

Analysis of Required Stability of Parameters of Radar Angle-Modulated Signals

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Abstract :- The main tactic-technical characteristics of the radar are defined to a considerable extent by parameters of sounding signal and by their processing system. Realization of potential tactic-technical characteristics of the radar with chosen signal parameters and processing system depends in the first place on stability of signals. In this case, it is expediently to take into account the following signals instability sources: former of sounding signals, sounding signals power amplifier, amplifying-converting section of receiving device, optimal filter (filter of compression) of a complex signal.

Key-Words: - Pulse compression, radar, stability, Doppler shift, sidelobe, quasi-continuous LFM

1 Introduction

For the researched radar the peak to mean ratio of the transmitted power is comparatively low as is common for phased array radar using solid-state transmitting modules. Therefore, in order to use the available mean radar power more efficiently long pulses should be transmitted in practically all modes of operation and for most acquisition ranges. To achieve required range resolution the transmitted pulses must be modulated with the time compression on receive.

The modulated transmitted pulses should satisfy the following requirements:

- Specified time resolution of the compressed pulse.
- Small signal to noise ratio losses for the main lobe of the compressed pulse over the range of likely target Doppler shifts.
- Minimum level of the range side lobes of the compressed signal over the range of possible Doppler shifts.
- The employed pulse modulation should allow for a flexible modification of the transmitted pulse length while preserving the required time resolution in order to adapt the radar power resources to target detection and tracking requirements.
- As far as possible, the adopted modulation technique should be easily realizable in hardware and computationally efficient pulse compression algorithms should be employed.

2 Forms and parameters of the compressed signal

Permissible distortions of parameters of compressed signals reflected from a quasi-point target, in comparing to undistorted signals, are most frequently using to state the influence value of signals instability on the main characteristics of the radar, such as detection probability P_D and false-alarm probability P_F , resolution on coordinates φ , ε , R , estimation accuracy of coordinates and their derivatives [5].

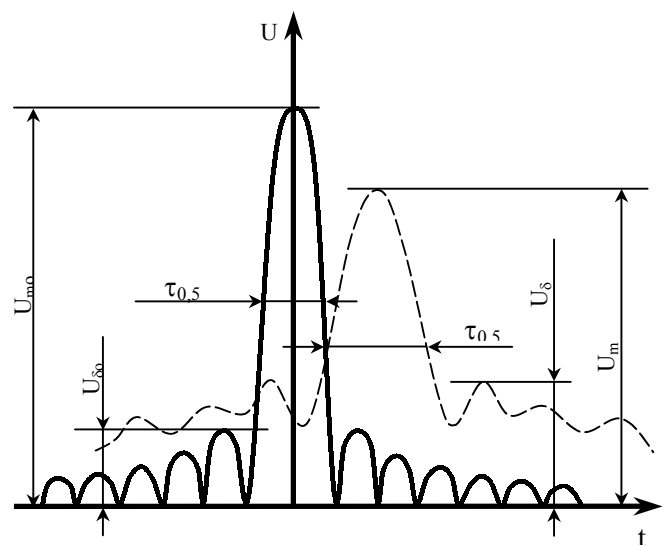


Fig.1: Forms and parameters of the compressed signal

3 Important parameters

Such parameters are the following (fig1):

1) expectation and variance of relative amplitude of the main pulse of compressed signals or of its permissible relative decrease

$$m_a = \overline{U_m}/U_{m0}; \sigma_a^2 = \sigma_{\Delta a}^2 = \frac{(\overline{U_m - U_{m0}})^2}{U_{m0}^2} \quad (1)$$

$$m_{\Delta a} = 1 - \left(\frac{\overline{U_m}}{U_{m0}} \right) \quad (2)$$

where

U_m and $\overline{U_m}$ – instantaneous and mean values of the main amplitude of compressed signal pulse in presence of instabilities;

U_{m0} – amplitude of this pulse in absence of instabilities;

2) expectation and variance of the relative level of the most sidelobes of compressed signal or of its permissible relative increase.

$$m_\delta = \frac{\overline{U_\delta}}{U_{m0}}; \quad m_{\Delta\delta} = \frac{\overline{U_\delta - U_{\delta0}}}{U_{m0}} \quad (3)$$

$$\sigma_\delta^2 = \delta_{\Delta\delta}^2 \left(\frac{(\overline{U_\delta - U_{\delta0}})^2}{U_{m0}^2} \right) \quad (4)$$

where

U_δ & $\overline{U_\delta}$ – instantaneous and mean values of sidelobes in presence of instabilities;

$U_{\delta0}$ – their level in absence of instabilities;

3) expectation and variance of permissible relative expansion of the main pulse of compressed signals

$$m_{\Delta\tau} = \frac{\overline{\tau'_{0.5}} - \tau_{0.5}}{\tau_{0.5}}; \quad \delta_{\Delta\tau} = \frac{(\overline{\tau'_{0.5}} - \tau_{0.5})^2}{\tau_{0.5}^2} \quad (5)$$

where

$\tau'_{0.5}$ и $\overline{\tau'_{0.5}}$ – instantaneous and mean values of the main compressed pulse duration at the level 0.5 in presence of instabilities,

$\tau_{0.5}$ – duration of this pulse in absence of instabilities;

4) permissible variance of compensation factor of passive interferences by instabilities

$$\sigma_\Gamma^2 = \frac{\Delta U_{out}^2}{U_{m0}^2} \quad (6)$$

where

ΔU_{out}^2 – variance of amplitude of uncompensated signal from stationary quasi-

point target on the compensation circuit output in presence of instabilities;

U_{m0} – amplitude of compressed signal from the same target on the compensation circuit output in absence of instabilities.

