A Variable Window Approach for Image Denoising

WIEM FOURATI[(1)] and M. SALIM BOUHLEL[(2)]

Ecole Nationale d'Ingénieurs de Sfax (ENIS) [(1)], Institut supérieur de biotechnologie de Sfax[(2)]
Sciences, Image Technologies and Telecommunications High Institute of Biotechnology, Sfax
TUNISIA

Abstract:- Several methods have been developed to enhance the performance of image denoising algorithms. In this letter we have developed an algorithm for image noise removal based on local adaptive window size/shape filtering. While rectangular windows are efficient, they yield poor results near object boundaries. We describe an efficient method for determining the variable size of the locally adaptive window using a region-based approach. A region including a denoising point is partitioned into disjoint subregions. The locally adaptive window for denoising is obtained by selecting the proper subregions. Our approach can be applied to several problems, including image restoration and visual correspondence. Comparison of the algorithm with the known techniques for noise removal from images shows the advantage of the new algorithm, both quantitatively and visually.

Key-Words: -image, wavelet, variable window, noise reduction , bishrink, .

1. Introduction

Many types of noise can be introduced when processing or transmitting images. The problem of image denoising is to recover an image that is "cleaner" than its noisy observation. Wavelet denoising techniques received wide attention mainly due to the breakthrough work by Donoho and Johnstone [1]. It has been shown that wavelet denoising with VisuShrink thresholding possesses nearly optimal rates of convergence over a broad family of Besov spaces [1]. Wavelet denoising with VisuShrink thresholding has been shown to be very successful for 1-D signals. However, its performance is not satisfactory for 2-D images [2].

To improve the performance of wavelet-based image denoising, several methods have been proposed in recent years [3, 4] It is found that models using the dependency between coefficients give better performance than those derived using an independence assumption [4]. A wavelet domain hidden Markov model was developed to exploit the statistical dependencies in [5, 6], also a joint shrinkage function using the neighboring coefficients was proposed in [7, 8]. A new Bivariate Shrinkage function using Bayesian and Maximum Posterior Estimator (MAP) was derived in [4]. In [9], a newly derived Bivariate Shrinkage function was used with locally adaptive estimated parameters for the function. A local adaptive algorithm was proposed and its performance was demonstrated in [9]. The performance of this algorithm was high improvement over others algorithms like BayesShrink [3], NormalShrink [10] etc. To implement the algorithm in [9], Sendur et al. have used square shaped window, Such windows poorly model the boundaries of real-world objects. This results in several well known problems; for example, corners tend to become rounded. In this paper, we describe a new denoising scheme for determining the variable shape of locally adaptive window using a region-merging method.

The paper is organized as follows: The bivariate shrinkage function is described in section 2. In section 3 we introduce our variable window solution. Section 4 is devoted to compare the performance of our proposed algorithm with the performance of other current wavelet-based denoising methods applied on a number of test images. Finally, section 5 draws conclusions and describes future work directions.

2. Bivariate Shrinkage Function

In the wavelet domain, if we use an orthogonal wavelet transform, the denoising problem can be formulated as

$$y = \omega + n \tag{1}$$

where y is the noisy wavelet, ω is the original coefficient, and n is the noise, which is independent Gaussian. Our aim is to estimate ω from the noisy observation, y. The MAP estimator [4],[9] will be used for this purpose.

$$\hat{\omega}_1 = \frac{\left(\sqrt{y_1^2 + y_2^2} - \frac{\sqrt{3}\sigma_n^2}{\sigma}\right)_+}{\sqrt{y_1^2 + y_2^2}} . y_1 \tag{2}$$

which can be interpreted as a bivariate shrinkage function. Here y_1 is the noise-corrupted image, y_2 parent image of the same signal but with one higher coarser scale, σ_n^2 is the variance of the noise-corrupted signal and σ^2 is the signal variance.

