

# Rocket Launching Simulation in Vertical-Type PM LSM Rocket Launcher System including Rotational Motion Control

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*Abstract:* - To reduce the cost of launching rocket, various researches are performed. A new vertical-type linear synchronous motor rocket launcher system is proposed by the first author Yoshida. The linear launcher is accelerated to a peak speed with loading the rocket, and after releasing the rocket, it is decelerated with no-load to be brought to rest at the upper end of underground guide tube near the earth surface. After the rocket is released from the launcher and takes off with a high initial-speed, the rocket continues to ascend with no control subject to the Coriolis force in the guide-direction under the force of gravity in the rise-direction.

This paper presents the rocket launching numerical simulation in vertical- type PM LSM rocket launcher system including rotational motion control. We have verified the stability of rolling, pitching and yawing motion control by the simulation results in a small experimental model on the ground.

*Key-words:* - PM LSM, Rocket launcher, Rotational motion, Simulation, Eulerian angles, Stability control

## 1 Introduction

Much practical feasibility study on linear synchronous motor ( LSM ) rocket launcher system is presented on a basis of dynamics simulations. A vertical-type superconducting linear synchronous motor rocket launcher system is proposed by the first author Yoshida [1][2][3].

In previous papers [5][6], we proposed a new vertical-type permanent magnet (PM) LSM rocket launcher system which is designed in our laboratory. It is composed of four acceleration guide-ways with armature-windings which are arranged symmetrically along a shaft of about 3.8m. The PM LSM rocket launcher is accelerated to a peak speed with loading the rocket, and after releasing the rocket, it is decelerated with no-load to be brought to rest at the upper end of guide tube. We analyzed the characteristics of three dimensional forces of PM LSM in the derived PM LSM rocket launcher system by using two-dimensional interpolation method [5]. Then, we derived the three-dimensional motion equations of launcher and simulated three-dimensional motion control of rocket launcher [6]. However, it was assumed that there were no rotary motions of the rocket launcher.

In fact, the rocket launcher in which rotary motions are not restricted makes them with three degree of freedom. In this paper, we derived the PM LSM rocket launcher analysis model, based on the PM LSM three

dimensional forces analysis results in such a case that the launcher produces complicated rotary motions. We also derived the three dimensional motion equations of PM LSM rocket launcher and proposed rotational motion control method of launcher. The rotational motion of rocket launcher can be controlled stably by using the Eulerian angles control method. The new control method has successfully damped rolling, pitching and yawing motions by using effective armature-current and mechanical load angle of each LSM. We simulated three dimensional rotational motion control of launcher with rocket on board. The stability of rocket launching has been verified by the rotational motion control simulation results.

## 2 PM LSM Rocket Launcher System

Figure 1 shows the vertical-type PM LSM rocket launcher system, which has the acceleration guide tube of about 3.8m on the ground.

Figure 2 shows a square configuration of the vertical-type PM LSM launcher system as a cross-section. In the figure, the linear launcher is deflected by “a” in the  $y$ -direction and by “b” in the  $z$ -direction.

The rocket launcher has 4 permanent magnets on each of four side-surfaces of the launcher, on which there are total mounted 16 PM's. The rated air-gap  $\delta_0$  between the fixed armature coil and the running launcher is 5mm in length.