Connection between the value σ_Γ^2 and factor of passive interferences compensation σ_k^2 can be expressed by relationship [5]:

$$\sigma_\Gamma^2 = \sigma_k^2 (\Delta K_{nb} - 1) \quad (7)$$

where

ΔK_{nb} – permissible decrease of subclutter visibility factor at the expense of instabilities.

For the MTI system with double canceller (type2 canceller):

$$\sigma_k^2 = 2 \cdot [3 - 4\rho(T_R) + \rho(2T_R)] \quad (8)$$

where

$\rho(n \cdot T_R)$ – the value of passive interference correlation factor at the time $n \cdot T_R$ (T_R – the radar pulse repetition period).

For the Gaussian form of passive interference correlation factor

$$\rho(T_R) = \exp \left[-2 \cdot \left(\frac{2\pi \cdot \sigma_v \cdot T_R}{\lambda} \right)^2 \right] \cdot \cos \left(\frac{4\pi \cdot V_I \cdot T_R}{\lambda} \right)$$

$$\rho(2T_R) = \exp \left[-2 \cdot \left(\frac{4\pi \cdot \sigma_v \cdot T_R}{\lambda} \right)^2 \right] \cdot \cos \left(\frac{8\pi \cdot V_I \cdot T_R}{\lambda} \right) \quad (9)$$

where

σ_v and V_I are accordingly mean square value of speed fluctuations and value of passive interference speed,

values of σ_k^2 for double canceller at $\sigma_\Gamma = 3\text{m/s}$; $V_I = 20\text{m/s}$, $T_{R1} = 200\mu\text{s}$, $T_{R2} = 400\mu\text{s}$, $\lambda = 10\text{cm}$ and $T_{R3} = 600\mu\text{s}$, are shown in the table 2.1.

Table 2.1

Repetition period	T_{R1}	T_{R2}	T_{R3}
σ_k^2	$3 \cdot 10^{-4}$	$1 \cdot 10^{-3}$	$1.12 \cdot 10^{-3}$

The values σ_Γ^2 at $\Delta K_{nb} = 1\text{dB}$ and $\Delta K_{nb} = 2\text{dB}$ are shown in the table 2.2.

Table 2.2

σ_Γ^2	ΔK_{nb}	T_{R1}	T_{R2}	T_{R3}
	1dB	$8 \cdot 10^{-5}$	$2.6 \cdot 10^{-4}$	$2.9 \cdot 10^{-4}$
2dB	$1.75 \cdot 10^{-4}$	$5.85 \cdot 10^{-4}$	$6.55 \cdot 10^{-4}$	

In the exciter (former) of pulse and quasi-continuous LFM and FSK signals, the main types of

instabilities are sufficiently characterized by variances of (fig.2 a, b, c, d):

- Inter-period instabilities of deviation value ($\sigma_{\Delta f}^2$) or frequency modulation rate (σ_{vf}^2), initial frequency ($\sigma_{f_0}^2$), duration ($\sigma_{\tau_c}^2$) and initial phase ($\sigma_{\phi_0}^2$) of signals, correlation time of which is more than signal duration ($\tau_{KS} > \tau_b$),
 - Intra-pulse accidental frequency deviations from linear law ($\sigma_{\Delta f}^2$), correlation time of which is less than signal duration ($\tau_{KS} < \tau_b$).
- In power amplifiers of LFM signals, phase instabilities, which are conditioned by instabilities of amplitude (U_{M0}) and of peak form ($\Delta U_M, \Delta U$) of modulating pulses in pulse power supply sources (fig.3 a), are prevailed. They are sufficiently characterized by variances of (fig.3 b, c, d):
- Inter-period instabilities of initial phase ($\sigma_{\phi_0}^2$) and also linear ($\sigma_{f\lambda}^2$) and square-law (σ_{fm}^2) phase accumulation, which result correspondingly to instabilities of initial (mean) frequency ($\sigma_{f_0}^2$; $\sigma_{f_{cp}}^2$) and of frequency deviation value ($\sigma_{\Delta f}^2$) with correlation time $\tau_{KS} > \tau_c$;
 - Intra-pulse accidental frequency deviations from square law ($\sigma_{\Delta\phi}^2$) that results to intra-pulse frequency instability with correlation time $\tau_{KS} < \tau_c$.

