Rectangular Slot Resonator With Four Dielectrics Layers

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***Abstract* — This work shows the development and the analysis of rectangular slot line resonator with four dielectrics layer, using the full wave Transversal Transmission Line – TTL method. Starting from the Maxwell's equations, obtaining a set of equations that represents the electromagnetic fields for the four Layers of rectangular slot line resonator, obtaining with a concise and effective procedures, the complex resonant frequency. This complex resonant frequency is calculated through double spectral variables.**

I. INTRODUCTIONS

The rectangular slot line resonator of four layers, consist of one rectangular slot line resonator, where there are two layers under and two layers over it, how it is shown in Fig. 1, with width w and length l. For the analysis through the method TTL, the basis function adequate and Gherkin's procedure is obtained the general equations of the fields electromagnetic allowing in concise and effective form, the calculation of the complex resonant frequency. This complex resonant frequency is calculated through double spectral variables, being the same, used in the elaboration of the efficiency and bandwidth's parameters.

With the aid of the system of Cartesian coordinates and the dimensional nomenclatures and electromagnetic as presented in Fig. 1.a (spatial view) and Fig. 1.b (traverse section of the structure), all are obtained referred them equations of fields, being considered despicable the thickness of the slot line.

II. FIELDS IN STRUCTURE

Due to your limitation in the length, the equations should be used for the analysis in the spectral domain in " x " and " z " directions as function. Therefore the field equations are applied for double Fourier transformed defined as:

$$\tilde{f}(\alpha_n, y, \beta_k) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y,z)\cdot e^{j\alpha_n x}\cdot e^{j\beta_k z}\, dxdz \quad (1)$$

Where α_n is the spectral variable in the "x" direction and β spectral variable in the "z" direction.

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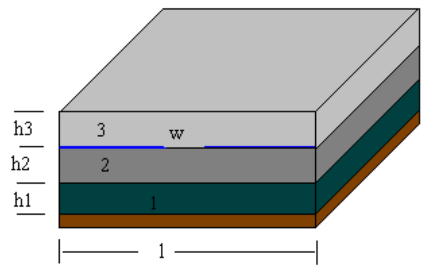


(a)

Fig. 1 – (a) spatial view of the four layer's slot line resonator where the fourth layer is the air.

After using the equations of Maxwell in the spectral domain, the general equations of the electric and magnetic fields in the method TTL, are obtained as:

$$\tilde{E}_{xi} = \frac{1}{\gamma_i^2 + k_i^2}\left[-j\alpha_n \frac{\partial}{\partial y}\tilde{E}_{yi} + \omega\mu\beta_k \tilde{H}_{yi}\right] \quad (2.1)$$

$$\tilde{E}_{zi} = \frac{1}{\gamma_i^2 + k_i^2}\left[-j\beta_k \frac{\partial}{\partial y}\tilde{E}_{yi} - \omega\mu\alpha_n \tilde{H}_{yi}\right] \quad (2.2)$$

$$\tilde{H}_{xi} = \frac{1}{\gamma_i^2 + k_i^2}\left[-j\alpha_n \frac{\partial}{\partial y}\tilde{H}_{yi} - \omega\varepsilon\beta_k \tilde{E}_{yi}\right] \quad (2.3)$$

$$\tilde{H}_{zi} = \frac{1}{\gamma_i^2 + k_i^2}\left[-j\beta_k \frac{\partial}{\partial y}\tilde{H}_{yi} + \omega\varepsilon\alpha_n \tilde{E}_{yi}\right] \quad (2.4)$$

Where:

i = 1, 2, 3,4 represent four dielectrics regions of the structure;

$$\gamma_i^2 = \alpha_n^2 + \beta_k^2 - k_i^2 \quad (2.5)$$

is the constant of the propagation in y direction; α_n is the spectral variable in "x" direction and β_k the spectral variable in "z" direction.

$k_i^2 = \omega^2\mu\varepsilon = k_0^2\varepsilon_{ri}^*$ is the number of wave of i^th term of Dielectric region;

$\varepsilon_{ri}^* = \varepsilon_{ri} - j\dfrac{\sigma_i}{\omega\varepsilon_0}$ is the dielectric constant relative of the material with losses;

$\omega = \omega_r + j\omega_i$ is the complex angular frequency;

$\varepsilon_i = \varepsilon_{ri}^* \cdot \varepsilon_0$ is the dielectric constant of the material;