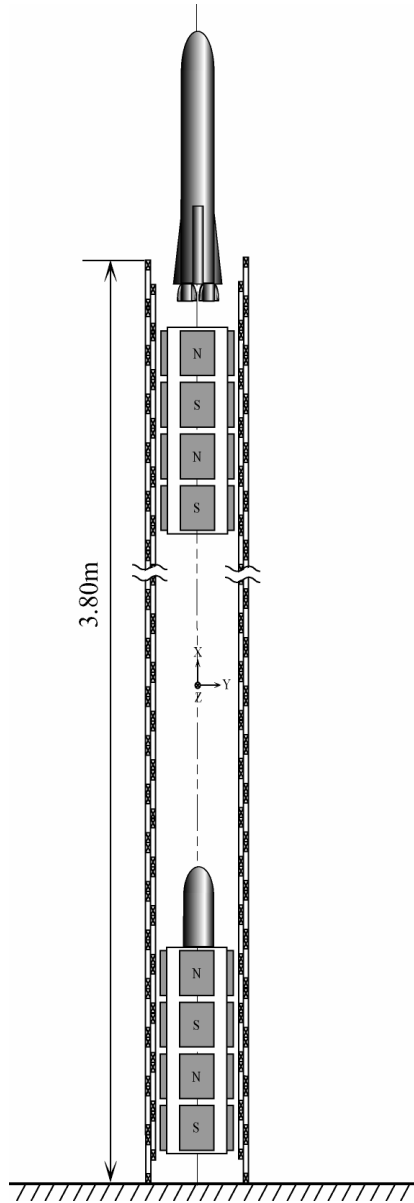


Fig.1 Vertical-type PM LSM rocket launcher system

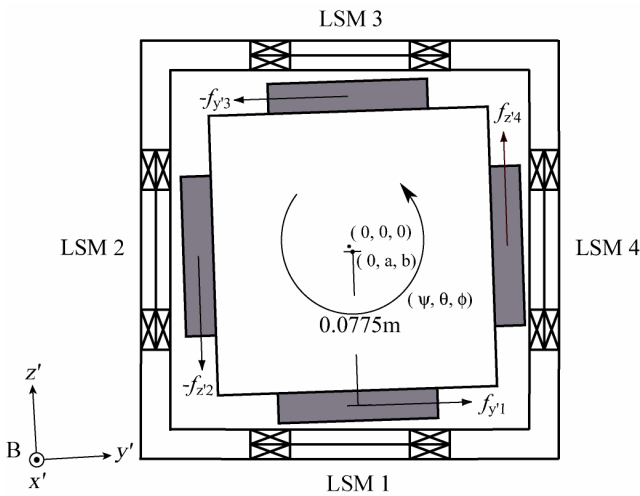


Fig.2 PM LSM rocket launcher in horizontal section

### 3 Three Dimensional Rotational Motions

#### 3.1 Three Dimensional Forces of PM LSM

Table 1 shows specifications of vertical-type PM LSM rocket launcher model.

Table 1 Parameters of PM LSM rocket launcher

Height of launcher	60cm
Width of launcher	17cm
Weight of launcher	50kg
Weight of rocket	20kg
Maximum mileage of launcher	3m
Air-gap	5mm
Pole pitch	15cm
Moment of inertia around $x$ -axis	0.2126kg·m <sup>2</sup>
Moment of inertia around $y$ - and $z$ -axes	1.5993kg·m <sup>2</sup>

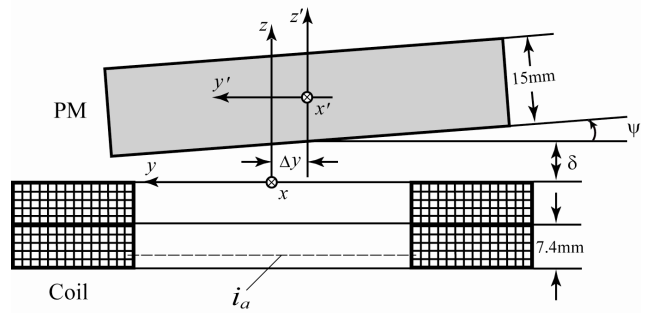


Fig.3 Cross-section of one-side PM LSM

Figure 3 shows cross-section of one-side PM LSM with deviated position  $\Delta y$  and rotation angle  $\psi$ .

In this PM LSM, the  $x$ -directed propulsion force  $f_x$  and the  $z$ -directed levitation force  $f_z$  are produced as  $x$ - and  $z$ -components of the tangential force and the  $y$ -directed guidance force  $f_y$  is also exerted as the normal force in the  $y$ -direction.

