A New Fuzzy Lyapunov Controller for Nonholonomic Mobile Vehicles

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Abstract: - This paper presents a new fuzzy Lyapunov controller for nonholonomic mobile vehicles. A symbiosis between classical backstepping techniques and fuzzy logic was realized. The control system ensures a good robustness with respect to outside perturbations. These perturbations can interact with the vehicle. They are sources of uncertainty for the system model and can perturb the validity of the nonholonomic constraints. Therefore the trajectory tracking problem with noise is considered. The asymptotic stability of the fuzzy kinematical control system is guaranteed by Lyapunov's method. The algorithm efficiency, error minimization and noises reject are confirmed through simulation examples in Matlab environment.

Key-Words: - Backstepping, Fuzzy controllers, Nonholonomic systems, Trajectory tracking control, Lyapunov's stability.

1 Introduction

In recent years much attention has been focused upon the motion control of nonholonomic mechanical systems [2], [3]. The mobile wheeled vehicle is usually studied as a typical nonholonomic system. Many approach have been proposed to treat the motion control on nonholonomic vehicles. In particular in [6], [8] a model of the mobile vehicle by using of the Lagrange-Euler method is developed. In [5], [13] some technical for the synthesis of kinematic controllers are presented. In [13] a control method for the trajectory tracking problem of a nonholonomic mobile robot, using a kinematic model with linearization, is developed. In [5] a solution based on a discrete time sliding mode controller is presented. In [9] and [6] a dynamical extension is proposed by referring to the backstepping kinematics into dynamics and the torque of the single driving wheels is obtained. However all these jobs have proved to be not very effective towards the adaptability to the component of uncertainty which characterizes the model. The neural net and/or fuzzy logic offer a solution to this problem. In [7] and [8] a neural net inside the classical controller which allows to estimate the entity of the random component is used. In [14] a kinematic fuzzy logic controller and heuristic rules are presented.

In this paper a mixed controller which uses a backstepping computed torque dynamic controller [6] and a new fuzzy mechanism which compensates the unmodelled dynamics is proposed. Pratically the proposed control system takes into account the effects instead of the causes which can give rise to errors on the vehicle position. These errors perturb the nonholonomics constraints. The fuzzy mechanism compensates these perturbations. Also the stability of the new Fuzzy kinematical control system is shown by the Lyapunov's method.

This paper is organized as follows. In section 2 the kinematic and dynamic model of the nonholonomic vehicle is presented. In section 3 the trajectory tracking problem with noise is defined. Section 4 shows the steps of the fuzzy computed torque controller design and proof of Lyapunov's stability has been developed. In particular the Fuzzy inference mechanism is planned to guarantee the Lyapunov's stability. Section 5 presents simulation tests in Matlab environment. The performances of the backstepping classical controller [6] and the new fuzzy controller of this paper are compared.

2 Mobile vehicle and dynamic model

Let the mobile vehicle with two independent driving wheels be rigid moving on the plane (see Fig. 1). The kinematic parameters are: *P* (push center), *C* (mass center), *d*, *r* and *R*. Furthermore the vehicle is also characterized by the dynamic parameters: *m* (mass) and *I* (inertia).

A mobile vehicle system having an *n*-dimensional configuration space with generalized coordinates $\boldsymbol{q} = (q_1, q_2, \ldots, q_n)$ and subject to *m* constraints can be described by use of d'Alembert-Lagrange form [6]:

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{V}_m(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{F}(\dot{\boldsymbol{q}}) + \boldsymbol{G}(\boldsymbol{q}) + \boldsymbol{\tau}_d = \boldsymbol{B}(\boldsymbol{q})\boldsymbol{\tau} - \boldsymbol{A}^T(\boldsymbol{q})\boldsymbol{\lambda} \quad (1)$$

$$\boldsymbol{A}(\boldsymbol{q})\dot{\boldsymbol{q}} = \boldsymbol{0} \quad (2)$$

where:

$\boldsymbol{M(q)} \in \Re^{nxn}$ inertia matrix ;

