CDMA Channel Estimation with Adaptive Fuzzy Filters

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Abstract: -This paper proposes a hierarchical system of three adaptive fuzzy filters into the RAKE receiver to estimate the channel coefficients in a downlink multipath fading CDMA channel. The simplest filter is proved to be equivalent with the alpha tracker. In tested circumstances (Rayleigh fading with AWGN) the two simpler filters proved to be most useful. The nested nature of the filters makes it possible to construct a 'hybrid' filter according to the situation.

Key-Words: -Communication, CDMA, Fuzzy Tracking, Rayleigh Fading, Multipath, RAKE

1 Introduction

In the code-division multiple access (CDMA), the channel estimation is an important task because it considerably increases the accuracy of a system. The fuzzy reasoning offers a one possibility to this problem.

The fuzzy logic dates back on the sixties when Zadeh published his article '*Fuzzy sets*' [6]. The idea in the fuzzy reasoning is to rule the uncertainty, to make decisions based on insufficient, faulty or contradictory information. The heuristic nature of the fuzzy inference offers a wide range of applications especially in systems, where the system model is complex or sometimes unknown.

In [4] we introduced a fuzzy filter to track the channel coefficients in a fading multipath downlink CDMA channel. In this paper we continue those studies by constructing a hierarchical group of three adaptive filters and by comparing their accuracy to each other. The trackers are implemented in the conventional RAKE receiver and the bit error rate (BER) values are counted in the multipath many users conditions. As a theoretical result it turns out that the simplest type of considered fuzzy filters includes in the channel estimation well known alpha tracker.

2 Data Model

The synchronous data of a user k, $k = 1,2,\ldots,K$ have the form

$$x_k(t) = b_k(t)s_k(t) \tag{1}$$

and the combination of all users equals

$$x(t) = \sum_{k=1}^{K} b_k(t)s_k(t) \tag{2}$$

where $b_k = b_k(t) \in \{\pm 1\}$ is the symbol and $s_k = s_k(t) \in \{\pm 1\}$ the signature waveform (code) transmitted by the kth user, respectively. The processing gain $C = T / T_c$ (T is the duration of a symbol and T_c the chip interval).

In the data channel the signal x is corrupted by the fading, multiple access interference (MAI) and noise. In addition, due to reflections from buildings, hills, mountains and so on the electromagnetic radiation proceeds along many paths to a receiver's antenna. The lengths of those paths are different and therefore the faded and noisy signals of each paths are delayed and influenced by the inter-path interference (IPI). So the received signal (chip flow) has the form

$$y(t)=\sum_{l=0}^{L-1} c_l(t)x(t-d_l)+n(t) \qquad (3)$$

where L is the number of paths, c_l $(l=0,1,...,L-1)$ time-dependent complex channel coefficients of each path, d_l the delay of the path l and $n(t)$ an additive zero-mean white Gaussian noise (AWGN). By combining (2) and (3) we have

$$y(t)=\sum_{l=0}^{L-1}\sum_{k=1}^{K} c_l(t)b_k(t-d_l)s_k(t-d_l)\ +n(t) \qquad (4)$$

3 Simulation Model

The symbols are transmitted in the packets of n information symbols. In addition, in front and behind of every packet m auxiliary symbols (unknown to the receiver) are used.

m auxiliary symbols	*n* data symbols	*m* auxiliary symbols

Fig. 1 The structure of a simulation symbol packet.

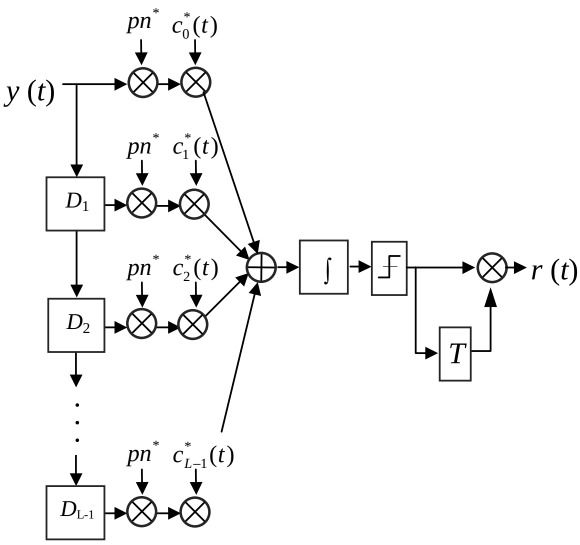

Fig. 2 The RAKE receiver.