We analyzed the propulsion force  $f_x$ , guidance force  $f_y$  and levitation force  $f_z$  of PM LSM using the analytical model of a PM LSM in such a case that the launcher produces complicated rotary motion within the range of the design. In the analytical results, three dimensional forces are influenced by the higher harmonic of the magnetomotive force spatial distribution and varied depending on launcher position  $x_2$ . The change of amplitude is dependent on the air-gap  $\delta$ , deviated position in the  $y$ -direction  $\Delta y$  and the Eulerian angles  $(\psi, \theta, \phi)$ . The phase is dependent on the mechanical load angle  $x_0$  and launcher position  $x_2$ .

Because the air-gap  $\delta$ , deflected position  $\Delta y$  and the Eulerian angles  $(\psi, \theta, \phi)$  are a function of the three dimensional forces, the propulsion, guidance and levitation forces can be transformed according to the theoretical analysis as follows:

(a) Propulsion force

$$f_x = K_{Fx1}(\delta, \Delta y, \psi, \theta, \phi) I_1 \sin\left(\frac{\pi}{\tau} x_0\right) + K_{Fx2}(\delta, \Delta y, \psi, \theta, \phi) I_1 \sin\left(\frac{\pi}{\tau} x_0 + \frac{6\pi}{\tau} x_2\right) \quad (1)$$

(b) Guidance force

$$f_y = K_{Fy1}(\delta, \Delta y, \psi, \theta, \phi) I_1 \cos\left(\frac{\pi}{\tau} x_0\right) + K_{Fy2}(\delta, \Delta y, \psi, \theta, \phi) I_1 \cos\left(\frac{\pi}{\tau} x_0 + \frac{6\pi}{\tau} x_2\right) \quad (2)$$

(c) Levitation force

$$f_z = K_{Fz1}(\delta, \Delta y, \psi, \theta, \phi) I_1 \cos\left(\frac{\pi}{\tau} x_0\right) + K_{Fz2}(\delta, \Delta y, \psi, \theta, \phi) I_1 \cos\left(\frac{\pi}{\tau} x_0 + \frac{6\pi}{\tau} x_2\right) \quad (3)$$

where  $\alpha$  is the pole pitch,  $I_1$  is effective armature-current,  $\delta$  is air-gap length between PM and armature coil,  $x_0$  is mechanical load angle,  $K_{Fx1}$  and  $K_{Fx2}$  are coefficients of propulsion force,  $K_{Fy1}$  and  $K_{Fy2}$  are coefficients of guidance force,  $K_{Fz1}$  and  $K_{Fz2}$  are coefficients of levitation force.

### 3.2 Equations of Motion

In the PM LSM rocket launcher system, neglecting air-resistance, the equations of three dimensional motions are simply described as follows:

(a) before the rocket separates from launcher

$$(M_L + M_R)a_{x_2} = f_{x1} + f_{x2} + f_{x3} + f_{x4} - (M_L + M_R)g \quad (4)$$

$$(M_L + M_R)a_{y_2} = f_{z2} - f_{z4} + f_{y1} - f_{y3} \quad (5)$$

$$(M_L + M_R)a_{z_2} = f_{z1} - f_{z3} + f_{y4} - f_{y2} \quad (6)$$

(b) after the rocket separates from launcher

$$M_L a_{x_2} = f_{x1} + f_{x2} + f_{x3} + f_{x4} - M_L g \quad (7)$$

$$M_R a_{x_R} = -M_R g \quad (8)$$

$$M_L a_{y_2} = f_{z2} - f_{z4} + f_{y1} - f_{y3} \quad (9)$$

$$M_R a_{y_R} = 0 \quad (10)$$

$$M_L a_{z_2} = f_{z1} - f_{z3} + f_{y4} - f_{y2} \quad (11)$$

$$M_R a_{z_R} = 0 \quad (12)$$

where  $a_{x_2}$ ,  $a_{y_2}$  and  $a_{z_2}$  are  $x$ -,  $y$ - and  $z$ -directed accelerations of the launcher, respectively.  $g$  is the acceleration of gravity.  $M_L$  and  $M_R$  is the mass of the launcher and rocket, respectively.