$\boldsymbol{V_m(q,\dot{q})} \in \Re^{nxn}$ centripetal and Coriolis matrix;

$\boldsymbol{F(\dot{q})} \in \Re^{nx1}$ denotes the surface friction;

$\boldsymbol{G(q)} \in \Re^{nx1}$ gravitational vector;

$\boldsymbol{\tau_d} \in \Re^{rx1}$ bounded unknown disturbances including unstructured unmodelled dynamics;

$\boldsymbol{B(q)} \in \Re^{nxr}$ torque transformation matrix;

$\boldsymbol{\tau} \in \Re^{rx1}$ input vector containing driving wheels torque;

$\boldsymbol{A(q)} \in \Re^{mxn}$ matrix associated with the constraints;

$\boldsymbol{\lambda} \in \Re^{mx1}$ constraint forces vector.

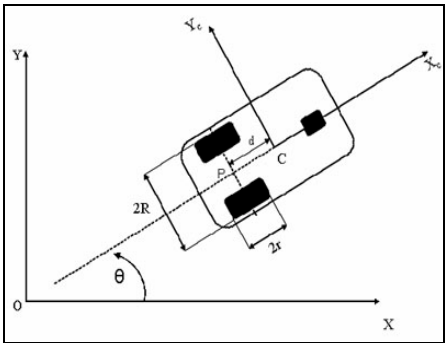

Fig.1 – Mobile vehicle

The tern {*O*, *X*, *Y*} (see Fig.1) is the earth fixed reference, while $\{C, X_c, Y_c\}$ is the body fixed reference. The position of the mobile vehicle in an inertial cartesian frame {*O*, *X*, *Y*} is completely specified by $\mathbf{q}=[x_c,\ y_c,\ \theta]^T$ if the body reference is fixed in C and $(x_c,\ y_c)$ are the coordinates of the reference point *C*, while is $\mathbf{q}=\begin{bmatrix}x_p & y_p & \theta\end{bmatrix}^T$ if the body reference is fixed in P. In any case θ is the orientation with respect to the inertial basis.Let $\mathbf{S(q)}$ be a full rank matrix (*n-m*) formed by a set of smooth and linearly independent vector fields spanning the null space of $\boldsymbol{A(q)}$, *i.e.*,:

$$\mathbf{S}^T(\mathbf{q})\mathbf{A}^T(\mathbf{q}) \tag{3}$$

according to (2) and (3), it is possible to find an auxiliary vector time function such that, for all *t*:

$$\dot{\mathbf{q}} = \mathbf{S(q)v}(t) \tag{4}$$

The nonholonomic constraint states that the vehicle can only move in the direction normal to the axis of the driving wheels. If the body reference is fixed in C, the mobile vehicle satisfies the conditions of pure rolling without slipping as follows [1], [16]:

$$\dot{y}_c \cos\theta - \dot{x}_c \sin\theta - d\dot{\theta} = 0 \tag{5}$$

The kinematic equations of motion of *C* in terms of its linear velocity and angular velocity are:

$$\begin{bmatrix}\dot{x}_c \\ \dot{y}_c \\ \dot{\theta}\end{bmatrix} = \begin{bmatrix}\cos\theta & -d\sin\theta \\ \sin\theta & d\cos\theta \\ 0 & 1\end{bmatrix}\begin{bmatrix}\upsilon \\ \omega\end{bmatrix} \tag{6}$$

Also, if the body reference is fixed in the point P (x_p, y_p) (see fig. 1) and C is the mass center, the kinematical model is:

$$\begin{bmatrix}\dot{x}_p & \dot{y}_p & \dot{\theta}\end{bmatrix}^T = \begin{bmatrix}\cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1\end{bmatrix}\begin{bmatrix}v \\ \omega\end{bmatrix} \tag{7}$$

