Comments on Magnetohydrodynamic Unsteady Flow of A Non-Newtonian Fluid Through A Porous Medium

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Abstract The governing equations describing the unsteady boundary layer flow of a power-law non-Newtonian conducting fluid through a porous medium past an infinite porous flat plate are transformed to a third order non linear ordinary differential equation. An additional boundary condition is written as $f(0) = 0$ by (Gamal M. Abdel- Rahman [1] (IASME transa-ctions 1(3), July 2004)). This boundary condition has not any physical meaning and is not matching to the mathematical analysis described. In this article, the second order non-linear ordinary differential equation with the appropriate boundary condition is solved analytically using the method of successive approximations and numerically using the shooting method.

1. Introduction

Gamal Abdel- Rahman [1] has investigated the formation of magnetohydrodynamic, unsteady flow of an incompressible, non-Newtonian power-law electrically conducting fluid past an infinite porous plate in a porous medium. By assuming that the magnetic Reynolds number is small and applying a similarity solutions, Gamal has obtained a third order non- linear ordinary differential equation for $f(\eta)$ (see equation (10) in [1]). The transformed boundary conditions for the problem are $f'(0) = 0$ and $f'(\infty) = 1$ He assumed an additional boundary condition $f(0) = 0$ (equation (11) in [1]) , which has not any physical meaning and is not matching to the mathematical analysis of the problem.

Also the special cases which were mentioned by Gamal [2-3] cannot be obtained from his analysis . For example case (1): for Newtonian fluid and non-porous medium, one obtains the equations of Takara, H.S., Nath, G. [2]. Takhar and Nath [2] studied the unsteady laminar incompressible boundary layer flow of an electrically conducting fluid in the stagnation region of two dimensional and axisymmetric bodies with an applied magnetic field while Gamal has studied the boundary layer over a plate. Case (2): In the absence of the Newtonian fluid, we obtained the equations of Helmy [2]. Helmy discussed the unsteady 2-dimensional laminar free convection flow of an incompressible, viscous, electrically conducting (Newtonian or polar) fluid through a porous medium bounded by an infinite vertical plane surface of constant temperature, while Gamal used a stationary plate. The aim of this comment is to correct the mathematical formulation and present an analytical and numerical solution for this problem.

2. Mathematical Formulation

Consider unsteady hydromagnetic flow of an incompressible, non-Newtonian power- law electrically conducting fluid past an infinite porous plate in a porous medium. In cartesian coordinate system, let x axis be alon the plate in the direction of the flow and y axis normal to it. A magnetic field is introduced normal to the direction of the flow. We assume that the magnetic Reynolds number is much less than unity so that the induced magnetic field is neglected compared to the applied magnetic field. Further, all the fluid properties are assumed constant. Under the above assumptions with the usual Boussinesq's approximation into account, the governing equations for continuity and momentum are:

$$\frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}=\frac{\partial U}{\partial y}+\frac{k}{\rho}\frac{\partial}{\partial y}[|\frac{\partial u}{\partial y}|^{n-1}\frac{\partial u}{\partial y}]$$

$$+\frac{\sigma M^2H^2}{\rho}(U-u)+\frac{K}{\rho\varepsilon}(U-u) \quad (2)$$

where u and v are the components of the velocity in $x-$ and $y-$direction, respectively, t is the time, ρ is the density of the fluid, ε is the permeability constant, k is the viscosity, μ is the magnetic permeability, σ is the electrical conductivity, H is the magnetic field strength and U is the outer flow velocity. Using the following notation

$$v=k/\rho \quad (3)$$

$$\frac{\sigma\mu^2H^2}{\rho}=N \quad (4)$$

Then equation (2) can be written as

$$\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}=\frac{\partial U}{\partial y}+$$

$$\nu\frac{\partial}{\partial y}[|\frac{\partial u}{\partial y}|^{n-1}\frac{\partial u}{\partial y}]+(N+v/\varepsilon)(U-u) \quad (5)$$

Integrating equation (1) we have $v=v_0(t)$ The initial and boundary conditions are

$$u=0,\ v=v_0(t) \text{ at } y=0,\ t>0$$
$$u\longrightarrow U \text{ as } y\longrightarrow\infty\ ,\ t>0 \quad (6)$$

where $v_0(t)$ is the velocity of injection at the infinite plate. Assume that

$$U=u_\infty\exp[\alpha t] \quad (7)$$

$$v_0=[(N+\alpha)/(\nu u_\infty^{n-1})]-\frac{1}{n+1}\exp[\frac{\alpha(n-1)}{n+1}t] \quad (8)$$

where α and u_∞ are constants.

