# Adaptive Sliding Mode Control for DC/DC Buck Converters

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*Abstract* - The considered DC/DC buck converter controller is based on classical (conventional) sliding mode control. However their dynamic and steady state performance deteriorate if the loading condition differs from the nominal loading condition. So it is added an adaptive scheme (gain scheduling scheme) to the sliding mode controller, which can optimize the dynamic performance of the converter during load variations.

*Key-Words:* - DC/DC converter, adaptive sliding mode controller, sliding surface.

## 1 Introduction

Switched Mode DC/DC converters are essential for efficient conversion of the battery voltage to various supply voltages, needed to perform every function with minimum power drain. With a DC/DC converter a variable battery supply voltage can be converted to an optimal supply voltage for an application. Both capacitors and inductors are used in switched mode DC/DC converters for temporary energy storage.

It is known that the switching DC/DC converters are highly nonlinear plants with uncertain parameters and inevitable and significant perturbations during operation. The control of switching DC/DC converters has been discussed in many contributions. However for high accuracy of a converter a digital controller should be implemented. The control in high performance DC/DC converters requires not only to ensure system stability but also to achieve a rapid response to sudden changes of the load, to achieve good regulation etc. The development of very high-speed systems on chip has made possible to obtain real time implementation of linear and nonlinear complex control laws.

In this contribution a DC/DC buck converter controller based on classical (conventional) sliding mode control (SMC) is considered [1] – [5]. The dynamics and steady state performance get worse if the loading condition differs from the nominal one. In this case, an adaptive scheme, which can optimize the dynamic performance of the converter during load variations, is added to the sliding mode controller.

The paper is organized as follows. In the next section we consider the analytical model of buck DC/DC converter. In section 3 we consider the basics of the conventional sliding mode control for buck DC/DC converters. In section 4 we consider an adaptive method for sliding mode controlled buck converter. We present Matlab (Simulink) model of the adaptive sliding mode controller of the converter in section 5 and the simulation results in section 6. The concluding remarks are given in section 7.

## 2 Analytical Model of the Converter

Figure 1 shows a schematic of a DC/DC sliding mode controlled buck converter.

We sense the output voltage $v_0$ multiplied by appropriate coefficient $\beta = R_2/(R_1 + R_2)$ that is subtracted from the reference voltage $V_{ref}$ (voltage error). That forms our first state variable $x_1$ (voltage error).

$$x_1 = V_{ref} - \beta v_0 \tag{1}$$

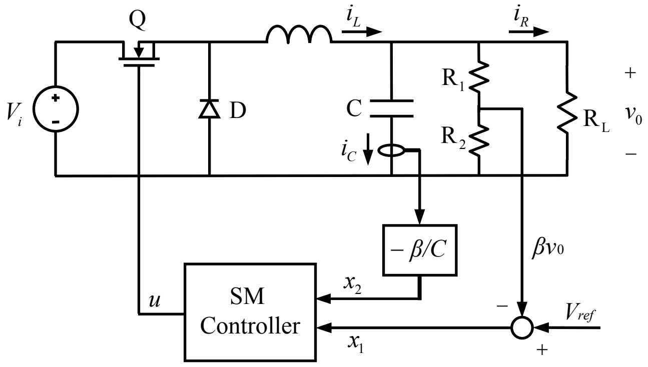


Fig.1. Sliding Mode (SM) Controlled Buck DC/DC Converter

The rate of change of the voltage error $x_2$ is our second state variable:

$$x_2 = \dot{x}_1 = -\beta \frac{dv_0}{dt} = -\beta \frac{i_C}{C} \tag{2}$$

The differentiation of equation (2) with respect to time gives

$$\dot{x}_2 = -\frac{\beta}{C}\frac{di_L}{dt} + \frac{\beta}{R_L C}\frac{dv_0}{dt} \tag{3}$$

Based on the state space averaging method [6], [7] and [8] we can write $v_L = uV_i - v_0$. Taking into account that $v_L = L(di_L/dt)$ we obtain

$$\frac{di_L}{dt} = \frac{v_L}{L} = \frac{uV_i - v_0}{L} \tag{4}$$

where $u$ is the control input that can be 1(switch Q is 'ON') or 0(switch Q is 'OFF').