Equation (6) or (7) can be written in form (4).
The following dynamical model is obtained:

$$\dot{\mathbf{q}} = \mathbf{S(q)\,v} \tag{8}$$

$$\overline{\boldsymbol{M}}\,\dot{\boldsymbol{v}} + \overline{\boldsymbol{V}}(\boldsymbol{v})\,\boldsymbol{v} + \overline{\boldsymbol{F}}(\boldsymbol{v}) + \tilde{\boldsymbol{\tau}}_d = \overline{\boldsymbol{B}}\boldsymbol{\tau} \tag{9}$$

where is:

$\boldsymbol{v}(t) \in \Re^{n-m}$ body fixed reference speeds vector;

$\overline{\boldsymbol{M}} \in \Re^{rxr}$ inertia matrix in body reference;

$\overline{\boldsymbol{V}}(\boldsymbol{v}) \in \Re^{rxr}$ Coriolis matrix in body reference;

$\overline{\boldsymbol{F}}(\boldsymbol{v}) \in \Re^{rx1}$ surface friction;

$\tilde{\boldsymbol{\tau}}_d \in \Re^{rx1}$ bounded unknown disturbances including unstructured unmodelled dynamics;

$\overline{\boldsymbol{B}} \in \Re^{rxr}$ torque transformation matrix;

$\boldsymbol{\tau} \in \Re^{r}$ input vector containing driving wheels torque.

Moreover if the body reference is fixed in C (see fig. 1) we have:

$$\overline{\boldsymbol{M}} = \begin{bmatrix}m & 0 \\ 0 & -md^2+I\end{bmatrix} \qquad \overline{\mathbf{V}} = \mathbf{0}$$

$$\overline{\boldsymbol{B}} = \frac{1}{r}\begin{bmatrix}1 & 1 \\ R & -R\end{bmatrix}$$

Also, if the body reference is fixed in P, it yields:

$$\overline{\mathbf{M}} = \begin{pmatrix}m & 0 \\ 0 & I\end{pmatrix} \qquad \overline{\boldsymbol{B}} = \frac{1}{r}\begin{bmatrix}1 & 1 \\ R & -R\end{bmatrix}$$

$$\overline{\mathbf{V}}(\mathrm{v}) = \begin{pmatrix}0 & -d\dot{\theta}m \\ d\dot{\theta}m & 0\end{pmatrix}$$

In any case the dynamical and kinematical parameters ($\overline{\boldsymbol{M}}$, $\overline{V}(\boldsymbol{v})$ and $\overline{\boldsymbol{B}}$) of the vehicle are exactly known, while the matrix $\overline{\boldsymbol{F}}(\boldsymbol{v})$ of the surface friction, is not well-known. Considering in a single term, $\overline{\boldsymbol{\tau}}_d$, all the uncertainty sources of the model, the following model form is proposed:

$$\dot{\mathbf{q}} = \mathbf{S\,v}$$

$$\overline{\mathbf{M}}\,\dot{\mathbf{v}} + \overline{\mathbf{V}}(\mathbf{v})\,\mathbf{v} + \overline{\boldsymbol{\tau}}_{\mathbf{d}} = \overline{\mathbf{B}}\boldsymbol{\tau} \tag{10}$$

Therefore the two most important features of model (10) are the nonlinearity and the high degree of uncertainty.

3 The trajectory tracking problem with noise

The trajectory tracking problem is definite as follows [4]:

Given a reference speed vector:

$$\boldsymbol{v_r}(t) = \left[\upsilon_r(t) \quad \omega_r(t)\right] \tag{11}$$

where

$\upsilon_r(t)$ is the linear velocity and $\omega_r(t)$ is the angular velocity, find a smooth velocity control input:

$$\boldsymbol{v_c}(t) = \boldsymbol{f_c}(\boldsymbol{e}(t), \boldsymbol{v}(t), \boldsymbol{K})$$

such that :

$$\lim_{t\to\infty}(\boldsymbol{q_r} - \boldsymbol{q}) = 0$$

where:

$$\boldsymbol{q_r} = \left[x_r \quad y_r \quad \theta_r\right] \tag{12}$$

and

$$\mathbf{e}^T(t) = \left[e_x \ \ e_y \ \ e_\theta\right] = \mathbf{q_r} - \mathbf{q} \tag{13}$$

are the reference position and the position error respectively.

