# AUV Controller Design and Analysis using Full-State Feedback

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*Abstract:* This paper presents the design and development of a system controller for our USM Autonomous Underwater Vehicle (USM-AUV). The state space design approach was used because this design approach is most suitable for nonlinear system and multi-input multi-output (MIMO) system set-up. The full state feedback control scheme under the state space design topic has been selected for the controller design. In the initial stage, the controller system was designed using the linearised equation of motion. For the USM-AUV vehicle design, the thruster motor output force was selected as the input with the depth position as the output. Analysis of the designed control system performances has been done via simulation in Matlab control simulation software. A satisfactory performance has been achieved from the designed controller system. Optimal parameters values have also been acquired for optimal performance of our USM-AUV platform.

Keyword:- Autonomous Underwater Vehicle, Control System Design, Full State Feedback Control Scheme

## 1 Introduction

In the initial stage, study about controller design has been focused on the diving system controller's design. In [1], a stern plane deflection has been used as input for the diving control system. In this paper we proposed the thrust force from thruster motor as the input to the controller. Currently, various underwater control systems have been proposed in the literature such as PID control [2] and sliding mode control [3 and 4]. The various control system have utilised different approaches in the control system design. Among them are the classical methods such as root locus technique or modern method like state-space technique. In our case, we have used the state-space technique for the diving controller system design. The state space design approach is used due to its suitability to nonlinear system and multi input multi output (MIMO) system set-up. Its computations also provide a powerful alternative to transfer function methods for the analysis and design of control systems [5].

The subsequent section presents the diving system model. This diving system model is an essential topic for the study and analysis of a diving system controller. The motion equation in vertical plane (diving mode) were derived in terms of rigid body dynamics, hydrodynamic forces and moments, and added mass. Examples of hydrodynamic forces are surface drag and sea current. From this equation, the state space equation of motions was derived. Section 3 in this paper describes the control system design via space-state approach. The full state feedback control scheme has been used, while the pole placement method was utilised to calculate the feedback gain, i.e. **K.** The last section presents the simulation and analysis of the control system design.

## 2. Diving System Modeling

## 2.1 Vehicle Description

Our USM AUV vehicle platform has a mass of 30 kg without accessories payload, and a dimension of 1m long and about 0.5m wide (see Figure 1). The USM AUV vehicle platform has been designed and developed to act as a test bed platform for a variety of research in underwater



Figure 1: USM Autonomous Underwater Vehicle technology, especially involving small-scale and low-cost underwater robots, sensor fusion and actuator control, monitoring and surveillance applications. The test-bed uses two thruster motors in horizontal plane for turning and heading propulsion, and another two thruster motors in vertical plane for the diving system. Depth and Gyro sensors were used to provide the feedback signals.

#### 2.2 Equation of Motion in Vertical Plane.

Four variables were considered (i.e. the heave velocity w, the pitch velocity q, the pitch angle  $\theta$ , and the depth position z) in order to develop the equation of motion in vertical plane. The forward speed u is assumed to be constant, while the sway and yaw modes were not considered.



Figure 2: The vehicle motion in the dive plane mode.

In [6], the full nonlinear model of motion for underwater vehicles can be found. By referring to Figure 2, the non-linear equations of motion in dive plane mode are:

$$m\left[\dot{w} - uq - x_{G}\dot{q} - z_{G}q^{2}\right] =$$

$$Z_{\dot{w}}\dot{w} + Z_{w}w + Z_{\dot{q}}\dot{q} + Z_{q}q + (W - B)\cos\theta + T_{v}$$

$$I_{v}\dot{q} - m\left[x_{G}\left(\dot{w} - uq\right) - z_{G}\left(\dot{u} + wq\right)\right] =$$

$$M_{\dot{w}}\dot{w} + M_{w}w + M_{\dot{q}}\dot{q} + M_{q}q$$

$$- (x_{G}W - x_{B}B)\cos\theta - (z_{G}W - z_{B}B)\sin\theta + x_{G}T_{v}$$

$$\dot{\theta} = q$$

$$\dot{z} = w\cos\theta - u\sin\theta$$
(1)

For the matrices, formed after the equations have been linearised (i.e. if the vehicle is neutrally buoyant):

$$M\dot{x} = A_o x + B_o u$$

$$\begin{cases} (m-Z_{\dot{w}}) & -mx_{G}-Z_{\dot{q}} & 0 & 0 \\ -mx_{G}-M_{\dot{w}} & (I_{y}-M_{\dot{q}}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \end{cases} \begin{bmatrix} \dot{\psi} \\ \dot{q} \\ \dot{\theta} \\ \dot{z} \end{bmatrix} = \\ \begin{cases} Z_{w} & (mu+Z_{q}) & 0 & 0 \\ M_{w} & M_{q}-mx_{G}u & (-z_{G}+z_{B})W & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & -u & 0 \\ \end{bmatrix} \begin{bmatrix} w \\ q \\ \theta \\ z \\ \end{bmatrix} \\ + \begin{cases} 1 \\ x_{G} \\ 0 \\ 0 \\ \end{bmatrix} T_{v} \qquad (2)$$

The equation can be written in state space equation form :

$$x(k + 1) = Ax(k) + Bu(k)$$
 (3)

where :

$$A = M^{-1}A_o$$
$$B = M^{-1}B_o$$

## **3.** Controller Design

#### 3.1 Full State Feedback Control

Basically, the state space equations are described by four matrices or vectors, i.e. the input control signal, **u**, the state, **x**, as well its time derivative,  $\dot{x}$ , and output signal, **y**. Its general form is given by:

$$\dot{x} = Ax + Bu$$
  
$$y = Cx + Du \tag{4}$$

where;

x is the state vector  $(n \ x \ 1)$ u is the system input  $(m \ x \ 1)$ y is the system output  $(l \ x \ 1)$ A is the state transition matrix  $(n \ x \ n)$ B is the input transition vector  $(n \ x \ m)$ C is the state observer vector  $(l \ x \ n)$ D is the feed-through matrix  $(l \ x \ m)$ 

Under the state feedback control (see Figure 3), the control action was achieved by introducing the feedback matrices, K, to produce the control input u(t).