We further define the following similarity variable

$$\eta=y[(N+\alpha)\nu u_\infty^{n-1}]^{\frac{1}{n+1}}\exp[\tfrac{\alpha(1-n)}{n+1}t]$$
$$u=Uf(\eta) \quad (9)$$

where f is the non dimensional velocity.

From equations (7)- (9) substituting in equation (5) and simplification leads to the following nonlinear ordinary differential equation.

$$nf''f'^{n-1}+[1+S/(1+M)](1-f)-$$
$$1/(1+M)][1/\alpha+\eta(1+n)/(1-n)]f'=0 \quad (10)$$

Where $M=N/\alpha$ is the magnetic number, $S=v/\alpha\varepsilon$ is the parameter of permeability and the prime denotes differentiation with respect to η the transformed boundary conditions are

$$\eta=0 \quad : \quad f=0$$
$$\eta\longrightarrow\infty \quad : \quad f\longrightarrow 1 \quad (11)$$

3. Analytical Solution

The successive approximations method is used to obtained the solution to (10), the different orders are obtained from the equation.

$$f''_{i+1}=f_i'^{(1-n)}[\{\tfrac{1}{\alpha n(M+1)}+\tfrac{(1-n)\eta}{n(1+n)(M+1)}\}f'_i-\{\tfrac{1}{n}+\tfrac{S}{n(M+1)}\}(1-f_i)],$$
$$i=0,1,2,...... \quad (12)$$

Assume that the zero-approximation solution may be written as follows

$$f_0=\alpha_0(1-\exp(-\beta\eta)) \quad (13)$$

where α_0 and β are two arbitrary constants chosen such that the boundary conditions are satisfied in the zero- approximation $f_0(\infty)=1$ and in the first approximation $f_1(0)=0$ i.e. $\alpha_0\beta=1$ and β is given as:

$$\beta^{n+1}n(n+1)(M+1)(2-n)^3+\beta\frac{(2-n)(1+n)}{\alpha}+$$
$$[2(1-n)-(2-n)(1+n)(M+1)-$$
$$S(1+n)(2-n)]=0 \quad (14)$$

Integrating (12), using the fact that $\frac{\partial u}{\partial y}\longrightarrow 0$ as $y\longrightarrow\infty$ and the boundary conditions (11) we have,

$$f_i(\eta)=1+\frac{1}{\beta^{(2+n)}n(n+1)(2-n)^3(M+1)}\{\beta^2$$
$$\frac{(2-n)(1+n)}{\alpha}+(2-n)(1+n)\eta]+\beta[2(1-n)-$$
$$(2-n)(1+n)(M+1)-[S(1+n)(2-n)]\}]\exp[-\beta(2-n)\eta] \quad (15)$$

4 .Numerical Solution

Equation (10) with the boundary conditions (11) were solved numerically, using the forth order Runge- Kutta method. The missing value of $f'(0)$ was determined by a shooting technique.

5 . Discussion

Equation (10), with the boundary conditions(11), has been solved numerically using the shooting method. The effect of the magnetic parameter on the velocity distribution are shown in figure.1. From this figure it is clear that the velocity increases with the increasing of the magnetic parameter M , which is contradicting the behavior achieved in the discussion of Gamal [1].

In figure 2 we compare our solution with the numerical solution using the shooting method. From this figure one finds that the velocity distribution obtained analytically are in good agreement with that obtained numerically.

The numerical investigation to the analytical solution are shown in figures 3-5. The velocity decreases as the magnetic parameter$M$ increase as shown in figure 3. From figure 4 one sees that the velocity distribution increases with the increasing of the parameter of permeability S . Figure 5 shows that the velocity distribution decreases as the power law index n increases.

From table 1 it is clear that both analytical and numerical values of $f'^{n}(0)$ are in good agreement. Table 2 illustrates that the skin- friction coefficient increases with the increasing of the magnetic parameter$M$ and the parameter of permeability S . The skin- friction coefficient decreases as the power law index n increases.

6 . Conclusion

The problem of unsteady magnetohydrodynamic boundary layer flow for a power-law non-Newtonian conducting fluid through a porous medium past an infinite porous flat plate is investigated . A similarity transformation is used to convert the governing partial differential equation to ordinary differential equations.The successive approximations method is used to solve the resulting non -linear ordinary differential equation and the results are compared with the numerical solution . It is found that the velocity distributions decrease as either the magnetic parameter is increased or the power law index increases . Also , the fluid velocity increases with the increasing of the permeability parameter . In addition , it is concluded that the skin-friction coefficient increases as either the magnetic parameter or the permeability increases , while it is decreased as the power law index is increased.