In the PM LSM rocket launcher system, the Eulerian angles  $(\psi, \theta, \phi)$  are necessary to describe the rotary motion of the rocket launcher. The rolling angle  $\psi$  is a rotational angle in the  $x$ -axis circumference. The pitching angle  $\theta$  is a rotational angle in the  $y$ -axis circumference. The yawing angle  $\phi$  is a rotational angle in the  $z$ -axis circumference. By the analytical result, the equations of three dimensional rotational motions including rolling, pitching and yawing motions are simply described, neglecting air-resistance, as follows:

$$I_x \ddot{\psi} = 0.0775 \times (f_{y1} - f_{z2} - f_{y3} + f_{z4}) \quad (13)$$

$$I_y \ddot{\theta} = -0.0775 \times (-f_{x1} + f_{x3}) + \frac{3\tau}{2}(f_{z1S1} + f_{z2S1} + f_{z3S1} + f_{z4S1}) + \frac{\tau}{2}(f_{z1N1} + f_{z2N1} + f_{z3N1} + f_{z4N1}) - \frac{\tau}{2}(f_{z1S2} + f_{z2S2} + f_{z3S2} + f_{z4S2}) - \frac{3\tau}{2}(f_{z1N2} + f_{z2N2} + f_{z3N2} + f_{z4N2}) \quad (14)$$

$$I_z \ddot{\phi} = 0.0775 \times (f_{x2} - f_{x4}) - \frac{3\tau}{2}(f_{y1S1} + f_{y2S1} + f_{y3S1} + f_{y4S1}) - \frac{\tau}{2}(f_{y1N1} + f_{y2N1} + f_{y3N1} + f_{y4N1}) + \frac{\tau}{2}(f_{y1S2} + f_{y2S2} + f_{y3S2} + f_{y4S2}) + \frac{3\tau}{2}(f_{y1N2} + f_{y2N2} + f_{y3N2} + f_{y4N2}) \quad (15)$$

where  $I_x$ ,  $I_y$ ,  $I_z$  are moments of inertia around  $x$ -,  $y$ - and  $z$ -axes.

## 4 Control Method

### 4.1 Control Method for Three Dimensional Motions

In the vertical-type PM LSM rocket launcher system, each LSM is controlled independently. The demand patterns of mechanical load angle  $x_0$  and effective armature-current  $I_1$  are obtained for the demand three-dimensional motion pattern from the motion equation. The three dimensional motions of the launcher are controlled by effective armature-current  $I_1$  and mechanical load angle  $x_0$  of each LSM.

In order to follow the demand patterns in the  $x$ -,  $y$ - and  $z$ -directions, the control law for  $F_x^*$ ,  $F_y^*$  and  $F_z^*$  based on PID regulator becomes as follows:

(a) before the rocket separates from launcher

$$F_x^* = (M_L + M_R)a_{x0} + (M_L + M_R)g + K_{Px}(x_{20} - x_2) + K_{Ix} \int (x_{20} - x_2)dt + K_{Dx}(\dot{x}_{20} - \dot{x}_2) \quad (16)$$

$$F_y^* = (M_L + M_R)a_{y0} + K_{Py}(y_{20} - y_2) + K_{Iy} \int (y_{20} - y_2)dt + K_{Dy}(\dot{y}_{20} - \dot{y}_2) \quad (17)$$

$$F_z^* = (M_L + M_R)a_{z0} + K_{Pz}(z_{20} - z_2) + K_{Iz} \int (z_{20} - z_2)dt + K_{Dz}(\dot{z}_{20} - \dot{z}_2) \quad (18)$$

(b) after the rocket separates from launcher

$$F_x^* = M_L a_{x0} + M_L g + K_{Px}(x_{20} - x_2) + K_{Ix} \int (x_{20} - x_2)dt + K_{Dx}(\dot{x}_{20} - \dot{x}_2) \quad (19)$$

$$F_y^* = M_L a_{y0} + K_{Py}(y_{20} - y_2) + K_{Iy} \int (y_{20} - y_2)dt + K_{Dy}(\dot{y}_{20} - \dot{y}_2) \quad (20)$$

$$F_z^* = M_L a_{z0} + K_{Pz}(z_{20} - z_2) + K_{Iz} \int (z_{20} - z_2)dt + K_{Dz}(\dot{z}_{20} - \dot{z}_2) \quad (21)$$

where  $F_x^*$ ,  $F_y^*$  and  $F_z^*$  are command three dimensional forces of launcher, respectively.  $K_{Pi}$ ,  $K_{Ii}$  and  $K_{Di}$  ( $i = x, y, z$ ) are feedback gains.  $x_2$  and  $x_{20}$  are  $x$ -directed position and its demand pattern of the launcher, respectively.