From (11) and (7), the vector (12) is the following:

$$\dot{x}_r = \upsilon_r \cos\theta_r \,;\, \dot{y}_r = \upsilon_r \, sen\theta_r \,;\dot{\theta}_r = \omega_r$$

In this paper a *trajectory tracking problem with noise* is developed. In fact such perturbations, perturb the nonholonomics constraints. Therefore the control system takes into account the effects instead of the causes which can cause errors on the vehicle position.

4 Design of fuzzy Lyapunov controller

Fig. 2 shows the proposed control system.

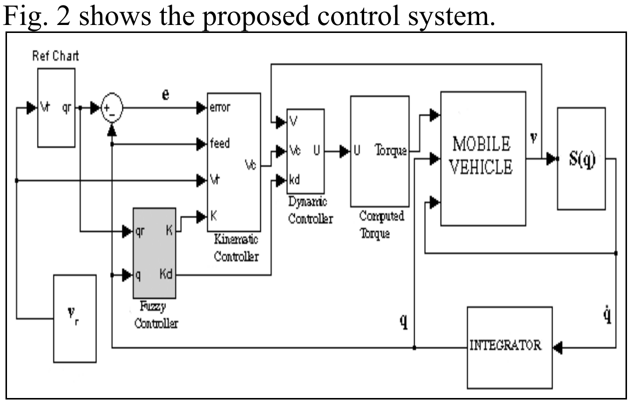

Fig.2 –Fuzzy computed torque controller

It has been obtained from a classical computed torque controller inserting a fuzzy controller that arranges for the determination of the parameters of the kinematical and dynamical controllers. The proposed kinematical control law is:

$$\mathbf{v_c} = \mathbf{f_c}(\mathbf{e}, \mathbf{v_r}, \mathbf{K}) = \begin{bmatrix} \upsilon_c \\ \omega_c \end{bmatrix} = \begin{bmatrix} \upsilon_r \cos e_\theta + k_x(t) e_x \\ \omega_r + k_y(t)\upsilon_r e_y + k_\theta(t)\upsilon_r sen\, e_\theta \end{bmatrix} \tag{14}$$

The control law (14) depends on the error vector (13), on the reference speed and on the kinematic parameter vector $\boldsymbol{K}$:

$$\mathbf{K} = \left[k_x(t) \quad k_y(t) \quad k_\theta(t)\right]^T \tag{15}$$

The parameters (15) are provided by the fuzzy controller and depend on the error vector. We observe that by substituting equation (14) into equation (7), the closed loop model results:

$$\dot{\mathbf{e}} = \left[\dot{e}_x \dot{e}_y \dot{e}_\theta\right]^T =$$

$$\begin{bmatrix} (\omega_r + u_r(k_y(t))e_y + k_\theta(t)\sin e_\theta)e_y - k_x(t)e_x \\ -(\omega_r + u_r(k_y(t)e_y + k_\theta(t)\sin e_\theta)e_x + u_r \sin e_\theta \\ -u_r(k_y(t)e_y + k_\theta(t)\sin e_\theta) \end{bmatrix} \tag{16}$$

The dynamic controller provides a control law for an auxiliary $\boldsymbol{u}$ vector. Using $\boldsymbol{u}$ vector and applying the nonlinear feedback, the following computed torque vector is obtained:

$$\boldsymbol{\tau} = \boldsymbol{f_\tau}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{v}, \boldsymbol{u}) = \overline{\boldsymbol{B}}^{-1}\left[\overline{\boldsymbol{M}}\,\boldsymbol{u} + \overline{\boldsymbol{V}}_{\boldsymbol{m}}\,\boldsymbol{v} + \overline{\boldsymbol{\tau}}_{\boldsymbol{d}}\right] \tag{17}$$

and the dynamic control problem can be convert into the kinematic control problem as follows:

$$\dot{\boldsymbol{q}} = \boldsymbol{S}\,\boldsymbol{v}$$

$$\dot{\boldsymbol{v}} = \boldsymbol{u} \tag{18}$$

The relation (18) is called “perfect velocity tracking” condition. Then the proposed nonlinear feedback acceleration control input is [16]:

$$\mathbf{u}(\mathbf{v}, \mathbf{v_c}, \mathbf{K_d}) = \dot{\mathbf{v}}_\mathbf{c} + \mathbf{K_d}(t)(\mathbf{v_c} - \mathbf{v}) \tag{19}$$

where:

$$\mathbf{K_d} = \begin{bmatrix} k_d(t) & 0 \\ 0 & k_d(t) \end{bmatrix}. \tag{20}$$