In (2), $(g)_+$ is defined as

$$(g)_+ = \begin{cases} 1, & if\ g < 0 \\ 0, & otherwise \end{cases} \tag{3}$$

To estimate the noise variance σ_n^2 from the noisy wavelet coefficients, a robust median estimator is used from the finest scale wavelet coefficients [1]

$$\hat{\sigma}_n^2 = \frac{\text{median}(|y_i|)}{0.6745} \quad y_i \in \text{subband } HH \tag{4}$$

Sendur et al. [9] proposed a low complexity but powerful scheme to denoise images by exploiting the dependencies between the coefficients and their parents in detail within each scale. In [9] reasonably good results were obtained. The signal variance σ^2 for the k^{th} wavelet coefficients is defined by

$$\sigma^2 = \sigma_k^2 - \sigma_n^2 \tag{5}$$

where σ_k^2 is the signal variance of noisy observations y_1 and y_2. σ_k^2 is estimed

$$\hat{\sigma}_k^2 = \frac{1}{M} \sum_{y_i \in N(k)} y_i^2 \tag{6}$$

where $N(k)$ is defined as all coefficients within a windows that is centered at k^{th} coefficient. The M is the size of the neighborhood. Then, σ can be estimated as

$$\hat{\sigma} = \sqrt{\left(\hat{\sigma}_k^2 - \hat{\sigma}_n^2\right)_+} \tag{7}$$

3. Varying Window Size Selection

In this section we describe our variable window algorithm. Our overall goal is to determine a reasonable window size in order to estimate the signal variance for each wavelet coefficient. Our approach is based on region merging. Assume that there is a region R including ω_k. Let $r_{k,0,} ,..., r_{k,Q-1.}$ be a partition of the region R, $r_{k,i} \cap r_{k,j} = \phi$, $i \neq j$, and $U_i r_{k,i} = R$. In addition, only a subregion $r_{k,0}$ includes ω_k.

Starting from the subregion $r_{k,0}$ including denoising point ω_k, the region in which to estimate the signal variance is expanded until the homogeneity of the variance is achieved. The measure of the homogeneity is defined according to the normalized difference of variances, that is

$$h_{k,q} = \frac{\left|\sigma_{k,q}^{2} - \sigma_{k,0}^{2}\right|}{\sigma_{k,0}^{2}}, q = 0,1,...,Q-1 \qquad (8)$$

where $\sigma_{k,q}^{2}$ is the local variance of region $r_{k,q}$. Since the local mean of wavelet coefficients is very small, $\sigma_{k,q}^{2}$ is approximately calculated by

$$\sigma_{k,q}^{2} = \frac{1}{\left|N_q(k)\right|} \sum_{y_i \in Nq(k)} y_i^2 \qquad (9)$$

where $N_q(k)$ is the set of all coefficients within $r_{k,q}$ and $\left|N_q(k)\right|$ is the cardinality of neighborhood $N_q(k)$.

Let $m_{k,q}$ be a binary factor indicating where the variance $\sigma_{k,q}^{2}$ is homogenous with $\sigma_{k,0}^{2}$, or not. That is

$$m_{k,q} = \begin{cases} 1, & if\ h_{k,q} < t_k \\ 0, & otherwise \end{cases} \qquad (10)$$

where t_k is the threshold defined by

$$t_k = \beta 2^{(J-j)}, j = 0,...,J \qquad (11)$$

In (11), β is a scaling constant, $j = 0$ indicates the finest scale, and $j = J$ indicates the coarsest scale of the wavelet decomposition. Then, the estimate of the signal variance $\hat{\sigma}^2$ is obtained by

$$\hat{\sigma}^2 = \left(\frac{\sum_{q=0}^{Q-1} \sigma_{k,q}^{2} . m_{k,q}}{\sum_{q=0}^{Q-1} m_{k,q}} - \sigma_n^2 \right)_+ \qquad (12)$$

Image denoising is achieved by employing (2) with the estimated signal variance $\hat{\sigma}^2$.

In our method, there are 2^{Q-1} differently shaped windows. We exploit a square-shaped region R and square shaped subregions for simplicity of computation. In this letter, we use $9 \times 9\ R$, and nine 3×3 subregions. Therefore, $Q = 9$ and there exist 256 differently shaped windows.

4. Results And Discussion

We tested our algorithm on a number of images, here, we only report the results for Shape, Anime, Bike and Resolution illustrated in Figure.1. We generated i.i.d. Gaussian noise at four different values of the noise variance, σ_n^2. All simulations were performed with a five-scale orthogonal wavelet transform by using Daubechies length-eight filter. For practical purposes, we use $9 \times 9\ R$, and nine 3×3 subregions. Therefore, $Q = 9$.The scaling constant of threshold t_k was set at $\beta = 0.1$ (our results were insensitive to this value). We compared the proposed algorithm to other effective systems in the literature, namely VisuShrink, BayesShrink, NormalShrink, and BiShrink. Performance analysis is done using the peak signal-to-noise ratio (PSNR) mesure. The PSNR is defined by

$$\text{PSNR} = 20.log_{10}\left(\frac{255}{\varepsilon}\right)$$

where ε is the root mean-squared error given by

$$\varepsilon = \sqrt{\frac{1}{N} \sum \left(\omega_k - \hat{\omega}_k\right)^2}$$

where N is the number of image pixels.