The sampled chip flow is received. In the reception the conventional RAKE receiver is used and the delays d_l of each path are assumed to be known. The channel coefficients $c_l(t)$ are assumed to be constant during the symbol period T.

4 Tracking Methods

The tracking methods considered in this paper are iterative in the sense that the estimated symbol $\hat{b}(i-1)$ is immediately used in the prediction of the coefficient $c_l(i)$. Especially in cases where the channel is faded, an erroneous estimate $\hat{b}(i-1)$ (+1 or –1) starts the process in which the signs of the following estimates $\hat{c}_l(i)$ and symbols $\hat{b}(i)$ are systematically changed in a long period. By using differential modulation we can ignore the described phenomena and avoid a series of bit errors. Therefore every symbol is differentially encoded before the spreading.

Alpha tracker

In the alpha tracker a channel coefficient $c_l(i)$ for the *i*th symbol is predicted with aid of the previous coefficient estimation $\hat{c}_l(i-1)$, the previous symbol estimation $\hat{b}(i-1)$ and measured data $y_l(i-1)$ separately for each path l:

$$\hat{c}_l(i)=\hat{c}_l(i-1)+(1-\alpha)(\tilde{c}_l(i-1)-\hat{c}_l(i-1)) \qquad (5)$$

where $\tilde{c}_l(i-1)=\hat{b}(i-1)y_l(i-1)$ is the measured coefficient.

The alpha tracker is adjusted by a single parameter only. In noisy circumstances the current estimation in heavily based on the previous estimation (alpha large, the correction term small) and vice versa. The difference $(\tilde{c}_l(i-1)-\hat{c}_l(i-1))$ is called the error term and it is denoted by $err_l(i-1)$.

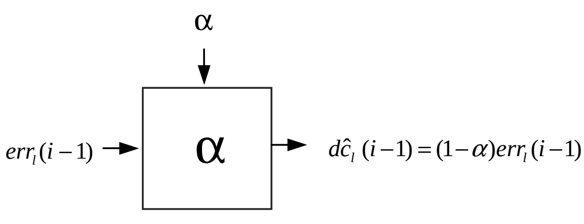

Fig. 3 The alpha tracker.

Fuzzy tracker

The fuzzy method applied here is based on two phases. In the first phase for each path the coefficients $\hat{c}_l'(i), i=1,2,...,n$ of a packet are estimated and the received chips are buffered. As in the case of the alpha tracker, to compute the coefficients $\hat{c}_l^{\prime}(i)$ the symbols $\hat{b}^{\prime}(i)$ must recursively be decided symbol-by-symbol. These temporary coefficients and symbols we call here the *pre-coefficients* and the *pre-symbols*, respectively.

Because of the tuning of the tracker pre-coefficients $\hat{c}_l^{\prime}(i)$ are time delayed versions of the true ones. In the second phase this disadvantage is corrected by the suitable opposite time-shift s:

$$\hat{c}_l(i) = \hat{c}_l^{\prime}(i+s) \qquad (6)$$

The coefficients $\hat{c}_l(i), i = 1,2,...,n$ together with the buffered chips of the packet under consideration are now used to estimate the final (differentially modulated) symbols:

$$\hat{b}(i) = \operatorname{sgn}\left(\sum_{l=0}^{L-1} \sum_{i_q=1}^{C} \hat{c}_l^{*}(i) s_k(i_q) y(C(i-1) + i_q + d_l) \right)$$

$$= \operatorname{sgn}\left(\sum_{l=0}^{L-1} \hat{c}_l^{*}(i) \sum_{i_q=1}^{C} s_k(i_q) y(C(i-1) + i_q + d_l) \right) \qquad (7)$$

By denoting $\mathbf{s}_k = \left(s_k(1),...,s_k(C) \right)^{\mathrm{T}}$, $\mathbf{y}_{il} = \left(y(C(i-1)+1+d_l,...,y(Ci+d_l) \right)^{\mathrm{T}}$ we have

$$\hat{b}(i) = \operatorname{sgn}\left(\sum_{l=0}^{L-1} \hat{c}_l^{*}(i) \mathbf{s}_k^{\mathrm{T}} \mathbf{y}_{il} \right) \qquad (8)$$

The final result can be obtained by demodulating symbols in (8).