The parameters of matrix (20) are provided by the fuzzy inference mechanism.

4.1 Fuzzy mechanism inference and Lyapunov’s stability

The controller is of Mamdani type [11] with three inputs and four outputs. The input is the error vector (13) and the output is the vector of k_x, k_y, k_θ and k_d values (cf. eqs. 15,19,20). The input and output membership functions, the set of rules and the defuzzification methods have been obtained by tests based on experimental simulations on the classical controller. Simulations have been developed submitting the system to several situations and changing the type of reference tracking, the form and the amplitude of the noise, the membership functions, the set of the rules and the defuzzification method. Fig.3 shows the membership function of e_x; three normalized membership functions, defined from the linguistic variables SMALL (S), MEDIUM

(M) and HIGH (H), have been defined; analogously the membership functions of e_y, and e_θ have been obtained. Fig.4 shows the defuzzification functions of K_x; analogously the defuzzification function of k_y, k_θ and k_d have been obtained. In table 1 the set of the controller rules is shown.

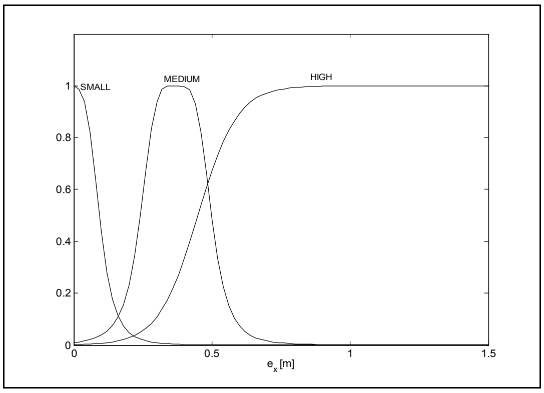

Fig.3 – Membership functions of e_x

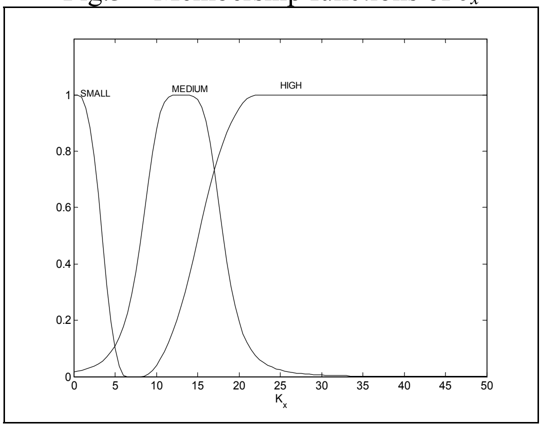

Fig. 4 – Defuzzification functions of k_x

n° rule	e_x	e_y	e_θ		K_x	K_y	K_θ	K_d
1	S	S	S+		S	S	S	S
2	S	M	S+		S	M	S	M
3	S	H	S+		M	H	S	H
4	M	S	S+		M	S	S	M
5	M	M	S+		M	M	S	M
6	M	H	S+		M	H	S	H
7	H	S	S+		H	M	S	H
8	H	M	S+		H	M	S	H
9	H	H	S+		H	H	S	H
10	S	S	M+		M	M	M	M
11	S	M	M+		M	M	M	M
12	S	H	M+		M	H	M	H
13	M	S	M+		M	M	M	M
14	M	M	M+		M	M	M	M
15	M	H	M+		M	H	M	H
16	H	S	M+		H	M	M	H
17	H	M	M+		H	M	M	H
18	H	H	M+		H	H	M	H
19	S	S	OPP		M	M	H	M
20	S	M	OPP		M	M	H	H
21	S	H	OPP		M	H	H	H
22	M	S	OPP		M	M	H	H
23	M	M	OPP		M	M	H	H
24	M	H	OPP		M	H	H	H
25	H	S	OPP		H	M	H	H
26	H	M	OPP		H	M	H	H
27	H	H	OPP		H	H	H	H
28	S	S	M-		M	M	M	M