When we substitute (4) in (3), we get

$$\dot{x}_2 = -\frac{\beta}{LC}(uV_i) - \frac{1}{LC}x_1 - \frac{1}{R_L C}x_2 + \frac{V_{ref}}{LC} \tag{5}$$

## 3 Sliding Mode Control for Buck DC/DC Converter

In sliding mode control the controller employs a sliding surface or line to decide its control input states $u$, which corresponds the turning on and off the power converter's switch, to the system:

$$S = \alpha x_1 + x_2 \tag{6}$$

where $\alpha$ is a positive quantity in some literature called a convergence factor and is taken to be

$$\alpha = \frac{1}{R_L C} \tag{7}$$

Graphically the sliding line is a straight line on the state plane with gradient $\alpha$ that determines the dynamic response of the system in sliding mode with a first order time constant $\tau = 1/\alpha$.

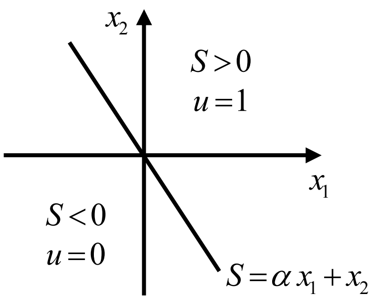


Fig.2. A diagram of sliding mode control.

To ensure that a system follows its sliding surface, a control law must be imposed:

$$u = \begin{cases} 1 = 'ON' & when\ S > 0 \\ 0 = 'OFF' & when\ S < 0 \end{cases} \tag{8}$$

The existing condition for sliding mode [3], [8] is:

$$\dot{S} = \begin{cases} \dot{S} < 0 & for\ S > 0 \\ \dot{S} > 0 & for\ S < 0 \end{cases} \tag{9}$$

Based on this we get the following expression for $\dot{S}$:

$$\dot{S} = \alpha \dot{x}_1 + \dot{x}_2 = \alpha x_2 + \dot{x}_2 = \\ = \alpha x_2 - \frac{\beta}{LC}(uV_i) - \frac{1}{LC}x_1 - \frac{1}{R_L C}x_2 + \frac{V_{ref}}{LC} \tag{10}$$

Depending on $S$ and u the state space is divided into two regions

region 1: $S > 0$ and $u = 1$

$$\dot{S}_1 = \left(\alpha - \frac{1}{R_L C}\right)x_2 - \frac{\beta}{LC}(uV_i) - \frac{1}{LC}x_1 + \frac{V_{ref}}{LC} < 0 \tag{11}$$

region 2: $S < 0$ and $u = 0$

$$\dot{S}_2 = \left(\alpha - \frac{1}{R_L C}\right)x_2 - \frac{1}{LC}x_1 + \frac{V_{ref}}{LC} > 0 \tag{12}$$

Sliding mode will only exist on the portion of the sliding line that covers both of the region 1 ($\dot{S}_1 < 0$) and region 2 ($\dot{S}_2 > 0$) [8].

From one side the speed of the system increases with increasing of $\alpha$ (sliding line become steeper), but from other side the existing region of the sliding mode decreases that can cause an overshoot in the voltage response ($\alpha >> 1/R_L C$) [8].

Figure 3 shows how $S = 0, \dot{S}_1 = 0, \dot{S}_2 = 0$ depend on $x_1$, $x_2$ and $\alpha$.

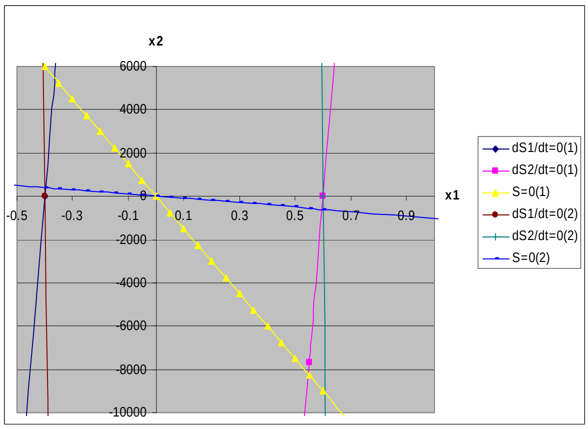


Fig.3. Regions of existence of SM in the state space.