5 Hierarchical System of Fuzzy Trackers

In the receiver the first symbol of a packet is simply guessed. The remaining $m-1$ auxiliary symbols are determined by using the alpha tracker with fixed alpha.

5.1 Three-input model

First we consider the three-input fuzzy tracker. The first one is the difference of the measured and the predicted coefficients $err_l(i-1) = \tilde{c}_l(i-1) - \hat{c}_l^{\prime}(i-1)$ (error). The second one is the chance of that difference $derr_l(i-1) = err_l(i-1) - err_l(i-2)$ (change in error) and the third one is the delayed output of the filter $d\hat{c}_l^{\prime}(i-2)$ (feedback). As an output the tracker gives the correction term $d\hat{c}_l^{\prime}(i-1)$ for the next coefficient (Figure 4). So we have

$$\hat{c}_l^{\prime}(i) = \hat{c}_l^{\prime}(i-1) + d\hat{c}_l^{\prime}(i-1) \qquad (9)$$

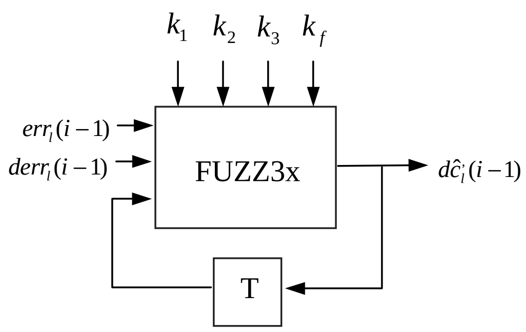

Fig. 4 The three-input fuzzy tracker.

Membership functions

For the input terms we use the triangle-shape membership functions. For the error term we have

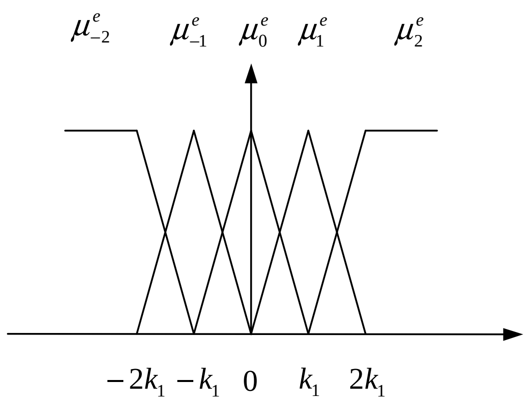

Fig. 5 The triangle-shape membership functions of error term ($n = 2$)

The membership functions for the change in error $\mu_j^{de}, j = -n,...,n$ and for the feedback $\mu_k^{dc}, k = -n,...,n$ are defined by the analogous way.

For the output we use singletons:

$$\mu_l^{out}(z) = \begin{cases} 1, & z = lk_f \\ 0 & \text{otherwise} \end{cases} \qquad (10)$$

$l = -3n, -3n+1 \ldots, 3n-1, 3n$

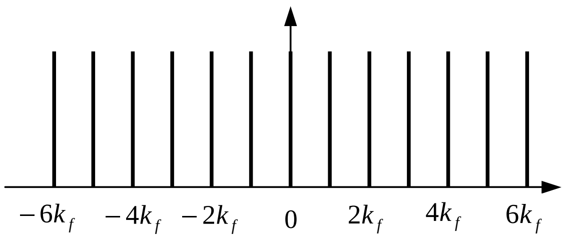

Fig. 6 The singletons for the output term ($n = 2$)

FAM

The fuzzy associative memory (FAM) used in this paper is the array

$$FAM(i, j, k) = k_f (i + j - k) \tag{11}$$

and it can be imagine as a three dimensional 'cubic'

Fuzzy inference and defuzzification

In the literature several fuzzy inference and defuzzification methods can be found [2]. In the following the *sum-product* method is applied. So

$$dc(p) = k_f \sum_{k=-n}^{n} \sum_{j=-n}^{n} \sum_{i=-n}^{n} (i + j - k) \mu_i^e(e(p)) \mu_j^{de}(de(p)) \mu_k^{dc}(dc(p-1))) \tag{12}$$

where $e(p) = err(p)$ is the error term, $de(p) = derr(p)$ the change in error and $dc(p-1)$ the delayed feedback term. The sum-product decision is a quite fast operation because of its simple algebraic structure and it gives the relative smooth control surfaces. Because the triangle-shape membership functions have the compact carrier, at most eight fuzzy rules are active simultaneously.