29	S	M	M-		M	M	M	M
30	S	H	M-		M	H	M	H
31	M	S	M-		M	M	M	M
32	M	M	M-		M	M	M	M
33	M	H	M-		M	H	M	H
34	H	S	M-		H	M	M	H
35	H	M	M-		H	M	M	H
36	H	H	M-		H	H	M	H
37	S	S	S-		S	S	S	S
38	S	M	S-		S	M	S	M
39	S	H	S-		M	H	S	H
40	M	S	S-		M	S	S	M
41	M	M	S-		M	M	S	M
42	M	H	S-		M	H	S	H
43	H	S	S-		H	M	S	H
44	H	M	S-		H	M	S	H
45	H	H	S-		H	H	S	H

Table. 1 – Controller rules

Theorem III.1: Let the kinematic model (7), fuzzy kinematic control laws (14), the linear reference velocity u_r positive and also:

$$\mathbf{K}^T(\mathbf{e}(t)) = \left(k_x(t) \quad k_y(t) \quad k_\theta(t)\right) = \mathbf{0} \Leftrightarrow \mathbf{e} = \mathbf{0}$$

$$0 \le k_x(t) \le k_{x\max} \quad ; \; 0 \le k_y(t) \le k_{y\max} \tag{21}$$

$$0 \le k_\theta(t) \le k_{\theta\max} \quad \left(\frac{dk_y}{d\mathbf{e}}\right)^T \left(\frac{d\mathbf{e}}{dt}\right) > 0 \; ,$$

then the equilibrium state of the non autonomous closed loop system (16) is asymptotically stable.

Proof: since for hypothesis **K(e)** (cf.eqs. 15 and 21) is equal to zero if only if **e** is equal to zero, the equilibrium state of the model (16) is the origin of the state space. The system (16) is non autonomous. The following Lyapunov function is chosen:

$$V_0 = \frac{1}{2}(e_x^2 + e_y^2) + (1 - \cos e_\theta)g(t) \tag{22}$$

where:

$$g(t) = 1/k_y(t) \tag{23}$$

and for hypothesis:

$$k_y(t) > 0 \Rightarrow \mathrm{g(t)} > 0 \; \forall t \tag{24}$$

Therefore Lyapunov function (22) is positive definite.

The time derivative of (22) is:

$$\dot{V}_0 = e_x \dot{e}_x + e_y \dot{e}_y + \dot{e}_\theta \sin e_\theta \, g(t) + (1 - \cos e_\theta)\dot{g}(t) \tag{25}$$

where:

$$\dot{g}(t) = \frac{d}{dt}\left[\frac{1}{k_y(t)}\right] = -\frac{dk_y/dt}{k_y^2(t)} = -\frac{(dk_y/d\mathbf{e})^T(d\mathbf{e}/dt)}{k_y^2(t)} \tag{26}$$

By substituting (16) into (25), it yields:

$$\dot{V}_0 = -k_x(t)e_x^2 - u_r k_\phi(t)\sin^2(e_\theta)g(t) + (1 - \cos e_\theta)\dot{g}(t) \tag{27}$$