## 4.2 Control Method for Rolling Motion

By the Eulerian angles control method, the rolling motion is controlled by using the controlled-value of levitation force  $f_{sc}$  that is generated by each LSM. To control rolling motion, the feedback control law becomes as follows:

$$f_{sc} = - \left[ K_{P\psi}(\psi_0 - \psi) + K_{D\psi} \frac{d(\psi_0 - \psi)}{dt} \right] \quad (22)$$

where  $\psi$  is rolling angle of the launcher and  $K_{P\psi}$ ,  $K_{D\psi}$  are feedback gains.

## 4.3 Control Method for Pitching and Yawing Motions

The control method of pitching motion and yawing motion is similar because there is a symmetry of the PM LSM rocket launcher system. The pitching motion is controlled by using the controlled-value of propulsion forces that is generated by LSM1 and LSM3. Similarly, the yawing motion is controlled by using the controlled-value of propulsion forces that is generated by LSM2

and LSM4. To control pitching and yawing motions, the controlled-value of propulsion forces  $f_{xc1}$ ,  $f_{xc2}$ ,  $f_{xc3}$  and  $f_{xc4}$  are shown below

$$(\theta \geq 0) \quad f_{xc1} = f_{x1} \quad (23)$$

$$f_{xc3} = f_{x3} - K_{P\theta}(\theta_0 - \theta) - K_{I\theta} \int (\theta_0 - \theta)dt - K_{D\theta} \frac{d(\theta_0 - \theta)}{dt} \quad (24)$$

$$(\theta < 0) \quad f_{xc1} = f_{x1} + K_{P\theta}(\theta_0 - \theta) + K_{I\theta} \int (\theta_0 - \theta)dt + K_{D\theta} \frac{d(\theta_0 - \theta)}{dt} \quad (25)$$

$$f_{xc3} = f_{x3} \quad (26)$$

$$(\phi \geq 0) \quad f_{xc2} = f_{x2} \quad (27)$$

$$f_{xc4} = f_{x4} - K_{P\phi}(\phi_0 - \phi) - K_{I\phi} \int (\phi_0 - \phi)dt - K_{D\phi} \frac{d(\phi_0 - \phi)}{dt} \quad (28)$$

$$(\phi < 0) \quad f_{xc2} = f_{x2} + K_{P\phi}(\phi_0 - \phi) + K_{I\phi} \int (\phi_0 - \phi)dt + K_{D\phi} \frac{d(\phi_0 - \phi)}{dt} \quad (29)$$

$$f_{xc4} = f_{x4} \quad (30)$$

where  $\theta$  is pitching angle of the launcher,  $\phi$  is yawing angle of the launcher and  $K_{P\theta}$ ,  $K_{I\theta}$ ,  $K_{D\theta}$ ,  $K_{P\phi}$ ,  $K_{I\phi}$ ,  $K_{D\phi}$  are feedback gains.

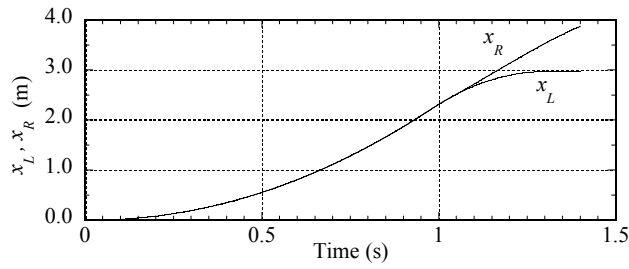
## 4.4 Instantaneous Armature-current

From command mechanical load angle  $x_{0n}^*$  and command effective armature-current  $I_{1n}^*$ , each LSM's command values of instantaneous armature-current can be derived. The instantaneous three phase armature-current of each LSM are shown below

