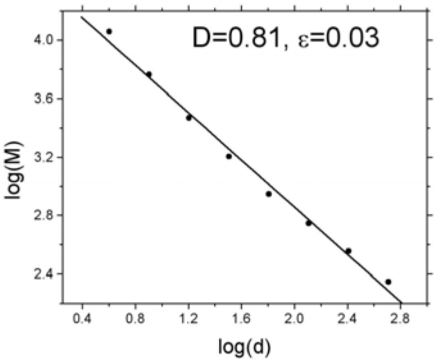
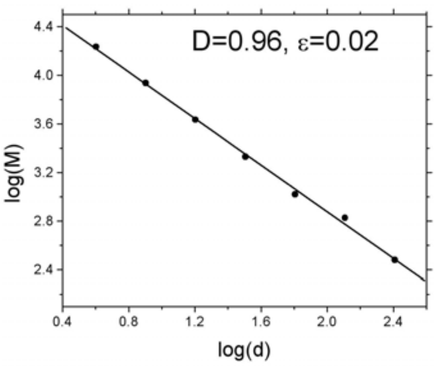


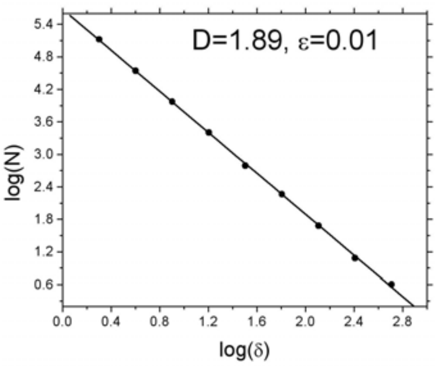
Figure 4 – Mandelbrot set.



(a)



(b)



(c)

Figure 5 – Calculation of dimension for Mandelbrot set: (a) vertical lines, (b) horizontal lines, (c) box-counting.

4 Conclusion

In this work we have presented a method for the calculation of the directional fractal dimension. This parameter is useful for the characterization of structures or systems with anisotropies or directional properties. This method is based on the well-known of parallel lines fractal dimension. Also, we compared the results with the box-counting method.

Acknowledgments

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