$S \setminus n$	0.7	0.9	1.2
1	An 0.874504	0.812834	0.683359
	Nu 0.870794	0.806049	0.75015
2	An 0.90422	0.888104	0.836751
	Nu 0.95892	0.903274	0.828464

Table 1. Comparison of analytical (An) and numerical (Nu) skin -friction coefficient for $\alpha = 0.3$ and $M = 3$

M	S	n	$(f'(0))^n$
3	2	0.9	0.888104
3	2	1.2	0.836751
0.5	2	0.75	0.608433
1	2	0.75	0.723065
3	0.1	0.8	0.745192
3	1.1	0.8	0.850028

Table 2. The skin- friction coefficient for different values of M, S and n for $\alpha = 0.3$

References

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[2] Takhar, H.S. and Nath, G., "Similarity solution of unsteady boundary layer equations with magnetic field", Meccanica, Inter. J. Italian Assoc. Theor. Appl. Mech., 32, No.2, 157, 1997.

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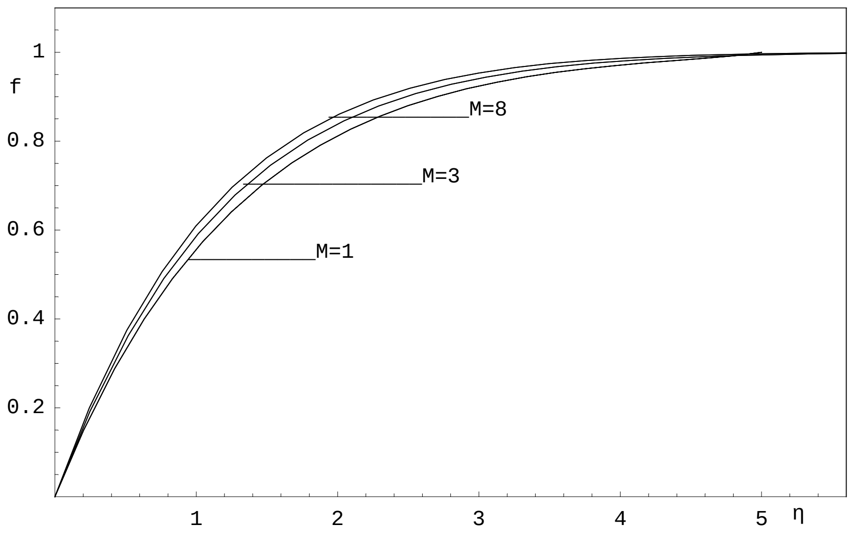

Figure 1. Velocity distribution for various values of M at $n = 1.2$, $S = 2$ and $\alpha = 0.3$

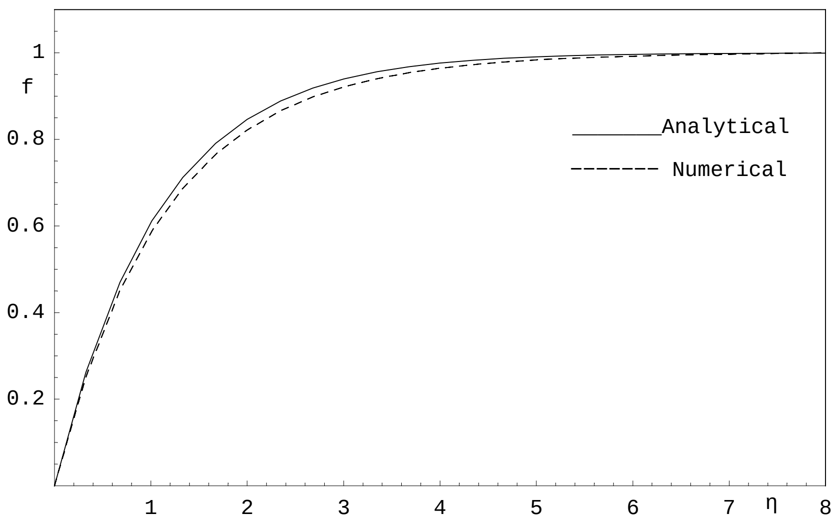

Figure 2. Velocity distribution for analytical and numerical profiles at $n = 0.9$, $S = 2$, $M = 3$ and $\alpha = 0.3$