5.2 Two-and single input model

If we drop the feedback term in the tracker illustrated in Figure 7 the fuzzy filter has only two inputs: error and the change in error.

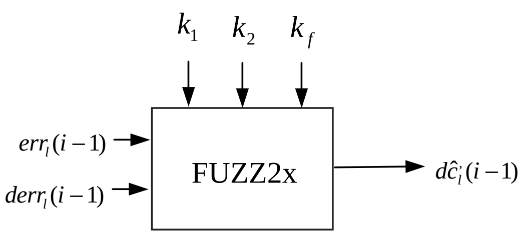

Fig. 7 The two-input fuzzy tracker.

By setting $dc(p-1) = 0$ in (12) we have $\mu_0^{dc}(dc(p-1)) = 1$, $\mu_k^{dc}(dc(p-1)) = 0, \quad k \neq 0$ which implies

$$dc(p) = k_f \sum_{j=-n}^{n} \sum_{i=-n}^{n} (i + j) \mu_i^e(e(p)) \mu_j^{de}(de(p)) \tag{13}$$

So the two-input filter is the special case of the three-input one. Furthermore, if $de \equiv 0$ in (13), the two-input model is reduced to a single-input-single-output (SISO) system.

$$dc(p) = k_f \sum_{i=-n}^{n} i \mu_i^e(e(p)) \tag{14}$$

Without loss of generality it can be assumed that the input error term is bounded, $|e(p)| \leq S$ for all p. Let $k_1 \geq S$ in (10). Now we get from (14)

$$dc(p) = \frac{k_f}{k_1} e(p) \tag{15}$$

By choosing $\frac{k_f}{k_1} = 1 - \alpha$ formula (15) gives the output of the alpha tracker. We have proved the following result: for a given alpha tracker it is always possible to construct a single-input fuzzy tracker such that their outputs coincide, i.e. *the alpha tracker is a special case of the fuzzy tracker (12).*

6 Comparison of Trackers

The three-input fuzzy tracker in Figure 4 includes four adjustable parameters k_1, k_2, k_3, k_f and the time-shift s (16). In addition, the number of fuzzy rules can vary. Both too few and too many rules decrease the results . In our simulations all input fuzzy variables include five terms ($n = 2$) implying 13 singletons in the output variable. Correspondingly, the two-input tracker has three parameters k_1, k_2, k_f, the time-shift term s and 9 output singletons ($n = 2$). As shown in (14), the alpha tracker can be considered a version of a single input fuzzy tracker with the single adjustable parameter α. No time-shift is used in the case of the alpha tracker.

In all cases the values of the parameters depend on the number of paths L, on the velocity v of the mobile, on the signal-to-noise ratio SNR and on the number of the competing users K. The signal structure of the last ones is not utilized in this paper but they are only considered as a type of noise.

To compare the different type trackers the parameters are numerically optimized to minimize the mean squared error (MSE)

$$MSE = \frac{1}{NL}\sum_{l=0}^{L-1}\sum_{i=1}^{N}\left| e_l(i)\right|^2 \tag{16}$$

in which L is the number of paths, N is the number of data symbols and $e_l(\cdot)$ is the error term: $\left|e_l(i)\right| = \min\left(\left|c_l(i)-\hat{c}_l(i)\right|,\left|c_l(i)+\hat{c}_l(i)\right|\right)$.

6.1 Three-and two input model

First the three-input fuzzy tracker with triangular membership functions is optimized in the case of $SNR = 0, 5, 10, 15, 20$ dB, $L = 1, 2, 3, 4$, $v = 50, 80, 100$ km/h and $s = 1, 2, 3, ..., 24$. All the path powers are equal (0 dB). Based on these measurements, the dependence of the parameters k_1, k_2, k_3, k_f on SNR, L, v and K is illustrated by the limited four-variable linear model:

$$\begin{aligned} k_1 &= 0.3800 - 0.0063SNR - 0.2507L \\ &+ 0.0290v - 0.1440K \quad (0.1 \le k_1 \le 1) \end{aligned} \tag{17}$$