The equations above are applied to the resonator being calculated, the fields E_y and H_y through the solution of the equations of wave of Helmoltz in the spectral domain [2]-[4]:

$$\left(\frac{\partial^2}{\partial y^2} - \gamma^2\right)\tilde{E}y = 0 \quad (3.1)$$

$$\left(\frac{\partial^2}{\partial y^2} - \gamma^2\right)\tilde{H}y = 0 \quad (3.2)$$

The solutions of equations Helmoltz's equations for the four regions of the structure are given for:

For region 1:

$$\tilde{E}_{y1} = A_{1e} \cdot \cosh \gamma_1 y \quad (4.1)$$

$$\tilde{H}_{y1} = A_{1h} \cdot sinh \gamma_1 y \quad (4.2)$$

For region 2:

$$\tilde{E}_{y2} = A_{2e} \cdot \operatorname{senh} \gamma_2 y + B_{2e} \cdot \cosh \gamma_2 y \quad (4.3)$$

$$\tilde{H}_{y2} = A_{2h} \cdot \operatorname{senh} \gamma_2 y + B_{2h} \cdot \cosh \gamma_2 y \quad (4.4)$$

For region 3:

$$\tilde{E}_{y3} = A_{3e} \cdot \operatorname{senh} \gamma_3 y + B_{3e} \cdot \cosh \gamma_3 y \quad (4.5)$$

$$\tilde{H}_{y3} = A_{3h} \cdot \operatorname{senh} \gamma 3 y + B_{3h} \cdot \cosh \gamma 3 y \quad (4.6)$$

For region 4:

$$\tilde{E}_{y3} = A_{3e} \cdot e^{-\gamma_3 y} \quad (4.7)$$

$$\tilde{H}_{y3} = A_{3h} \cdot e^{-\gamma_3 y} \quad (4.8)$$

Substituting these solutions in the equations of the fields (2.1) the (2.4), in function of the unknown constants A_{21}, A_{22}, B_{21} and B_{22} is obtained, for example, for the region 2:

$$\tilde{E}_{x2} = \frac{-j}{K_2^2 + \gamma_2^2}\begin{bmatrix}(jw\mu_0\beta_k B_{21} + \alpha_n\gamma_2 A_{22})\cosh(\gamma_2 y)) + \\ (jw\mu_0\beta_k B_{22} + \alpha_n\gamma_2 A_{21})\operatorname{senh}(\gamma_2 y))\end{bmatrix} \quad (4.9)$$

$$\tilde{H}_{x2} = \frac{-j}{K_2^2 + \gamma_2^2}\begin{bmatrix}(jw\varepsilon_2\beta_k A_{21} + \alpha_n\gamma_2 B_{22})\cosh(\gamma_2 y)) + \\ (jw\varepsilon_2\beta_k A_{22} + \alpha_n\gamma_2 B_{21})\operatorname{senh}(\gamma_2 y))\end{bmatrix} \quad (4.9.1)$$

For the determination of the unknown constants, they are applied the conditions of close contour the regions 1, 2 and 3, or be:

For the regions 1 e 2: y = h1

$$\tilde{E}_{x1} = \tilde{E}_{x2} \quad (5.1)$$

$$\tilde{E}_{z1} = \tilde{E}_{z2} \quad (5.2)$$

$$\tilde{H}_{x1} = \tilde{H}_{x2} \quad (5.3)$$

$$\tilde{H}_{z1} = \tilde{H}_{z2} \quad (5.4)$$

For the regions 2 e 3: y =d- ; (g=h1+h2)

$$\tilde{E}_{x2} = \tilde{E}_{x3} = \tilde{E}_{xg} \quad (5.5)$$

$$\tilde{E}_{z2} = \tilde{E}_{z3} = \tilde{E}_{zg} \quad (5.6)$$

After several calculations it is obtained, for two region :

$$A_{21} = \frac{\varepsilon_1 \cosh(\gamma_2 y)}{\varepsilon_2\gamma_1 \operatorname{senh}(\gamma_1 g_1)\cosh(\gamma_2 g_2) + \gamma_2\frac{\varepsilon_1}{\varepsilon_2}\cosh(\gamma_1 g_1)\operatorname{senh}(\gamma_2 g_2)} * \left[j(\alpha_n \tilde{E}_{xg} + \beta_k \tilde{E}_{zg}\right] \quad (6.1)$$

$$A_{22} = \frac{\gamma_1 \operatorname{senh}(\gamma_2 y)}{\gamma_2\gamma_1 \operatorname{senh}(\gamma_1 g_1)\cosh(\gamma_2 g_2) + \gamma_2\frac{\varepsilon_1}{\varepsilon_2}\cosh(\gamma_1 g_1)\operatorname{senh}(\gamma_2 g_2)} * \left[j(\alpha_n \tilde{E}_{xg} + \beta_k \tilde{E}_{zg}\right] \quad (6.2)$$