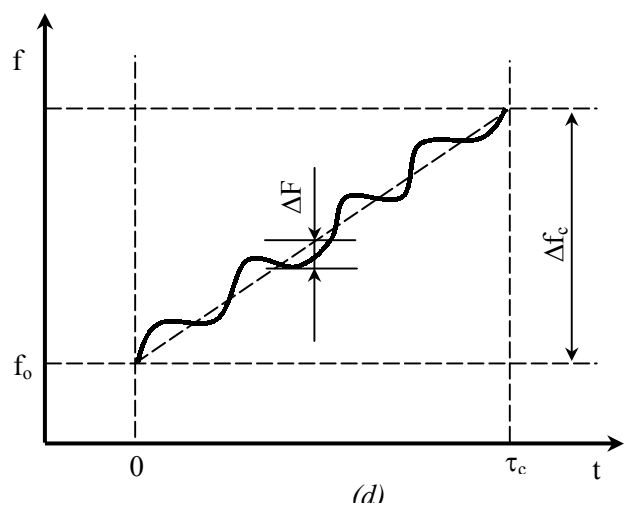
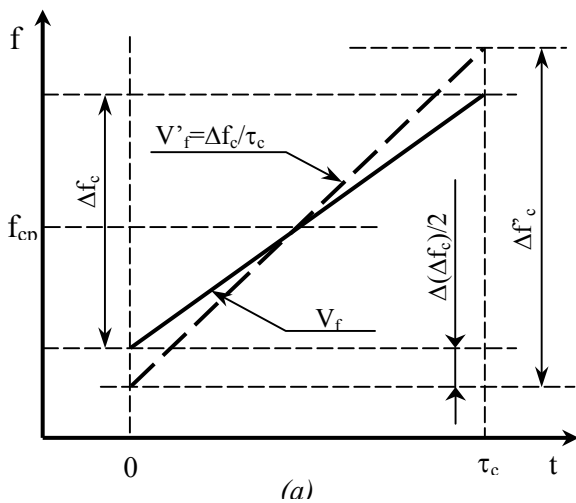
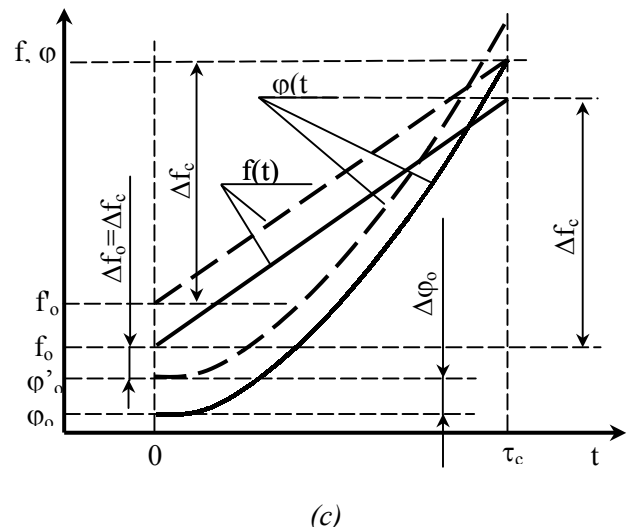
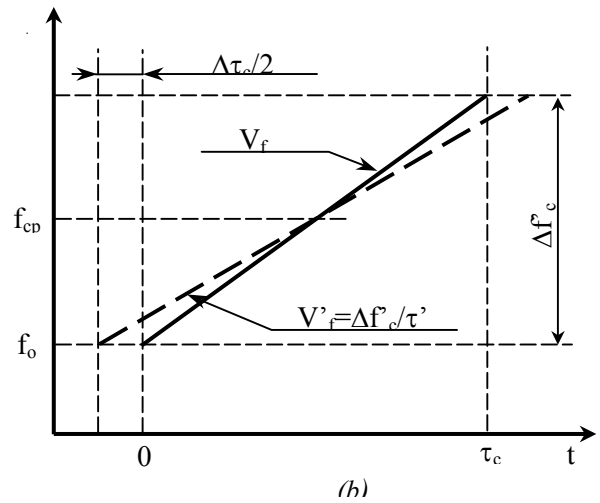


Fig.2: Kinds of instabilities pulse and continuous LFM signals arising in shapers

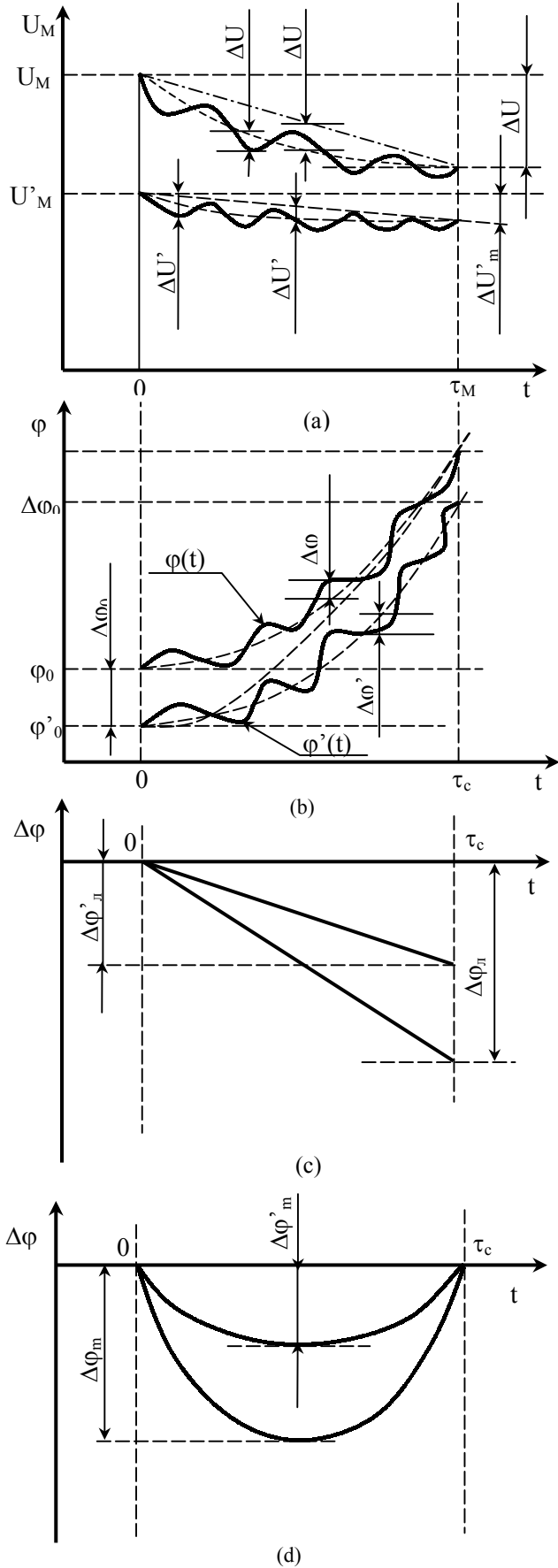


Fig.3: Kinds of instabilities appearing in capacity amplifiers of pulse LFM signals.