Under the hypothesis of the theorem, function (27) is negative semidefinite, because it does not depend on e_y error. Since it results:

$$V_0 = \frac{1}{2}(e_x^2 + e_y^2) + (1 - \cos e_\theta)g(t) \le \\ \le \frac{1}{2}(e_x^2 + e_y^2) + (1 - \cos e_\theta)g_{\max} \tag{28}$$

where:

$$g_{\max} = \max[g(t)], \tag{29}$$

then the function (22) is a decrescent function. Therefore vector (13) is bounded and the equilibrium state of the closed loop system (16) is stable. It is also possible to calculate the second time derivative of Lyapunov function (28). Since the second time derivative of (28) depends on bounded variables, it is a bounded function. Therefore function (27) is uniformly continuous. From Lyapunov-like version of Barbalat's Lemma [9], it yields:

$$\lim_{t\to\infty}\dot{V}_0(t)=0 \qquad (30)$$

From equations (27) and (30), e_x and e_θ converge to zero. From equations (16), $\dot{e}_y$ function converges to zero. Therefore the steady state error e_y is constant. It results:

$$\dot{e}_\theta(\infty)=-u_r k_y(\infty)\bar{e}_y \qquad (31)$$

where $\bar{e}_y$ is the steady state value of e_y . Since e_θ converges to zero, e_y converges to zero. We observe that k_y converges to zero if $\bar{e}_y$ converges to zero. Therefore the equilibrium point of the closed loop system (16) is asymptotically stable (Q.E.D.).

The control surfaces of fuzzy inference mechanism are chosen (see Fig. 5) so that the hypothesis on k_y (cf. eqs. 21) can be verified.

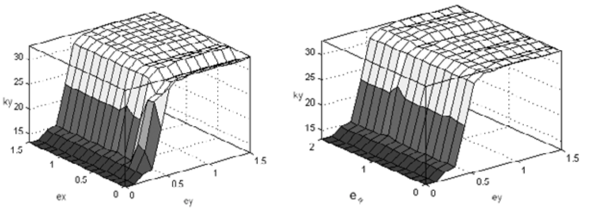

Fig. 5. k_y versus e_x and e_y , k_y versus e_θ and e_y

From Fig. 5 we can observe that the control surfaces are continuous function and it is results:

$$\left(\frac{dk_y}{d\mathbf{e}}\right)^T\left(\frac{d\mathbf{e}}{dt}\right)\cong\frac{dk_y}{de_y}\frac{de_y}{dt}>0 \quad \forall t\,.$$

5 Simulation results

Simulations have been developed in Matlab simulink environment and the performances of a classical backstepping controller and the fuzzy Lyapunov controller of this paper are compared. The parameters of the vehicle for the simulations are as follows:

r = 0,1m ; R = 0,4 m ; d = 0,5 m

m = 30 kg ; I = 15 $kg\cdot m^2$

The classical backstepping controller [6] is characterized by the following parameters:

$$\boldsymbol{K}_c=\begin{bmatrix}20 & 20 & 5\end{bmatrix} \qquad \boldsymbol{K}_d=\begin{bmatrix}30 & 0\\ 0 & 30\end{bmatrix}$$

The case of a mixed reference trajectory is simulated (see Fig. 6). The vehicle is subject to a triangular noise (see Fig. 7) on the X direction.

The following initial conditions have been considered:

υ(0) = 0 m/s ; ω(0) = 0 rad/s

x(0) = 0 m ; y(0) = 0 m ; θ(0) = 0 rad.

