How to Achieve the Stock Control of a Corporation

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Abstract: - This paper investigates the capital structure of large corporations and shows how to take over control of them, spending only the necessary financial resources. In the first part, we define an enterprise as a controlled or colligated company, which is the basis for understanding how a strategy can be developed for either keeping or taking control, depending on the goal of the investors. After that, using Linear Programming (LP) and Integer Programming, we show the equations for an LP problem that can be solved for any LP package. We also show the algorithm that produces the equations for any shareholder capital structure.

Key-Words: - corporate governance, stock control, controllers of a company, finance, operational research, linear programming, integer programming.

1 Introduction

Corporate Governance is a complex subject, since corporations are structured in different ways. The United States, Canada, and Australia have corporations with similar ownership structures [1]. In these countries, usually, the shares are widely held, and no single investor holds more than a small fraction of these shares and a takeover is unthinkable. However, countries like Brazil and Germany have corporations with shareholders (natural persons and other companies) who hold a substantial amount of shares [2]. In these scenarios, in which a great amount of shares are not widely held, a takeover from an outsider investor is possible.

First of all, for someone who aims to get the control of a company or to get rid of an unexpected takeover, a *sine qua non* requirement is to understand the capital structure of the ownership of that company. Determining the ownership of a large company may be a very complicated task, due to the transitive nature of the control [3].

A large company or corporation is composed of its stock shareholders, represented by natural or artificial persons. Determining the control of a corporation is an intricate task, because a natural or an artificial person may appear as a stock shareholder in several companies, which in turn may also be shareholders of the corporation. For example: a corporation A and a natural person NP1 hold stocks of a corporation B. Besides, NP1 holds stocks of corporation A. Finding out the controller of corporation B is known as *Sum of Subsets*, which is a backtracking algorithm [4]. However, more important than determining the controller(s) of a corporation is to establish a strategy for getting or keeping its control.

By using Linear and Integer Programming, this paper shows how to get, with minimum cost, the control of a corporation, assuming that all of its capital structure of ownership is completely known. If the corporation has artificial persons as shareholders, it is also assumed that the ownership of these shareholders is known, too. Furthermore, in order to establish the best strategy to take over control, the algorithm needs to know the price per share and the amount of shares available of all the artificial persons involved.

2 The Control of A Corporation and How to Achieve it

The control of a corporation is based on the ownership of the stock shares, and it is possible to have two types of controllers: a natural person and an artificial person. The controller will be the one who has more than fifty percent of the stock shares. If this condition is satisfied, the company will have a unique controller. It should be pointed out that only stock shares with the right to vote are being taken into account. This remark is relevant because in some markets, as in the Brazilian market, there are two types of stock shares, where only one of them has the right to vote.

When there is not a shareholder with more than fifty percent, it will be assumed that the corporation is not a controlled company, but a colligated one (according to Brazilian laws, companies are colligated when the parent participates with more than 10% of the other's capital, without controlling it [5]. As a matter of simplicity, we are considering all investments of this type as colligated companies). In a colligated

company, the controllers will be determined by an agreement among shareholders that guarantees the majority of stock shares (more than a half). This condition must only be theoretically held, because the stock shares of large companies are dispersed among numerous shareholders that cannot meet together to make use of their power. However, the criterion of majority must always be held.

Having the control of a corporation implies that, according to Brazilian law [5], the controller can elect the administrators of the corporation. Consequently, the power stems from this control. In this case, a strategy to keep the company will be necessary for the one who holds its power.

2.1 How to Establish the Controller of a Corporation

As mentioned before, a corporation can be controlled or colligated. In Figure 1, it is presented an example with only controlled companies. In the following examples, for simplicity, it is always assumed that the amount of stock shares of any artificial person is 1000.

The controller of the corporation AP-A is AP-C, which holds 510 stock shares of AP-A. The AP-C's controller is AP-E, which holds 510 stock shares of AP-C. Therefore, the controller of corporation AP-A is AP-E's controller. For someone who wants to get the control of corporation AP-A, the first step is to take a look inside the shareholders of AP-E.

AP-B is controlled by AP-D and the natural person NP-1 controls AP-D.

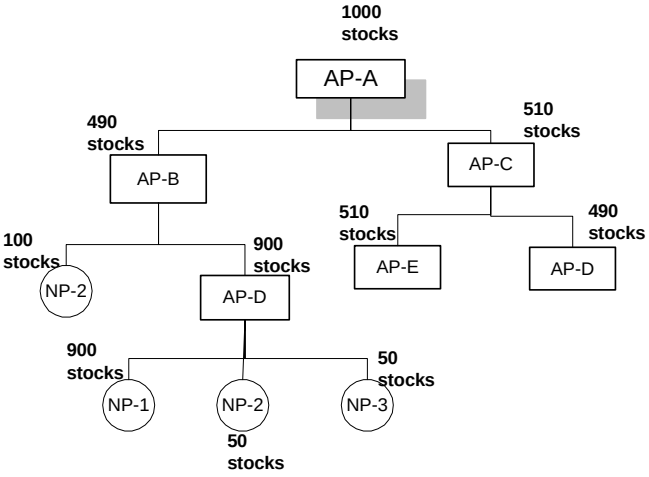

Fig 1 – **Shareholders of controlled corporations**.