4 Instability of compressed signals parameters

In [1] we obtain dependencies that define connection between distortions of compressed signals parameters and permissible instabilities of initial frequency f_0 , of frequency deviation Δf_c , of compressed pulse duration τ_c and of initial phase φ_0 .

At the bandwidth-duration product of LFM radio-pulses $n_c = \Delta f_c \tau_c \geq 30$, at normal law of instabilities distribution and in absence of their inter-period correlation, these dependencies are the following:

- 1) to estimate permissible relative variances of inter-period instabilities of initial frequency f_0 , frequency deviation Δf_c , pulse duration τ_c and initial phase φ_0 ;

$$\frac{\sigma_{f_0}^2}{\Delta f_c^2} \approx \begin{cases} (0.2 + 4 \cdot n_c^2)^{-1} \cdot \sigma_{\Gamma}^2 \cdot B_s \cdot (A_{\kappa} \cdot r \sum_1^s B_s)^{-1}, \\ 7.3 \cdot \sigma_{\Delta a}^2 \cdot B_s \cdot (r \sum_1^s B_s)^{-1}, \\ 31 \sigma_{\Delta \delta}^2 \cdot B_s \cdot (r \sum_1^s B_s)^{-1}, \\ 48.6 m_{\Delta \delta}^2 \cdot B_s^2 \cdot (r \sum_1^s B_s)^{-2}; \end{cases} \quad (10)$$

$$\frac{\sigma_{\Delta f}^2}{\Delta f_c^2} \approx \begin{cases} 37.3 \sigma_{\Gamma} \cdot n_c^{-2} \cdot B_s^{0.5} \cdot (A_{\kappa} \cdot r \sum_1^s B_s)^{-0.5}, \\ 44.4 \sigma_{\Delta a} \cdot n_c^{-2} \cdot B_s^{0.5} \cdot (r \sum_1^s B_s)^{-0.5}, \\ 79.7 \sigma_{\Delta \delta} \cdot n_c^{-2} \cdot B_s^{0.5} \cdot (r \sum_1^s B_s)^{-0.5}, \\ 138.4 m_{\Delta \delta} \cdot n_c^{-2} \cdot B_s \cdot (r \sum_1^s B_s)^{-1} \\ 20.6 \sigma_{\Delta \tau} \cdot n_c^{-2} \cdot B_s^{0.5} \cdot (r \sum_1^s B_s)^{-0.5}; \end{cases} \quad (11)$$

$$\frac{\sigma_{\tau_c}^2}{\tau_c^2} \approx \begin{cases} \sigma_{\Gamma} \cdot n_c^{-2} \cdot B_s \cdot (A_{\kappa} \cdot r \sum_1^s B_s)^{-0.5}, \\ 44.4 \sigma_{\Delta a} \cdot n_c^{-2} \cdot B_s^{0.5} \cdot (r \sum_1^s B_s)^{-0.5}, \\ 79.7 \sigma_{\Delta \delta} \cdot n_c^{-2} \cdot B_s^{0.5} \cdot (r \sum_1^s B_s)^{-0.5}, \\ 138.4 m_{\Delta \delta} \cdot n_c^{-2} \cdot B_s \cdot (r \sum_1^s B_s)^{-1} \\ 20.6 \sigma_{\Delta \tau} \cdot n_c^{-2} \cdot B_s^{0.5} \cdot (r \sum_1^s B_s)^{-0.5}; \end{cases} \quad (12)$$

$$\sigma_{\varphi_0}^2 \approx \sigma_{\Gamma}^2 \cdot B_s \cdot (A_K \cdot r \sum_1^s B_s)^{-1}, \quad (13)$$

where

A_K – factor that is defined by divisibility of over-period subtract of MTI system ($A_K = 6$ for type2 canceller, $A_K = 20$ for type3 canceller);

$r = 1, 2, 3, \dots$ – number of taking into account noncorrelated sources of instabilities;

$s = 1, 2, 3, \dots$ – number of taking into account sources of instabilities;

B_s – weight factor that is defined permissible deposit of instabilities of s-type, which are conditioned by r-source, into the total value σ_i^2

or m_i ;

Charts designed on these parities are shown in Fig. 5.

- 2) to estimate permissible relative variance of inter-pulse frequency instabilities with Gaussian form of correlation function, we obtain the expression:

$$\frac{\sigma_{\Delta f}^2}{\Delta f_c^2} = 0.8 \sigma^2(m_{\Delta\delta})_{\Delta\varphi} \cdot n_c^{-2} \cdot C_{\kappa\varphi}^2, \quad (14)$$

where

$\sigma^2(m_{\Delta\delta})_{\Delta\varphi}$ – permissible variance of inter-pulse phase instabilities with given value $m_{\Delta\delta}$ that is defined by diagrams fig.4,

$$C_{\kappa\varphi} = \frac{2\tau_{\kappa\varphi}}{\tau_c} \text{ – relative correlation time of}$$

inter-pulse phase instabilities ($C_{\kappa\varphi} < 1$).

Suppose, the range of $n_c = \Delta f_c \cdot \tau_c$ lies within the limits: from 40 up to 300.

The values of permissible relative instabilities of initial frequencies f_0 , of frequency deviation Δf_c , of pulse duration τ_c and of initial phase φ_0 were calculated by the formulas given above. They are shown in the diagrams fig.6 ...fig.9.