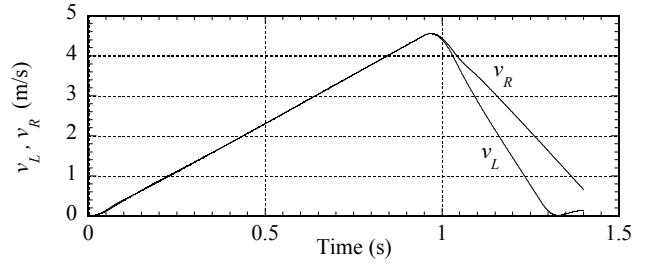
$$\left. \begin{aligned} i_{1n}^* &= \sqrt{2}I_{1n}^* \cos\left(\omega t + \frac{\pi}{\tau}x_{0n}^*\right) \\ i_{2n}^* &= \sqrt{2}I_{1n}^* \cos\left(\omega t + \frac{\pi}{\tau}x_{0n}^* - \frac{2}{3}\pi\right) \\ i_{3n}^* &= \sqrt{2}I_{1n}^* \cos\left(\omega t + \frac{\pi}{\tau}x_{0n}^* - \frac{4}{3}\pi\right) \end{aligned} \right\} \quad (31)$$

$$\omega t = \frac{\pi}{\tau}x_2(t) \quad (32)$$

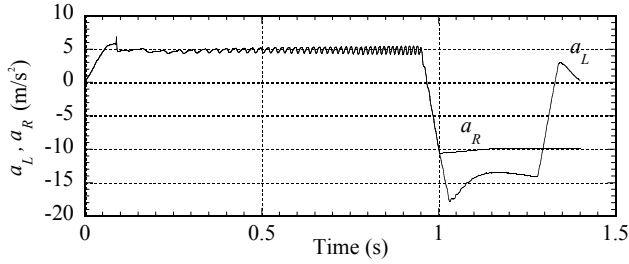
where  $i_{1n}^*$ ,  $i_{2n}^*$  and  $i_{3n}^*$  ( $n = 1, 2, 3, 4$ ) are each LSM's command instantaneous armature-current.



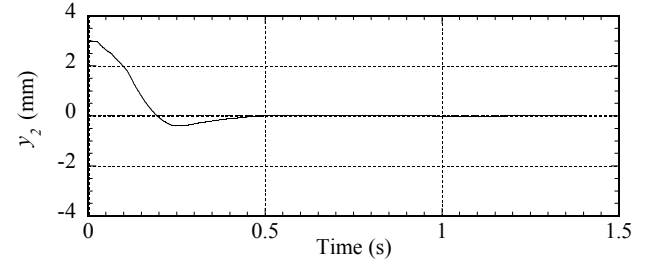
(a) Positions of launcher and rocket



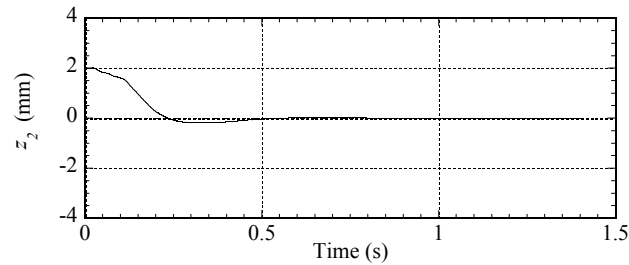
(b) Speeds of launcher and rocket



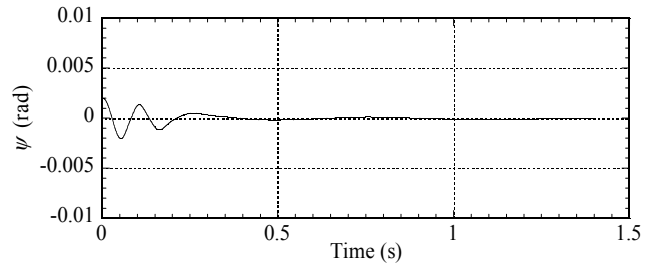
(c) Accelerations of launcher and rocket