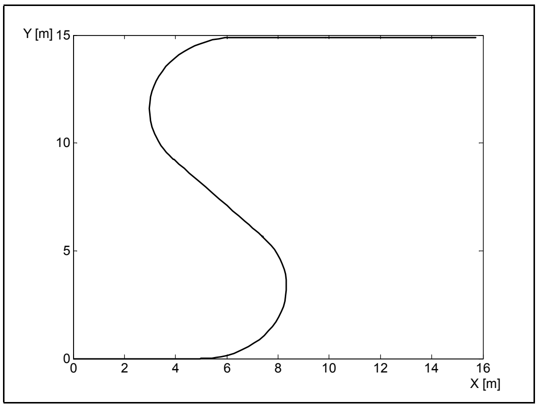

Fig 6 - Reference trajectory

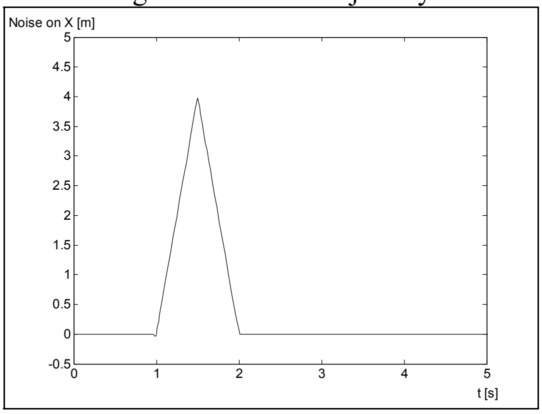

Fig 7 – Noise

Figs 8-10 show the tracking errors and the actual trajectories for the classical backstepping controller [6] and the new fuzzy Lyapunov controller proposed in this paper.

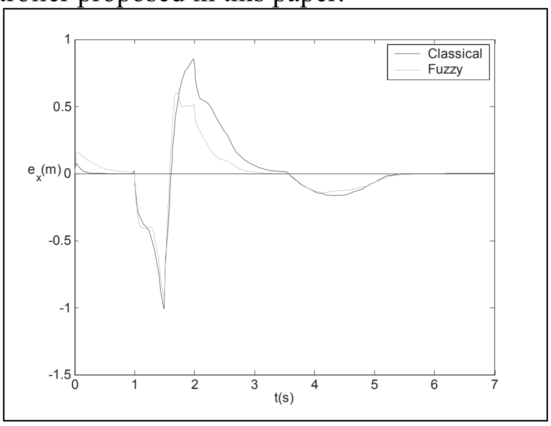

Fig. 8 - e_x [m]

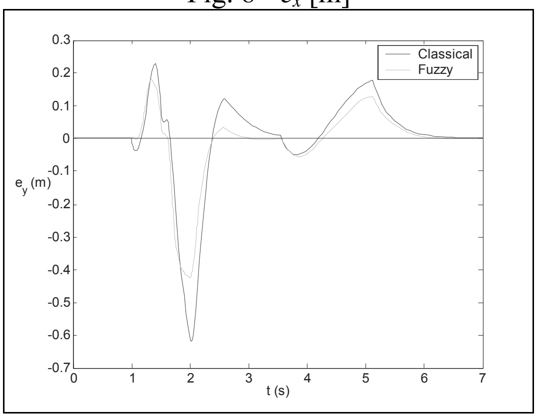

Fig. 9 – e_y [m]

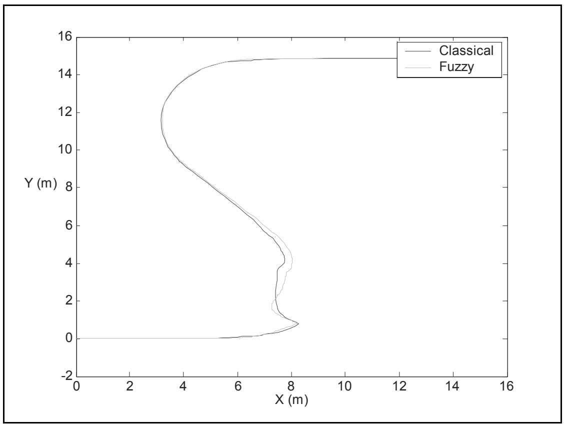

Fig. 10 – Actual trajectories

From Figures 8 and 9 we observe peak value, delay time and response time reductions with respect to the classical backstepping without fuzzy mechanism.

6 Conclusion

In this paper a tracking control problem with noise of a mobile vehicle driven by two independent wheels has been solved by using a new fuzzy Lyuapunov dynamical computed torque controller. The fuzzy Lyapunov controller has been developed supposing known the vehicle features, as the kinematic and dynamic parameters, and taking into account boundeness noise which can perturb the nonholonomic constraints. The fuzzy controller supplies the kinematical and dynamical parameters of the classical controller and it is based on classical backstepping control. Simulations results in Matlab environment have shown that the Fuzzy Lyapunov controller has better performances with respect to a classical backstepping controller.

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