In calculations of $\frac{\sigma_{f_0}^2}{\Delta f_c^2}$, $\frac{\sigma_{\Delta f}^2}{\Delta f_c^2}$, $\frac{\sigma_{\tau_0}^2}{\tau_c^2} = \varphi(\sigma_{\Delta a})$ we

assume that $r = 3$, $s = 4$, $B_s = 1/4$ (enumerated instability types have approximately the same deposits). In calculations of these dependencies as function of σ_{Γ} , $m_{\Delta\delta}$, $\sigma_{\Delta\delta}$, we assumed that $r = 4$, $s = 5$, $B_s = 1/5$. Besides that, we assumed that worsening of detection probability at the expense of instabilities has to be not more than $\Delta P_D = P_D - P'_D < 0.05$, that corresponds to the values $\Delta K_{\text{inb}} = 1 \dots 2\text{dB}$. Expectation of the relative decrease of compressed pulse amplitude, which are

connected to reachable improvement of MTI filter, is $m_{\Delta a} = -60 \text{ dB}, \dots, -30 \text{ dB}$. Expectation of permissible relative expansion of the main pulse of compressed signals is changed from $6.3 \cdot 10^{-3}$ up to 0.2, that corresponds to the power of uncompensated remainders on the outputs of the MTI devices at the expense of deviation of compressed signal duration, from -60dB up to -30dB.

From the shown diagrams, we can see that requirements to stability of LFM signals parameters, in modes of operation with MTI, are by an order stricter

than in modes without MTI. So the value $\frac{\sigma_{f_0}}{\Delta f_c} \leq$

$3.23 \cdot 10^{-5}$ in the MTI mode and $\frac{\sigma_{f_0}}{\Delta f_c} \leq 2 \cdot 10^{-4}$ in the

mode without MTI. It means that short-term (up to 3...5ms) carrier frequency stability has to be not worst than $2 \cdot 10^{-9}$. Very high claims are laid to the initial phase stability. It has to be not worst than 0.025^0 within the limits of coherent packet.

The analysis of permissible inter-period and intra-impulse phase instabilities is also interest to us. In [1], the analysis of permissible instabilities was listed. Here we limit ourselves by separate results of this analysis. It was written above that phase distortions are sufficiently characterized by variances of inter-pulse phase instabilities $\sigma_{\Delta\varphi}^2$ and also by variances of inter-period initial phase instabilities $\sigma_{\varphi_0}^2$ and of amplitudes of linear and square-law phase accumulations $\sigma_{\varphi \cdot 1}^2$, $\sigma_{\varphi m}^2$.

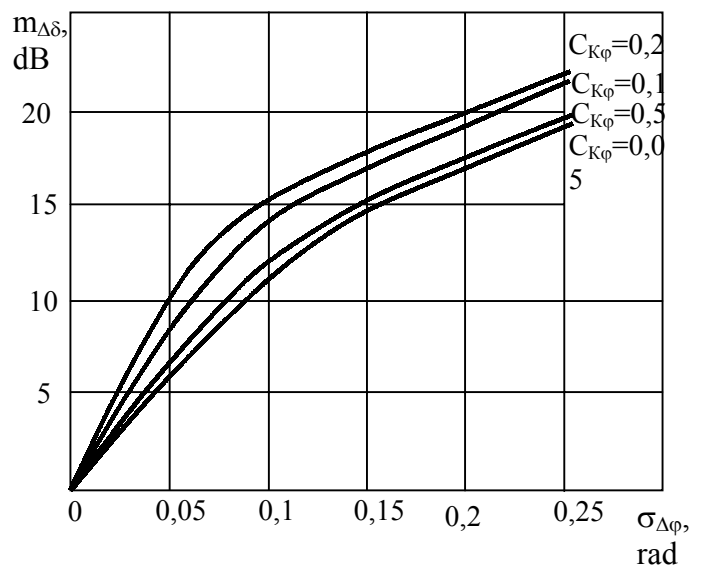


Fig. 4: Dependence of a relative sidelobe level of compressed LFM signal from intra-pulse phase instability $\Delta\varphi$ (see fig. 3. b) of LFM signal.

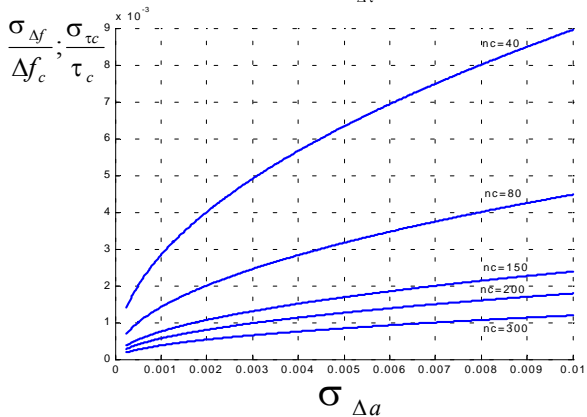
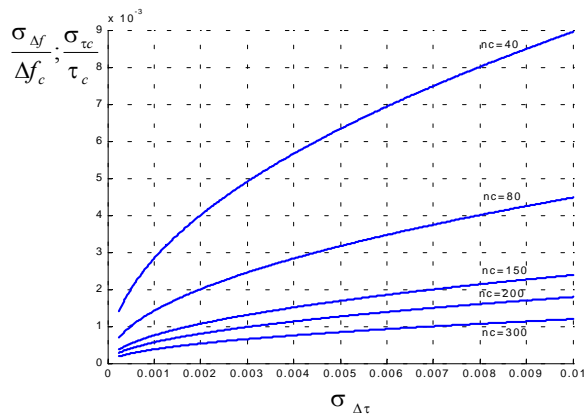
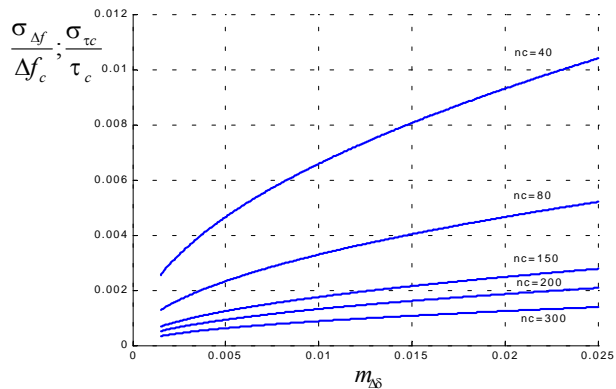
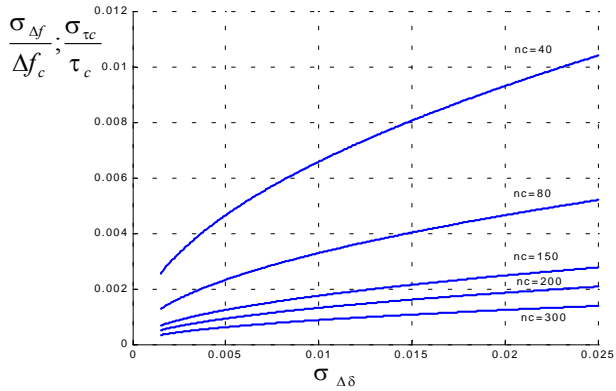