Calculations are made for two different α: (1) $\alpha_1$=15000: (2) $\alpha_2$=1000, and for $R_L$=10Ω; L=10μH; C=50μF; $V_i$=2V; $V_{ref}$=0.6V; β=0.5.

## 4 An Adaptive Method for Sliding Mode Controlled Buck Converter

In the design of the sliding mode controller, $\alpha$ is typically set as a constant parameter ($\alpha_{(nom)}$), corresponding to a nominal operating condition. This makes the sliding line static irrespective of the operating condition.

However from (7) we see that $\alpha$ is inversely proportional to load resistance $R_L$, which may not constant:

$$\text{if } \begin{array}{l} R_{L(dy)} > R_{L(nom)} \; than \; \alpha_{(dy)} < \alpha_{(nom)} \\ (under - loaded\ condition) \end{array} \tag{13}$$

$$\text{if } \begin{array}{l} R_{L(dy)} < R_{L(nom)} \; than \; \alpha_{(dy)} > \alpha_{(nom)} \\ (over - loaded\ condition) \end{array} \tag{14}$$

Hence there is a mismatch between the required sliding line and the actual one.

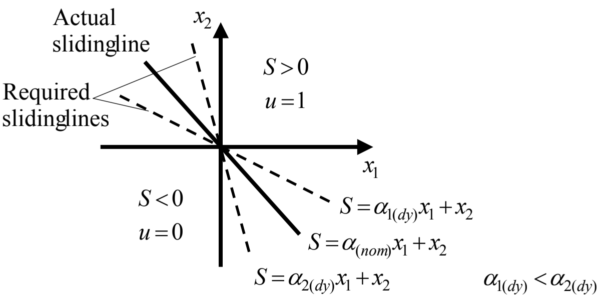


Fig.4. Sliding line of under-loaded and over-loaded conditions.

When the converter operates in an under-loaded condition (13), the result is a reduced sliding mode existence region causing overshoots in the transient response.

Conversely, if it operates in an over-loaded condition (14), the response of the converter will be slower than it should be.

Therefore, to alleviate the problems associated with the deviation of loading conditions, it is necessary to manipulate the sliding line so as to fit to the dynamic operating conditions. Based on the relation

$$\frac{\alpha_{(dy)}}{\alpha_{(nom)}} = \frac{R_{L(nom)}}{R_{L(dy)}} \tag{15}$$

we can find an expression for the instantaneous sliding line gradient $\alpha_{(dy)}$:

$$\alpha_{(dy)} = \alpha_{(nom)} \frac{R_{L(nom)}}{R_{L(dy)}} \tag{16}$$

Since it is not possible to measure the instantaneous loading resistance directly, we can use the following relationship to obtain it:

$$R_{L(dy)} = \frac{v_0}{i_R} \quad where\ i_R \neq 0 \tag{17}$$

The substitution of (17) in (16) leads to the following expression for $\alpha_{(dy)}$:

$$\alpha_{(dy)} = \alpha_{(nom)} \frac{i_R R_{L(nom)}}{v_0} \quad where\ v_0 \neq 0 \tag{18}$$

# 5 Matlab (Simulink) Model of the Adaptive Sliding Mode Controller for Buck Converter

Based on the control law and on the above calculations we designed the schematic of the DC/DC converter with an adaptive sliding mode controller in Simulink.

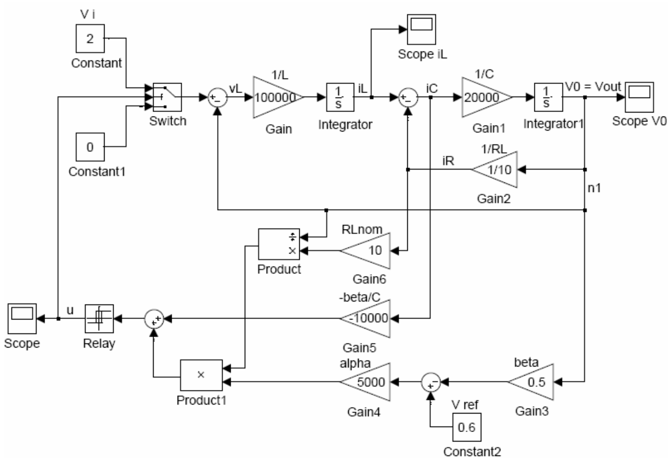


Fig.5. Simulink model of the adaptive SMC for buck DC/DC converter The parameters of the converter are: $V_i$ =2V; L=10□H; C=50µF; $R_{L(nom)}$=10Ω.