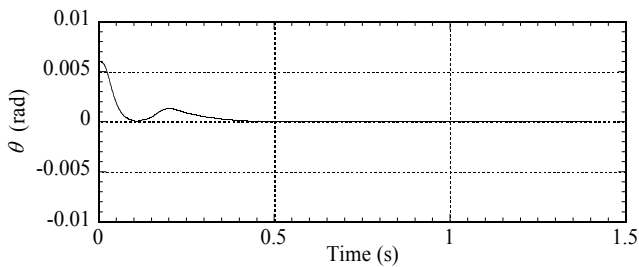
(d) Deflection of launcher in the y-direction



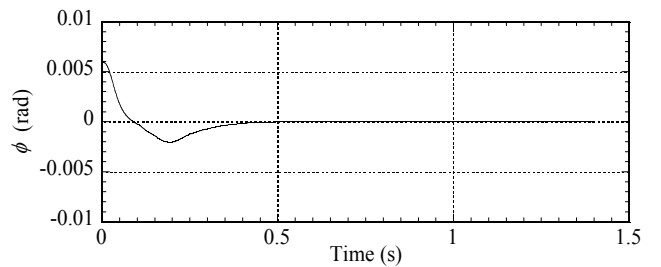
(e) Deflection of launcher in the z-direction