Each PSNR value is averaged over five runs. The results can be seen in Table 1. In this table, the highest PSNR value among five algorithms is emphasized with a star (*). As seen from the results, our algorithm mostly outperforms the others.

Some illustrative images are given in Figure 2. Figure 2.(a) show the noisy image (shape), with a PSNR value of 24.59. The denoised image obtained using VisuShrink, BayesShrink, NormalShrink, BiShrink and our model are illustrated in Figures 2.(b, c, d, e, f) respectively and have PSNR values of 29.57, 29.78, 30.48, 32.53, and 32.98, respectively. We can see that our model has a significant performance improvement in both PSNR and visual quality.

5. Conclusions And Perspective

In this paper, we proposed a new algorithm for determining the variable size of a locally adaptive window using a region-based approach. Because our approach uses a nearly arbitrarily shaped window to obtain more accurate local statistics of images, the denoising method using a varying window size according to the homogeneity of variance is an efficient algorithm in removing white Gaussian noise from image. The comparison suggests the new denoising results outperform the best wavelet-based results reported in the literature.

Other new algorithm for determining the variable size of a locally adaptive window are under way and the results are postponed to a subsequent publication.

References:

[1] D. L. Donoho and I. M. Johnstone, Ideal spatial adaptation via wavelet shrinkage, *Biometrika,* vol. 81, 1994, pp. 425-455.

[2] P. Moulin and J. Liu, Analysis of multiresolution image denoising schemes using generalized Gaussian and complexity priors, *IEEE Trans. Info. Theory*, vol. 45,1999, pp. 909-919.

[3] S. Chang, B. Yu, and M. Vetterli, Adaptative wavelet thresholding for image denoising and compression, *IEEE Trans Processing*, vol. 9, 2002, pp. 1532-1546.

[4] Sendur, L; Selesnick, I. W., A bishrinkage function for wavelet based denoising, *IEEE Trans on Signal Processing*, vol. 50, 2000, pp. 2744-2756.

[5] Crouse, M. S.; Nowak, R. D.; Baraniuk, R, G., Wavelet bases signal processing using hidden markov models, *IEEE Trans on Signal Processing*, vol. 46, 1998, pp. 886-902.

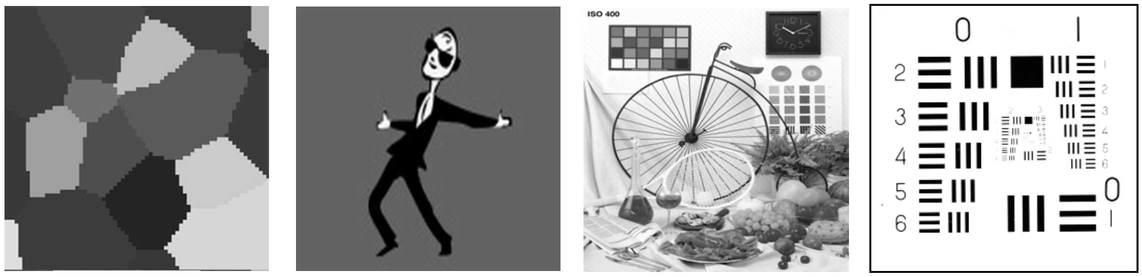
[6] H. Choi, J. Romberg, R. Baraniuk, and N. Kingsbury, Hidden Markov tree modeling of complex wavelet transforms, In: *Proc. IEEE Int. Conf. Acoustics, Speech, Signal Processing, Istanbul*, 2000 Turkey.

[7] E. P. Simoncelli, Bayesian denoising of visual images in the wavelet domain, Bayesian Interference in Wavelet Bases Models.eds. P Miller and B Vidakovic. Springer-Verlag, , 1999, In: *Lecture Noted in Statistics,* 141.

[8] E. P. Simoncelli. Modelling the joint statistics of images in the wavelet domain, *Proc. SPIE*, 313(1), , 1999, pp. 188-195.