The most surprising thing, in this example, is that the relation between control and ownership is not as strong as someone could imagine at first. In terms of possession, NP-1 has 62,18 % of the corporation AP-A due to the indirect possession of stock shares of company AP-D. It is obvious that NP-1 has made a very poor investment decision in the past. He or she has made the major investment inside the company, but its control belongs to the controller of corporation AP-E. If we suppose that AP-E has a shareholder with 51% of its stock shares, this shareholder would possess only the miserable 13,26% of the company AP-A. Nevertheless, this shareholder would exercise his or her influence by electing directors and approving the major AP-A corporate decisions.

Figure 2 presents another example of a controlled company. In this example, corporation AP-A is controlled by NP-1, because he or she has 49% of direct control plus 20% of indirect control; NP-1 has the majority of stock shares in corporation AP-B. From this example, we can envisage, how difficult the definition of the controller may be in large corporations. Although AP-A does not have a single shareholder with more than fifty percent of its ownership, AP-A is controlled, not colligated. Looking over all the company structure is then required to determine whether or not the company is controlled and, if this is the case, the controller.

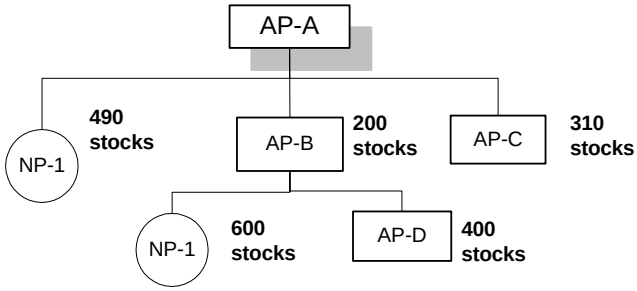

Fig 2 – **Shareholders of another controlled corporation AP-A.**

In Figure 3, another distribution of shareholders for corporation AP-A is shown. As it can be seen from this example, there is not a single shareholder with more than 51% at first level; NP-1 has 49%, NP-2 controls AP-B, and NP-4 controls AP-C. Therefore, any combinations with two shareholders would define the AP-A's controller.

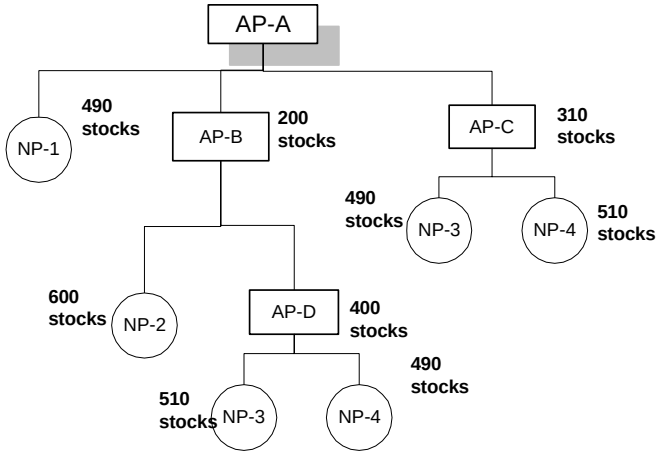

Fig 3 – **Shareholders of the colligated company AP-A.**

2.1.1 How to Achieve the Control of a Corporation

From the examples above, the reader should be able to envisage that it is essential to define a strategy for either getting the control of a corporation or keeping it, avoiding an unexpected takeover by a shareholder who could take advantage of a corporation's vulnerability.

This fact opens a range for optimization because it is possible to take over control of a corporation by investing more or less financial resources. It does not mean that it will always be possible to get the control of a corporation. We have just stated that a company may have a structure of shareholders that makes a takeover nearly impossible; this is one of the purposes of this paper – shed light on potential vulnerability in the capital structure of large corporations. Let's revisit the examples from the previous section.

Considering the example of the Figure 1, it is clear that this company is "well structured", since the corporation is a controlled one. The only way to get the control of corporation AP-A is by taking the control of AP-E. Anything can be made, except holding the majority shares of AP-E. In spite of this, the control of AP-A could be achieved with the possession of only about 26% of AP-A. Investing in AP-D is useless, considering the goal of controlling AP-A. The reason is that AP-D does not control AP-C , which in turn controls AP-A.