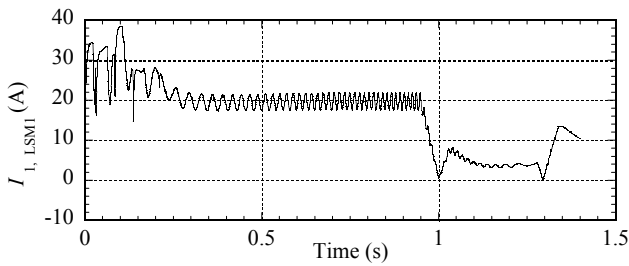
(f) Rolling angle of the launcher



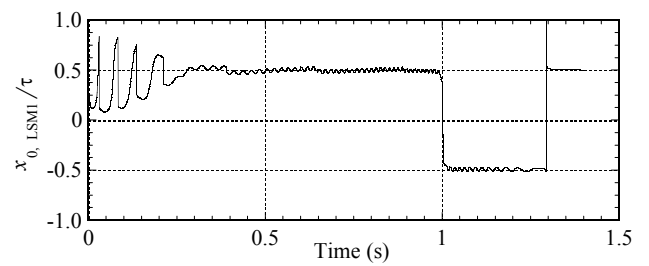
(g) Pitching angle of the launcher



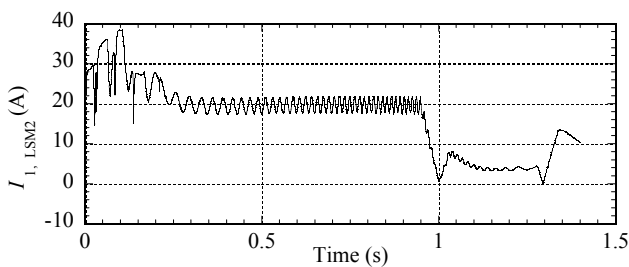
(h) Yawing angle of the launcher



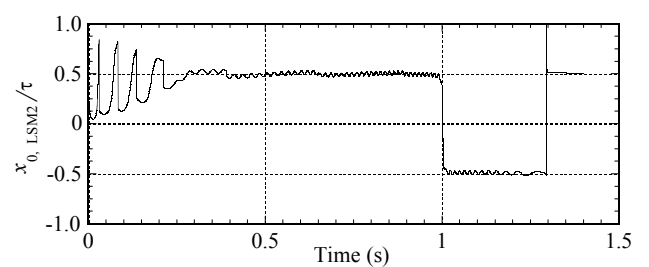
(i) Effective current of LSM1



(j) Mechanical load angle of LSM1

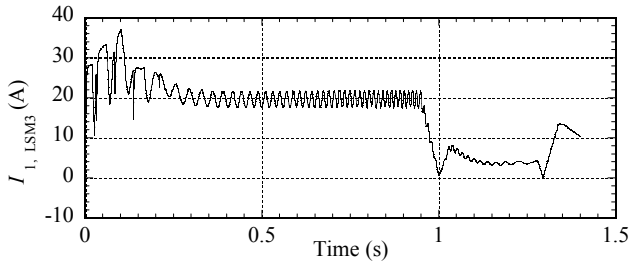


(k) Effective current of LSM2

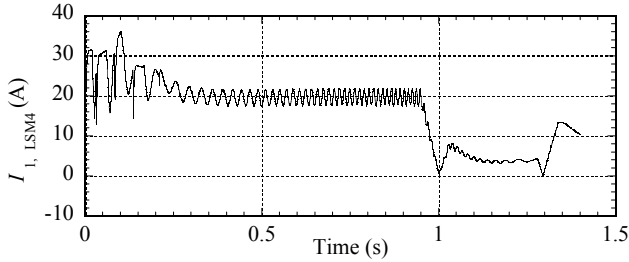


(l) Mechanical load angle of LSM2

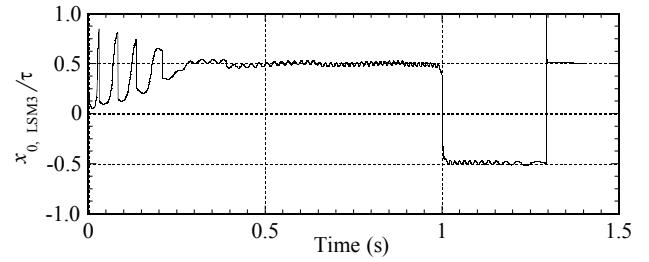
Fig.4 Simulation results



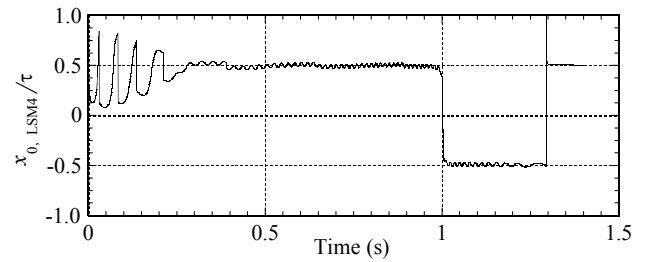
(m) Effective current of LSM3



(o) Effective current of LSM4



(n) Mechanical load angle of LSM3



(p) Mechanical load angle of LSM4

Fig.4 Simulation results

## 5 Rocket Launching Simulation

Figure 4 shows rocket launching simulation results. Figure 4 (a) ~ (c) show the positions, speeds and accelerations of the launcher and rocket, respectively. From these figures, we can see that the launcher follows very well the demand patterns of position, speed and acceleration.

Figure 4 (d) ~ (h) show deflections in the  $y$ - and  $z$ -directions, rolling angle, pitching angle and yawing angle of the launcher, respectively. To verify the stability of 3-D rotational motion control, we simulated rocket launcher starting from a lot of different positions. From the results, the launcher can be controlled stably in the short time when it starts from horizontal and rotational displacements.

Figure 4 (i) ~ (p) show results of the mechanical load angle  $x_0$  and the effective armature-current  $I_1$  of each LSM. It is found from Fig. 4 (i) that the maximum value of effective armature-current oversteps 38 A when launching rocket starts from horizontal displacements with (0, 3mm, 2mm) and rotational displacements with (0.002rad, 0.006rad, 0.006rad).

## 6 Conclusion

In the vertical-type PM LSM rocket launcher system proposed here, propulsion, levitation and guidance motions have been controlled simultaneously by four LSM's. The Eulerian angles control method proposed by us has successfully applied in the rotational motion control simulation, including rolling, pitching and yawing motions.

We have verified the stability of three dimensional

rotational motions control including rolling, pitching and yawing motions in the PM LSM rocket launcher system with launching rocket.

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