[9] Sendur, L; Selesnick, I. W., Bivarite shrinkage with local variance estimation, *IEEE Signal Processing Letters*, vol. 9, 2002, pp. 438-441.

[10] Lakhwinder Kaur, Savita Gupta, and R. C. Chauhan, Image denoising using wavelet thresholding, . *Third Conference on Computer Vision, Graphics and Image Processing, India* , Dec 2002 16-18.



Shape Anime Bike Resolution

Figure1. original images test

Table.1

PSNR values of denoised images for different test images and noise levels of Noisy, VisuShrink, BayesShrink, NormalShrink, BiShrink , Our Model.

Image	σ_n	Noisy	VisuShrink	BayesShrink	NormalShrink	BiShrink	Our Model
Shape	10	28.1	32.85	33.29	33.62	35.65	36.10 *
	15	24.59	29.57	29.78	30.48	32.53	32.98 *
	20	22.13	27.23	27.66	28.18	30.35	30.75 *
	25	20.26	25.89	26.48	26.73	28.67	29.07 *
Anime	10	28.13	32.5	33.33	33.55	35.33	35.54 *
	15	24.61	29.24	30.37	30.43	32.2	32.44 *
	20	22.11	27.11	28.13	28.07	30.05	30.29 *
	25	20.18	25.35	26.37	26.33	28.42	28.63 *
Bike	10	28.11	29.88	30.32	30.56	33.95	34.07 *
	15	24.6	27.71	28.61	28.93	31.63	31.86 *
	20	22.13	24.07	25.61	25.79	29.72	29.79
	25	20.22	23.61	24.09	24.78	28.63 *	28.56
Resolution	10	28.23	31.08	31.14	31.71	33.18	33.99 *
	15	24.7	28.06	28.21	28.8	30.08	30.73 *
	20	22.19	25.76	26.15	26.73	27.88	28.39 *
	25	20.24	23.97	24.58	24.69	26.17	20.58 *

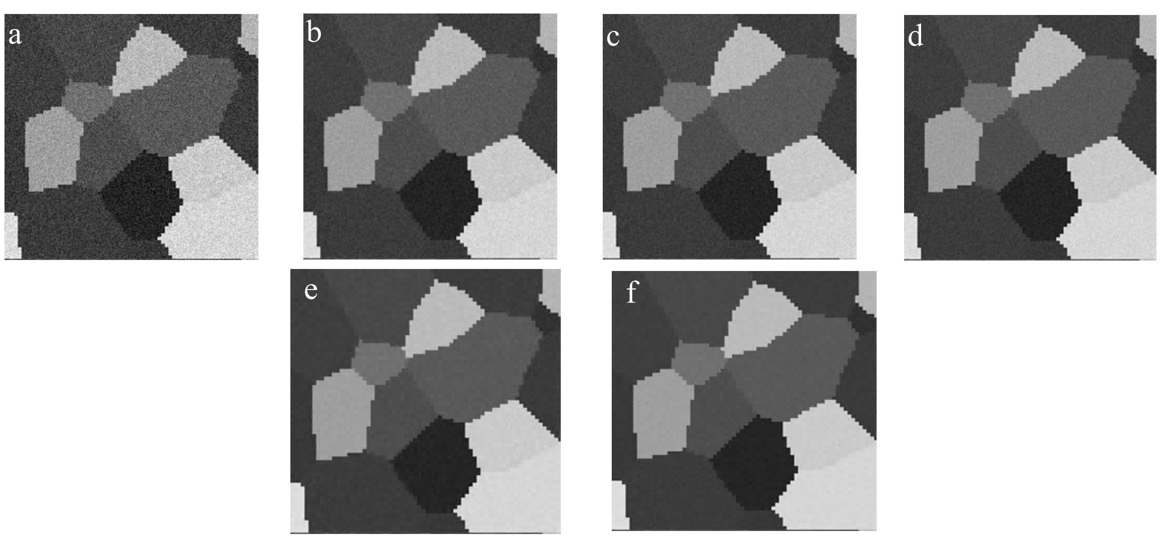


Figure 2. (a) noisy images (PSNR=24.59**), (b)**VisuShrink (PSNR=29.57), **(c)**BayesShrink (PSNR=29.78), **(d)**NormalShrink (PSNR=30.48), **(e)**BiShrink (PSNR=32.53), **(f)**Our model (PSNR=32.98).