The value of k_2 is kept constant $k_2 = 1$ because the ratio k_1/k_2 is more essential than the absolute values of k_1 and k_2. For k_3 and k_f we have

$$\begin{aligned} k_3 &= 1.1800 - 0.0163SNR - 0.2507L \\ &+ 0.0290v - 0.1440K \quad (0.1 \le k_3 \le 2) \end{aligned} \tag{18}$$

$$\begin{aligned} k_f &= 0.0280 - 0.0004SNR - 0.0045L \\ &+ 0.0007v - 0.0010K \ (0.01 \le k_f \le 0.1) \end{aligned} \tag{19}$$

Finally

$$s = \left[\begin{matrix} 22.35 - 0.1383SNR + 0.0553L \\ -0.0040v + 1.0800K \end{matrix}\right]^{1} \tag{20}$$
$$(0 \le s \le 24)$$

To test the influence of the feedback term in the three-dimensional model we compared its results to the two-input filter which was obtained by omitting k_3.

6.2 Single-input model (alpha tracker)

The alpha tracker is optimized independently by using the same test data as in the case of the three-input filter. For alpha we estimated the formula

$$\begin{aligned} \alpha &= 0.8767 - 0.0060SNR + 0.0553L \\ &- 0.0040v + 0.0140K \ (0.1 \le \alpha \le 0.9) \end{aligned} \tag{21}$$

6.3 Results

All three-type of trackers were tested in the area of $0 \le SNR \le 20$ dB, $0 \le L \le 4$, each path equal power 0 dB, $50 \le v \le 100$ km/h and $5 \le K \le 15$. In each tested (SNR, L, v, K)-cell 10000 bits (20 frames) were used.

All the three trackers gave similar results in order of magnitude. However, we can see that the two-input model is the best one in relative noisy circumstances, say $0 \le SNR \le 10$ dB. For $10 < SNR \le 20$ dB the single input model i.e. the adaptive alpha tracker is the winner. In all cases the three-input model gave worse results than the another ones.

SNR \ L	1	2	3	4
0	0.2494	0.1726	0.1534	0.1646
5	0.1262	0.0672	0.0659	0.0878
10	0.0479	0.0273	0.0380	0.0580
15	0.0168	0.0174	0.0275	0.0489
20	0.0086	0.0138	0.0236	0.0476

BER

Table 1 Two-input fuzzy tracker, 15 users, each path 0 dB for all users, v= 100 km/h

SNR \ L	1	2	3	4
0	0.2593	0.1760	0.1737	0.1832
5	0.1189	0.0759	0.0850	0.0925
10	0.0485	0.0332	0.0450	0.0573
15	0.0164	0.0214	0.0351	0.0473
20	0.0060	0.0187	0.0335	0.0446

BER

Table 2 Alpha tracker (single input fuzzy tracker), 15 users, each path 0 dB for all users, v= 100 km/h

[1] [x] is the smallest integer less or equal than x.

SNR	L = 1	L = 2	L = 3	L = 4
0	0.2457	0.1274	0.0787	0.0555
5	0.1023	0.0316	0.0174	0.0095
10	0.0375	0.0044	0.0036	0.0030
15	0.0141	0.0022	0.0014	0.0022
20	0.0061	0.0010	0.0006	0.0018

BER

Table 3 Two-input fuzzy tracker, 5 users, each path 0 dB for all users, v= 50 km/h

SNR	L = 1	L = 2	L = 3	L = 4
0	0.2241	0.1363	0.0800	0.0620
5	0.1047	0.0289	0.0175	0.0124
10	0.0399	0.0055	0.0017	0.0025
15	0.0117	0.0009	0.0002	0.0002
20	0.0046	0.0002	0.0000	0.0002

BER

Table 4 Alpha tracker (single input fuzzy tracker) , 5 users, each path 0 dB for all users, v= 50 km/h

7 Conclusion

Among the considered three type trackers the two- and single input models proved to be the most accurate systems in the considered circumstances (Rayleigh fading CDMA channel with AWGN). In the three input model the feedback term did not introduced any benefits to the results. In [4] we used filters with fixed parameters together with the moving average of ten coefficients. By using the adjustable parameters the moving average is not needed and still the results improved.

The fact that the single input system is a special case of the two-input one makes possible easily to construct a ‘hybrid’ filter which changes its role according to the noise circumstances: in noisy situations the change in error input will be switch on and in clear situations off.

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