# 6 Simulation Results

The simulations are made at two different $\alpha$: $\alpha_1$=5000 and $\alpha_2$=1000, in two different cases: sliding mode controller with adaptive scheme and without adaptive scheme.

Figure 6 shows the output voltage response in the case of an adaptive sliding mode controller when $\alpha_1$=5000. The other parameters of the controller are: $V_{ref}$=0.6V, $\beta$=0.5.

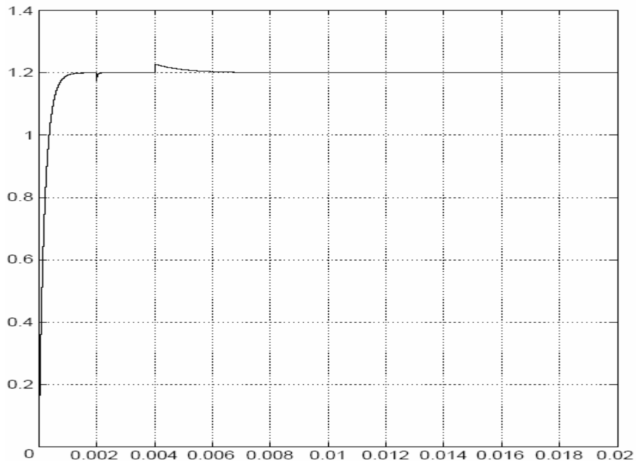


Fig.6. Output voltage response in the case of an adaptive SMC ($\alpha_1$=5000)

We start the simulation with a nominal load of 10Ω, than apply a step load change to 2Ω at time $t = 2$ ms and step load change to 50Ω at time $t$ = 4 ms.

Figure 7 shows the changes in the inductor current .

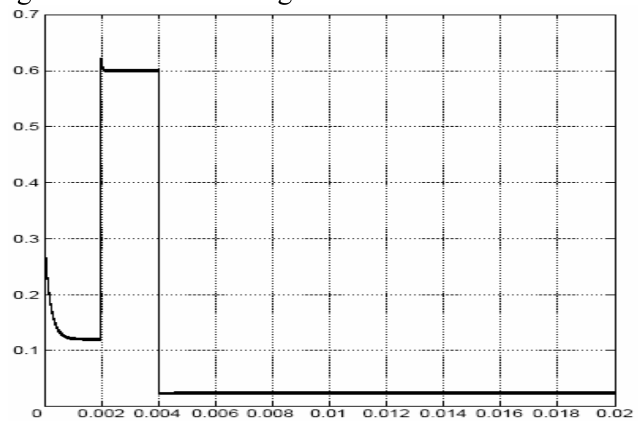


Fig.7. Inductor current changes in the case of an adaptive SMC ($\alpha_1$=5000)

There is an overshoot in the inductor current at a step load change from 10Ω to 2Ω, but there isn't an undershoot in the inductor current at a step load change from 2Ω to 50Ω.

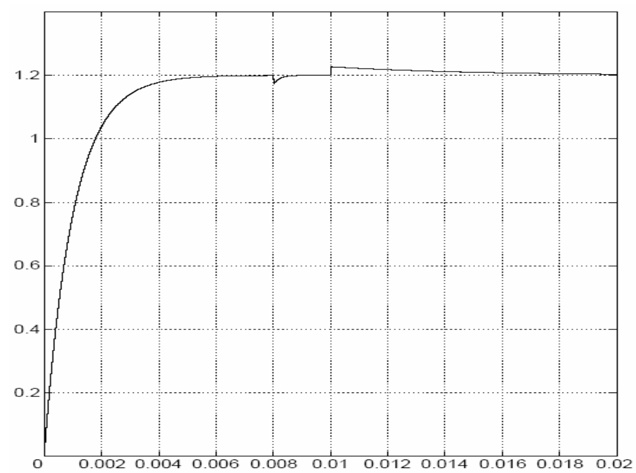


Fig.8. Output voltage response in the case of an adaptive SMC ($\alpha_2$=1000)