Figure 3: Full State Feedback Control Block Diagram

#### 3.2 Pole Placement Method

From equation (4), by applying the feedback gains, K, the equation became:

$$\dot{x} = Ax + Bu = Ax + B(-Kx + r)$$
$$= (A - BK)x + Br$$
$$y = Cx$$
(5)

The following steps have been taken in applying the pole placement methodology in phase variable form [7]:

- *i.* Represent the plant in phase variable form controller canonical form.
- *ii. Feed back each phase variables to the input of the plant through a gain K<sub>i</sub>.*
- *iii. Find the characteristic equation for the close loop system represented in step ii.*
- *iv.* Decide upon close loop pole locations and determine an equivalent characteristic equation.
- v. Equate like coefficients of the characteristic equation from step iii and iv and solve K<sub>i</sub>.

The  $K_i$  value can easily be solved by using simulation software such as  $MatLab^{\text{(B)}}$ . In  $MatLab^{\text{(B)}}$ , the function 'acker' was used for the SISO system, whereas for the MIMO system the function 'place' applies.

### 4. Simulation and Analysis

The depth control system transfer function for USM\_AUV is:

$$G_P(s) = \frac{0.02082s^2 + 0.03958s + 11.09}{s^4 + 2.18s^3 + 556.7s^2 + 8.925s}$$

Our design objectives were:

- *i.* 5% overshoot.
- *ii. 1 second settling time &*
- *iii.* Steady state error,  $e_{ss} = 0$  %

By using the pole placement method, we have selected poles at p = -40, -41, -4.0000 + 4.1960i, and -4.0000 - 4.1960i in order to get 5% overshoot and 1s settling time. The selection was made through trial-error approach. The **K** values by using function '*place*' are:

$$K = 419 \quad 0.0706 \quad -0.9333 \quad 1.2730$$

After applying the feedback gain, *K*, the response as shown in Figure 4 was acquired. The system performances using step response are:

Overshoot,  $M_o = 4.2081 \%$ Peak Time,  $t_p = 0.9 s$ Rise Time,  $t_r = 0.4353 s$ Settling Time,  $t_s = 1.2 s$ Steady State Error,  $e_{ss} = 99.9214 \%$ 

From Figure 4, it can be seen that the steady-state error is too large. To eliminate the steady state error, we have applied the constant value called  $\overline{N}$  after the reference.



Figure 4: The Step Response for The Close Loop System with K Controller



Figure 5: The Step Response for The Close Loop System with K Controller and  $\overline{N}$  value

As shown in Figure 5, the system performances after applying the  $\overline{N} = 1273$ , are:

Overshoot,  $M_o = 4.2081 \%$ Peak Time,  $t_p = 0.9 s$ Rise Time,  $t_r = 0.4353 s$ Settling Time,  $t_s = 1.2 s$ Steady State Error,  $e_{ss} = 4.3521e-012 \%$ 

## 5. Conclusion

This paper has shown the result of using the full state feedback control scheme in the design of diving mode controller for our USM AUV. We have calculated the feedback matrix gain K using the pole placement method. Steady state error of the system can be eliminated by introduced the  $\overline{N}$  value after the reference input. In the design using pole placement methods, we have assumed all state variables are measured. If the states are not available, an observer can be used to acquire the state variables. Estimated states, rather than actual states are then feed to the controller. For an actual nonlinear system utilisation, the sliding mode control scheme is proposed.

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#### References:

- Ezril Hisham Mat Saat and Omar Yaakob, *Design and Implementation of Electronic Control System for UTM-AUV*, Tencon Proceedings, Vol II, pp 332-337, September 2000.
- [2] Allen, B.; Stokey, R.; Austin, T.; Forrester, N.; Goldsborough, R.; Purcell, M.; Von Alt, C. REMUS: A Small, Low Cost AUV; System Description, Field Trials And Performance Results, OCEANS '97. MTS/IEEE Conference Proceedings, Page(s): 994-1000 Vol.2. 1997
- [3] K.J Walckho, D. Novick and M.C Nechyba, Development of a Sliding Mode System with Extended Kalman Filter Estimation for Subjugator, Florida Conference on Recent Advanced in Robotics, FAU, Dania Beach, FL, May 8-9, 2003.
- [4] K.H Oh, J.Y Kim, I.W Park, J. Lee and J.H Oh, A Study on the Control of AUV's Homing and Docking, IEEE Conference on Mechatronics and Machine Vision in Practice, Chiang Mai, Thailand, 2002
- [5] Noami Ehrich Leonard, William S.Levine, Using Matlab to Analyze and Design Control Systems second Edition, The Benjamin/Cummings Publishing Company, Inc. 1995.
- [6] Logan, C.L, A Comparison between H-Infinity/Mu-Synthesis Control and Sliding-Mode Control for robust Control of a Small Autonomous Underwater Vehicle, Autonomous Underwater Vehicle Technology, 1994. AUV '94, Proceedings of the 1994 Symposium on, July 1994.
- [7] Norman S. Nise, *Control System Engineering*  $3^{rd}$  *Edition*, John Wiley & Sons INC, 2000.