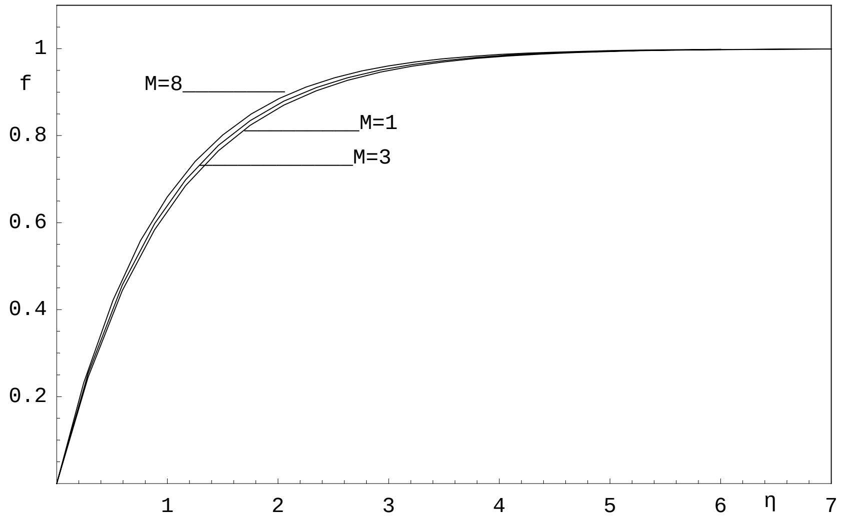

Figure 3. Velocity distribution for various values of M at $n = 0.75$, $S = 2$ and $\alpha = 0.3$

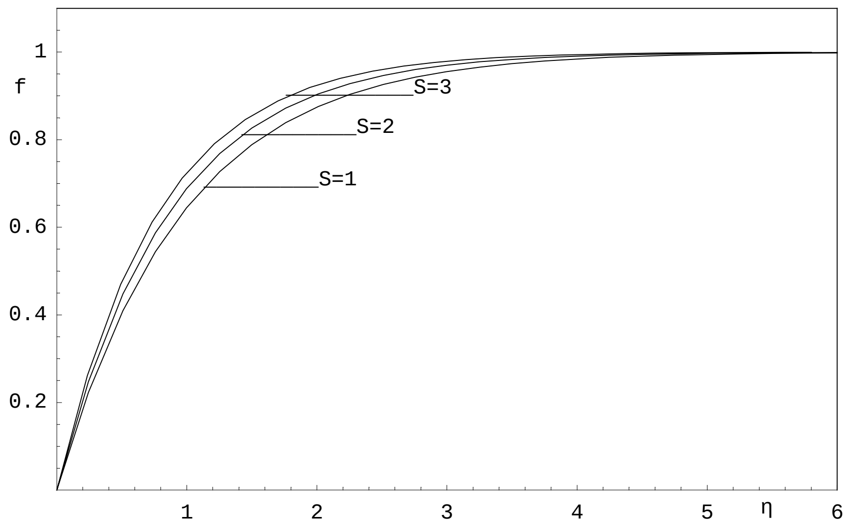

Figure 4. Velocity distribution for various values of S at $n = 0.75$, $M = 3$ and $\alpha = 0.5$

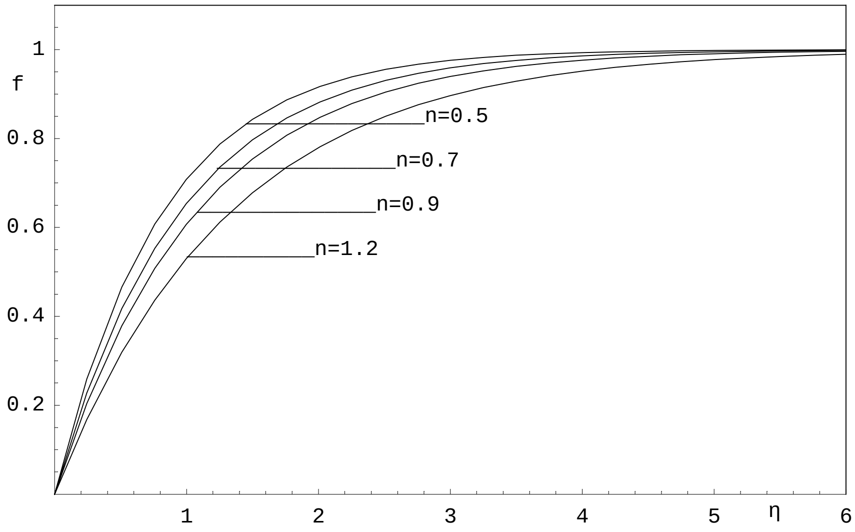

Figure 5. Velocity distribution for various values of n at $\alpha = 0.3$, $M = 3$ and $S = 2$