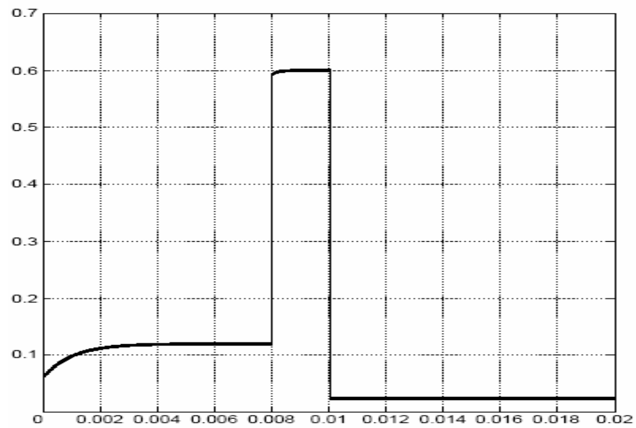


Fig.9. Inductor current changes in the case of an adaptive SMC ($\alpha_2$=1000)

The next two graphs are related to the case of an adaptive SM controller when $\alpha_2$=1000. $V_{ref}$=0.6V, $\beta$=0.5 are the same. Figure 8 shows again the output voltage response. We start the simulation with a nominal load of 10Ω, than apply a step load change to 2Ω at time $t$ = 8 ms and step load change to 50Ω at time $t$ = 10 ms.

Figure 9 shows the changes in the inductor current in the above case. The next two graphs are related to the case of an nonadaptive SM controller when $\alpha_1$=5000, $V_{ref}$=0.6V, $\beta$=0.5. The output voltage response is shown on Figure 10. We start the simulation with a nominal load of 10Ω, than apply a step load change to 2Ω (over-loaded condition) at time $t$=2 ms and step load change to 50Ω (under-loaded condition) at time $t$=4 ms.

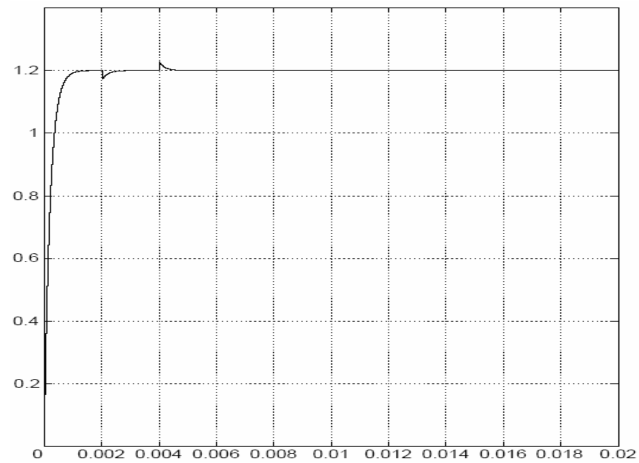


Fig.10. Output voltage response in the case of an nonadaptive SMC ($\alpha_1$=5000)

Figure 11 shows the changes in the inductor current in that case. There is an undershoot [8] in the inductor current at a step load change from 2Ω to 50Ω.

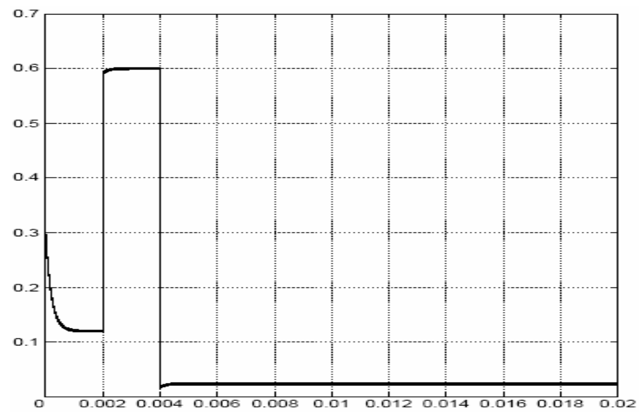


Fig.11. Inductor current changes in the case of an nonadaptive SMC ($\alpha_1$=5000)

The last two graphs are related to the case of an nonadaptive SM controller when $\alpha_2$=1000, $V_{ref}$=0.6V, $\beta$=0.5. The first of this graphs (Figure 12) shows the output voltage response. We start the simulation with a nominal load of 10Ω, than apply a step load change to

2Ω at time $t$ = 8 ms and step load change to 50Ω at time $t$ = 12 ms.