Fig. 5: Dependence of relative instability of deviation of frequency and duration of LFM signal from parameters distortions of the compressed signal $\sigma_{\Delta a}$, $\sigma_{\Delta \tau}$, $\sigma_{\Delta \delta}$, $m_{\Delta a}$.

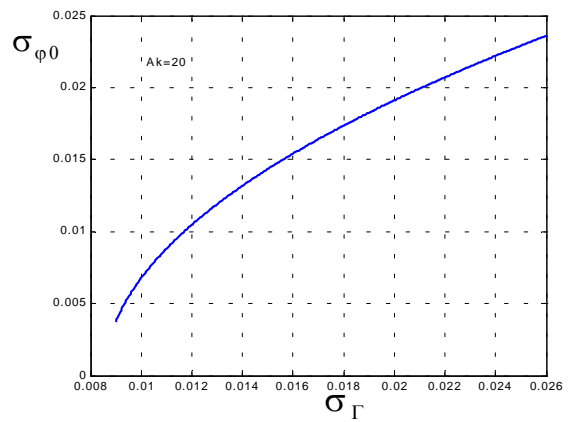
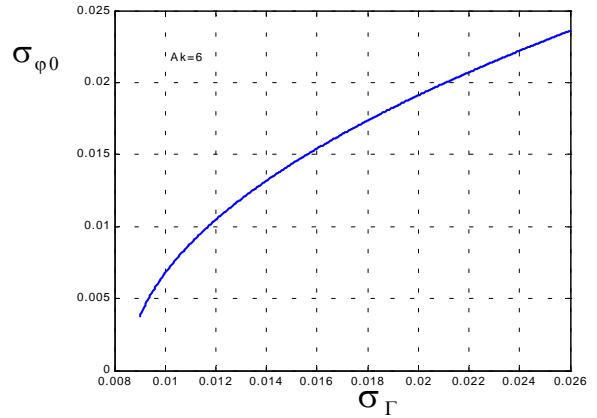


Fig. 6: Dependence of instability of initial phase of LFM signal $\sigma_{\varphi 0}$ from clutter compensation factor σ_{Γ} .

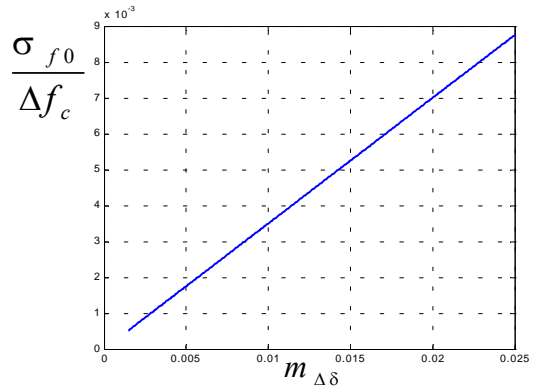
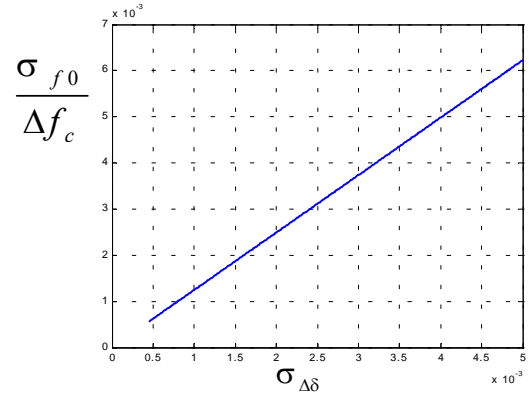


Fig. 7: Dependence of relative instability of initial frequency of LFM signal from sidelobe relative level of the compressed signal $m_{\Delta \delta}$ and its relative increase $\sigma_{\Delta \delta}$.