Taking into consideration the example of Figure 2, there is more than an option to achieve the control of AP-A. Nevertheless, the investor has to persuade NP-1 to sell his or her stock shares of AP-A or AP-B. The cheapest alternative depends on the price of stock shares of AP-A, AP-B and AP-C. In any case, it is useless to take over control of AP-D since this company does not control AP-B.

Figure 3 presents the most interesting example, since corporation AP-A is a colligated company. Thus, its control can be achieved by buying stock shares of any corporation in the structure. The AP-A controller will be defined by any combination of two shareholders at the first level of AP-A. Which combination should the investor select? It will only depend on the price per share of each company and whether these shares are available in the stock market. One possibility would be become the controller of AP-C and AP-B, supposing that the stock price per share of AP-A is too expensive. In addition, in order to become the AP-B controller, there are two possibilities: buying at least 501 shares from NP-2, or taking the control of AP-D and buying at least 101 shares from NP-2.

Analyzing manually all the alternatives is time consuming and prone to errors. For large corporations with complex capital structure of shareholders, it is necessary to develop a method to calculate computationally the best strategy. That method is presented in the following section

3 The Solution

The solution we propose uses Linear Programming [6] and Integer Programming [7]. A C++ program was developed; it reads all the capital structure of the shareholders of a corporation and the stock prices per share of each corporation involved, and then generates the linear equations to solve the problem,. After generating the equations, the program calls LINGO [8] to solve the linear equations. LINGO is a tool designed to build and solve linear, nonlinear, and integer optimization models. However, any linear and integer program package could be used, with minimal adjusts.

3.1 The Linear Equations

In this section, the example shown in Figure 3 will be used to demonstrate the necessary equations that allow us to establish the best strategy to achieve the control of AP-A.

The Linear Programming Problem (LPP) developed in this work uses the variables whose brief description is given in Table 1. The developed objective function of the LPP is shown in Table 2.

@BIN(CtrAP-A);	**(3)**
TotalCtrAP-A>=((TotalSharesAP-A/2)+1)*CtrAP-A;	**(4)**
TotalCtrAP-A=BuySharesAP-A+200*CtrAP-B+310*CtrAP-C;	**(5)**
BuySharesAP-A<=490;	**(6)**

Table 4. **Constraints for a corporation that is not at the leaf of the capital structure.**

Variable	Description
PriceAP-X	The stock price per share of corporation AP-X.
BuySharesAP-X	The stock shares of corporation AP-X that must be bought. The range value is from 0 to the minimum to get the control of AP-X.
CtrAP-X	It is a binary integer variable that indicates whether or not stock shares of AP-X must be bought.
TotalSharesAP-X	The total shares of AP-X with voting right.
TotalCtrAP-X	The total shares that an investor have to take in order to control AP-X

Table 1. **Variables used in the LPP.**

MIN =PriceAP-A * BuySharesAP-A + PriceAP-B * BuySharesAP-B + PriceAP-C * BuySharesAP-C +PriceAP-D * BuySharesAP-D;	**(1)**

Table 2. **The AP-A Objective Function of the LPP.**

Generally, for every capital structure we have the formula:

$Min = \sum_{i=A}^{every\ company} \Pr iceAP-i * BuySharesAP-i$	**(2)**

Table 3. **The Objective function of the LPP for every company.**

The purpose is to minimize the total investment that an investor has to do in order to achieve the control of AP-A.

There is a set of constraints of the LPP for each corporation in the capital structure. However, these constraints differ, depending on whether the corporation in the structure is at the leaf or not.

At first, taking into consideration corporation AP-A, which is not at a leaf, its constraints are shown in Table 4.

The reason for constraint (3) is to indicate to Lingo software that the variable CtrAP-A must be treated as a binary integer variable, a special case of an integer variable that is required to be either zero or one. If CtrAP-A is equal to zero, it means that the investor should not buy shares of AP-A. The variable CtrAP-A is linked to the inequality constraint (4) and the equality constraint (5). On the contrary, if CtrAP-A is equal to 1, the investor should take over control of AP-A and the constraint (4) guarantees that the total shares that this investor has to take in order to control AP-A is more than fifty percent of the total stock shares with the right to vote. The constraint (5) is the summation of all the possibilities to take the control of AP-A. The investor can buy shares of AP-A, which is represented by *BuySharesAP-A*. In addition, the investor can take over control of corporation AP-B, which adds 200 votes and, finally, he or she can take over control of AP-C. The last constraint only establishes the limit of shares the investor can buy from any natural person AP-A's shareholder. An artificial person may appear at a leaf in the capital structure tree for two reasons: the shareholders are only natural persons, or the capital structure of that corporation is unknown. In this case, only two constraints are necessary. In Table 5 we show the constraints for AP-C that is at a leaf.