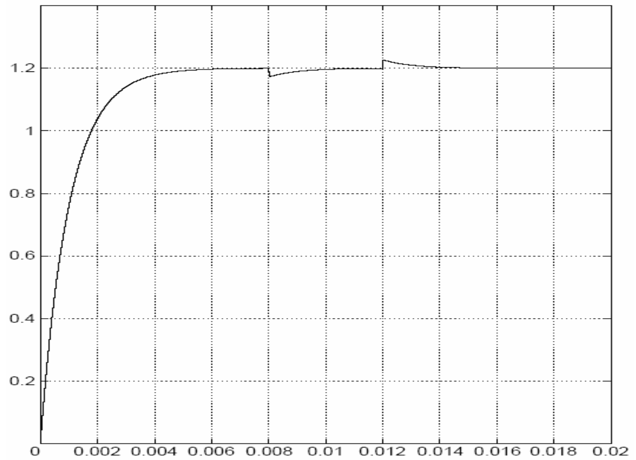

Fig.12. Output voltage response in the case of an nonadaptive SMC ($\alpha_2$=1000)

The second one (Figure 13) shows the changes in the inductor current in that case. There is an undershoot in the inductor current at a step load change from 2Ω to 50Ω.

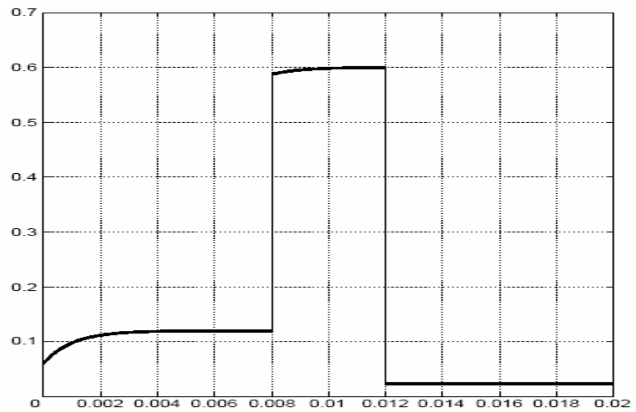

Fig.13. Inductor current changes in the case of an nonadaptive SMC ($\alpha_2$=1000)

Simulation results show that the adaptive SM controlled buck converter has faster dynamic response at over-loaded conditions and it eliminates undershoots in the transient response at under-loaded conditions.

| **over-loaded cond.** | **adaptive** | **nonadaptive** |
|---|---|---|
| $\alpha_1$=5000 | $\approx 0.3*10^{-3}$ | $\approx 0.9*10^{-3}$ |
| $\alpha_2$=1000 | $\approx 0.9*10^{-3}$ | $\approx 4*10^{-3}$ |

Table 1. The approximate settle down time in the case of over-loaded conditions.

The speed of the response for nonadaptive and adaptive SM controllers is given in Table 1 and Table 2. It is obvious that this speed is proportional to the magnitude of the coefficient $\alpha$.

| **under-loaded cond.** | **adaptive** | **nonadaptive** |
|---|---|---|
| $\alpha_1$=5000 | $\approx 4*10^{-3}$ | $\approx 0.9*10^{-3}$ |
| $\alpha_2$=1000 | $\approx 12*10^{-3}$ | $\approx 4*10^{-3}$ |

Table 2. The approximate settle down time in the case of under-loaded conditions.

# 7 Conclusions

Due to some physical limitations the straight sliding line is not the optimal one. The objective of the further research is theoretically to find out the optimal sliding surface (curve) and base on it to design a sliding mode controller for the converter considered.

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