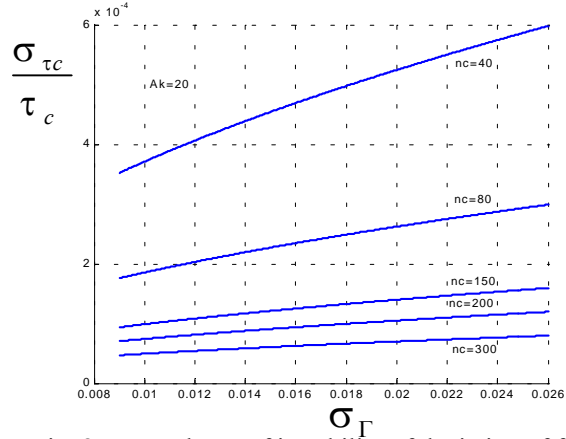
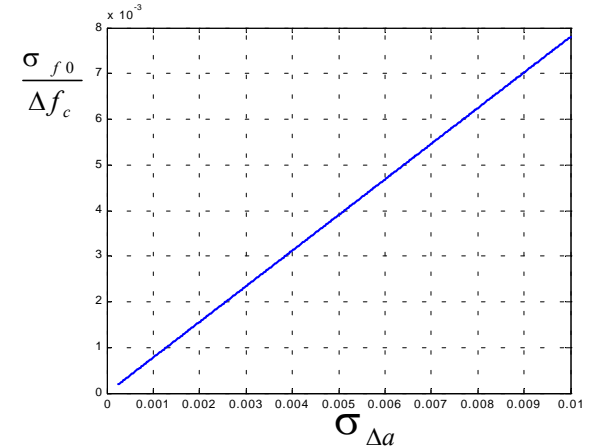
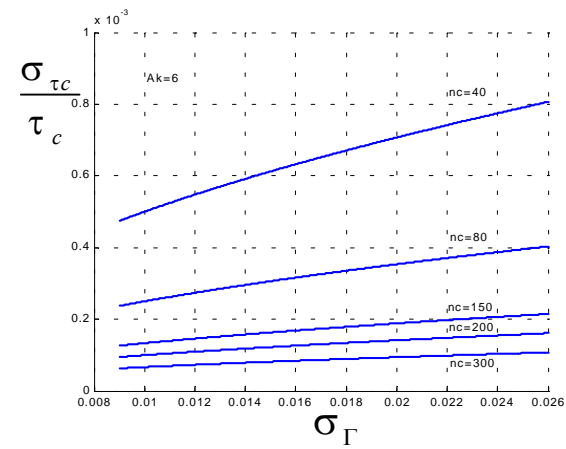
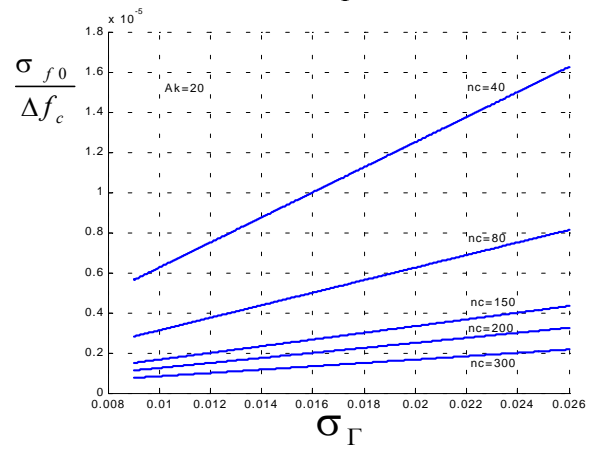
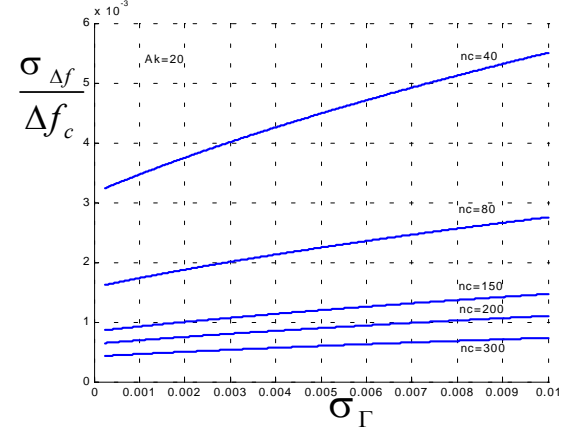
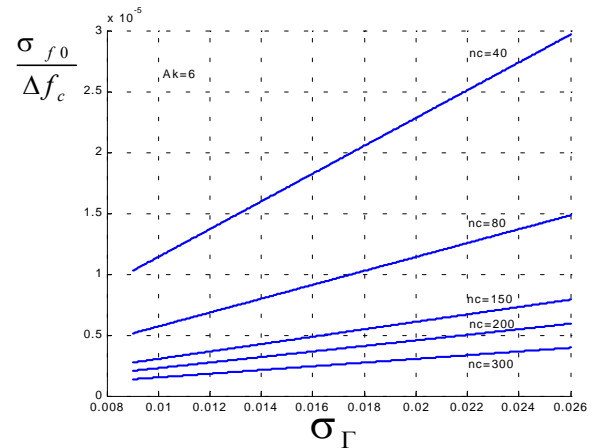
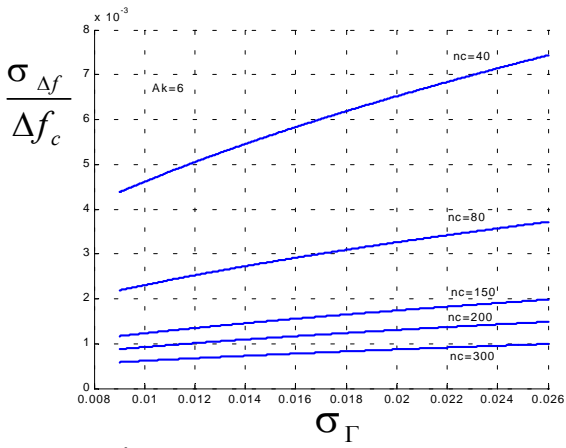


Fig. 8: Dependence of instability of deviation of frequency Δf and duration of LFM signal τ_c from nonlinearities of clutter compensation factor

Fig. 9: Dependence of relative instability of initial frequency of LFM signal from clutter compensation factor σ_Γ and on distortions of amplitude of the compressed signal $\sigma_{\Delta a}$.