BuySharesAP-C>=((TotalSharesAP-C/2)+1) * CtrAP-C;	**(7)**
@BIN(CtrAP-C);	**(8)**

Table 5. **Constraints for a corporation that is at the leaf of the capital structure.**

If the company is at the leaf, that means the investor can only buy shares of the company itself, so *BuySharesAP-C* must be more than half of the total shares of the corporation, assuming that the control of this corporation is needed in order to achieve the control we are fighting for. An A *BuySharesAP-C* upper limit is not necessary because the variable appears at the objective function. If this variable increases more than necessary, the objective function will get worse, so it will remain the minimum possible for a better result.

The data section of the Lingo file for the example considered is shown in Table 6.

LPPConstruction(corporation AP-X)

if (corporation has already been introduced into the LPP)
// a corporation may appear in several parts in the capital structure
return
add to the objective function "PriceAP-X * BuySharesAP-X"
add the price per share of X (PriceAP-X) and the amount of shares available (TotalSharesAP-X) into the Data Section of the model
add constraint **(3)** - "@BIN(CtrAP-X)
add constraint **(4)** – "TotalCtrAP-X>=((TotalSharesAP-X/2)+1)*CtrAP-X"
start to mount the constraint **(5)** – "TotalCtrAP-X=BuySharesAP-X+"
for (each shareholder - S_i - of corporation X that is an artificial person)
add to constraint **(5)** the gain for controlling AP-X if we get the control of S_i.
if (S_i has an any artificial person as shareholder)
// a recursive call to generate the equations for S_i
call **LPPConstruction(corporation** S_i**)**
else
// the shareholder S_i is at the leaf in the capital structure
if (S_i has not already been introduced into the LPP)
// a shareholder may S_i appear in several parts in the capital structure
add to the objective function "PriceS_i * BuySharesS_i"
add the price per share of S_i (PriceS_i) and the amount of shares available (TotalSharesS_i) into the Data Section of the model
add constraint **(7)** – "BuySharesS_i >=((TotalSharesS_i /2)+1)*CtrS_i "
add constraint **(8)** – "@BIN(CtrS_i) "
end
end
end
add constraint **(6)** "BuySharesAP-X<=the amount of shares in possession of natural persons"
end

Figure 4. **The outline of the recursive function LPPConstruction that generates the LPP**.

The stock price per share and the total shares available of each company is provided. In addition, the equality *CtrAP-A=1* establishes that the goal is to take over control of AP-A.

```
DATA:
CtrAP-A=1;
PriceAP-A=200;
TotalSharesAP-A=1000;
PriceAP-B=100;
TotalSharesAP-B=1000;
PriceAP-D=10;
TotalSharesAP-D=1000;
PriceAP-C=1;
TotalSharesAP-C=1000;
```

Table 6. **Data Section of the LPP.**

After running Lingo to solve the LPP, taking into account the prices and the amount of shares above, the results found are shown in Table 7.

Variable	Value
BuySharesAP-B	**101**
BuySharesAP-C	**501**
BuySharesAP-D	**501**

Table 7. **The results of non zero variables.**

The main reason for these results is the price of stock shares of AP-A, which is too expensive. So the solution is to try to control the other shareholders. To control AP-C and AP-D, it is necessary to buy 501 shares of each, since they are at the leaf of the capital structure. But controlling AP-D is not enough to control AP-B, so it is necessary to buy 101 shares of natural person NP-2 in order to take over control of the majority shares of AP-B. It is obvious that if the prices change, the result may be completely different. In the next section, the function that generates the LPP is outlined.

3.1 Algorithm to Construct the Equations.

In order to achieve the results of this research, a whole system was developed, which includes the input of data, the determination whether a corporation is controlled or colligated, and the percentage of possession and control of each shareholder in the capital structure. However, in this paper only the algorithm that constructs the LPP for any corporation is shown (Figure 4). After the construction of the whole LPP, Lingo is called in order to solve it, and the results are presented to the user.

The LPPConstruction function is a recursive one. Usually, it starts at the root of a capital tree structure and produces the equations to the whole capital structure tree.

We believe that the algorithm is clear enough, but a point deserves to be mentioned: if the capital structure of an artificial person is unknown, this corporation is treated as a colligated company, and it will appear at a leaf of the capital structure. In this case, we assume that it is possible to buy its shares if it is necessary to achieve the desired control.

4 Conclusion

Possession does not necessarily mean control and, if this is true, and it really is, there is a range for optimization. It is possible to get the control of a corporation spending more or less money. It depends on the strategy adopted by the investor. In addition, if it is possible to spend less money to hold the power, why should a smart investor not do it? The saved money could be used to make another investment, perhaps taking over control of another company.

However, as important as taking over control of a corporation is keeping that control. If the corporation is not “well structured”, it can be the target of a hostile takeover, and the actual controller may be replaced by another. In fact, after analyzing some real capital structures, we found out that a takeover could be possible.

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