The requirements to the permissible value $\sigma_{\varphi_0}^2$ are interest for working operations with MTI and they are valued by formulas listed above as well as for amplifiers and for complex signal formers.

Linear phase accumulation with amplitude $\Delta\varphi_\Lambda$ results to permanent frequency drift within the LFM radio-pulse, which can be conveyed through the initial frequency drift:

$$\Delta f_0 = \frac{\Delta\varphi_\Lambda}{2\pi\tau_c} \quad (15)$$

Square-low phase accumulation with amplitude φ_m results to change of the radio-pulse frequency deviation. In this case the $\sigma_{\varphi_A}^2$ and $\sigma_{\varphi_m}^2$ values are

$$\Delta\varphi_m = 2\pi \int_0^{\frac{\tau_c}{2}} \Delta(\Delta f_c) \left(\frac{\tau_c - 2t}{2\tau_c} \right) 2t = \frac{\pi\tau_c}{4} \Delta(\Delta f_c) \quad (16)$$

Hence, for relations between variances of non-correlated inter-pulse instabilities we have:

$$\sigma_{\varphi_{\cdot 1}}^2 = 4\pi^2 \cdot n_c^2 \cdot \frac{\sigma_{f_0}^2}{\Delta f_c^2}; \quad (17)$$

$$\sigma_{\varphi_m}^2 = 6.25 \cdot 10^{-2} \cdot \pi^2 \cdot n_c^2 \cdot \frac{\sigma_{\Delta\varphi}}{\Delta f_c^2}.$$

Substituting the $\frac{\sigma_{f_0}^2}{\Delta f_c^2}$ and $\frac{\sigma_{\Delta f}^2}{\Delta f_c^2}$ values from

the formerly given relations for the $\sigma_{\varphi_{\cdot 1}}^2$ and $\sigma_{\varphi_m}^2$ into the expression, we can obtain expressions that associate permissible inter-period instabilities of linear and square-low phase accumulations in amplifying sections and the main parameters of signals (σ_r , $\sigma_{\Delta a}$, $\sigma_{\Delta\delta}$, $m_{\Delta\delta}$, $\sigma_{\Delta\tau}$).

The relations for calculation of permissible variance of inter-pulse phase instabilities $\sigma_{\Delta\varphi}^2$ at $C_{K\varphi} < 1$, at normal distribution low of their probabilities and at Gaussian form of correlation function, are the following:

$$\sigma_{\Delta\varphi}^2 = \begin{cases} 0.83\pi^2 \sigma_r \cdot C_{K\varphi} \cdot B_s^{0.5} \cdot (A_k \cdot r \sum_1^s B_s)^{-0.5}, \\ \sigma_{\Delta a} [C_{K\varphi} \cdot (0.312 - 0.43C_{K\varphi})]^{-0.5} \cdot B_s^{0.5} \cdot (r \sum_1^s B_s)^{-0.5}, \end{cases} \quad (18)$$

The diagrams calculated by these relationships for $r = 3$, $s = 4$, $B = 0.25$, $A_k = 6$, are shown in the fig. 10. From analysis of these diagrams we can see that, at inter-pulse phase instability $\sigma_{\Delta\varphi} \leq 0.02$ rad, relative sidelobe level of a compressed signal $m_{\Delta\delta}$ increases not more than by 3dB.

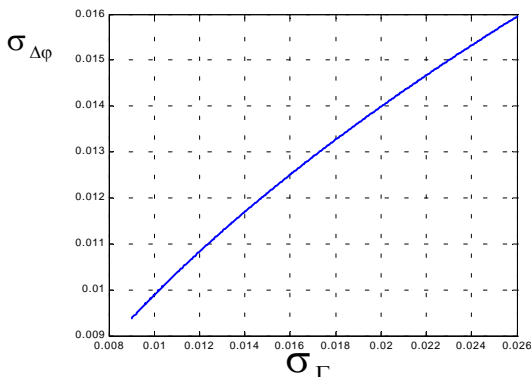


Fig.10(a)

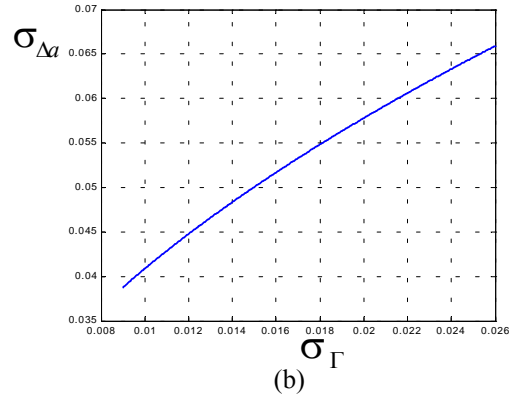


Fig. 10: Dependence intra-pulse instability of phase of LFM signal $\sigma_{\Delta\varphi}$ from instability of clutter compensation factor

5 Conclusions

Thus, using the relationships and the diagrams listed in this paper, we can state the value permissible instabilities of frequency, time and phase parameters of LFM radio-pulses in excitors and power amplifiers of radar. In addition, we can do it for amplifiers and frequency converters of receiving section of the radar with radio-pulses compression, binding it with permissible decrease of the main tactic-technical characteristics of the radar. We have to take into account the exposed requirements to stability of LFM radio-pulses parameters in the choice of sounding signal forming methods and in the development of T/R module power amplifiers and of amplifiers in receiving sections of the radar.

Besides that, it is necessary to give particular attention to the choice of the secondary power supply sources. Their ripple, particularly at frequencies 40...400kHz, results to parasitic phase modulation of amplifiers and can considerably worsen the radar characteristics.

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