$$B_{21} = -\frac{\operatorname{senh}(\gamma_1 g_1)}{\omega\mu_0 \operatorname{senh}(\gamma_1 g_1)\cosh(\gamma_2 g_2) + \frac{\gamma_1}{\gamma_2}\cosh(\gamma_1 g_1)\operatorname{senh}(\gamma_2 g_2)} * \left[-\beta_k \tilde{E}_{xg} + \alpha_n \tilde{E}_{zg}\right] \quad (6.3)$$

$$B_{22} = -\frac{\gamma_1}{\gamma_2}\frac{\cosh(\gamma_1 g_1)}{\omega\mu_0 \operatorname{senh}(\gamma_1 g_1)\cosh(\gamma_2 g_2) + \frac{\gamma_1}{\gamma_2}\cosh(\gamma_1 g_1)\operatorname{senh}(\gamma_2 g_2)} * \left[-\beta_k \tilde{E}_{xg} + \alpha_n \tilde{E}_{zg}\right] \quad (6.4)$$

They are obtained the general equations of the fields electromagnetic then in the structure in function of and that are the components of the fields electric tangents in the region of the chap line antenna.

III. CALCULATION OF THE ADMITANCE MATRIX

The following equations (7.1) and (7.2) they relate the current densities in the sheets ($\tilde{J}_{xt}$ and $\tilde{J}_{zt}$) and the magnetic fields in the interface y = h1+h2:

$$\tilde{H}_{x2} - \tilde{H}_{x3} = \tilde{J}_{zt} \quad (7.1)$$

$$\tilde{H}_{z2} - \tilde{H}_{z3} = -\tilde{J}_{xt} \quad (7.2)$$

Being made the substitutions of the equations of the magnetic fields, given in function, and after some calculations it was obtained,

$$Y_{xx}\tilde{E}_{xg} + Y_{xz}\tilde{E}_{zg} = \tilde{J}_{zg} \quad (8.1)$$

$$Y_{zx}\tilde{E}_{xg} + Y_{zz}\tilde{E}_{zg} = \tilde{J}_{xg} \quad (8.2)$$

That in matrix's form is:

$$\begin{bmatrix} Y_{xx} & Y_{xz} \\ Y_{zx} & Y_{zz} \end{bmatrix} \begin{bmatrix} \tilde{E}_{xg} \\ \tilde{E}_{zg} \end{bmatrix} = \begin{bmatrix} \tilde{J}_{zg} \\ \tilde{J}_{xg} \end{bmatrix} \quad (9)$$

The " Y " admittance functions are the functions dyadic of Green of the antenna and they are given for:

$$Y_{xx} = -\frac{j}{\varpi\mu 0(\gamma 2^2 + k2^2)}\left[-\beta k^2\gamma 2.E + k2^2\alpha n^2.F\right] + \frac{j}{\varpi\mu 0\gamma 3}\left[\alpha n^2 k3^2 E - \beta k^2\gamma 3^2.D\right] \quad (9.1)$$

$$Y_{xz} = \frac{-j\alpha n\beta k}{\varpi\mu 0\left(\gamma 2^2 + k2^2\right)}\left[A + k2^2.(B)\right] - \frac{\alpha n\beta k}{\varpi\mu 0\gamma 3\left(k3^2 + \gamma 3^2\right)}\left[k3^2.C + \gamma 3^2.D\right]$$

$$Y_{zx} = \frac{-j\alpha n\beta k}{\varpi\mu 0\left(\gamma 2^2 + k2^2\right)}\left[A + k2^2.(B)\right] - \frac{\alpha n\beta k}{\varpi\mu 0\gamma 3\left(k3^2 + \gamma 3^2\right)}\left[k3^2.C + \gamma 3^2.D\right] \quad (9.2)$$

$$Y_{zz} = \frac{j}{\varpi\mu 0\left(\gamma 2^2 + k2^2\right)}\left[\alpha n^2.A - \beta k^2.k2^2.\mathbf{B}\right] - \frac{j}{\varpi\mu 0\gamma 3\left(k3^2 + \gamma 3^2\right)}\left[\alpha n^2\gamma 3^2.C - \beta k^2\gamma 3^2.D\right] \quad (9.3)$$

$$A = \frac{\gamma 1\cdot\gamma 2}{\gamma 2tgh(\gamma 1\cdot h1) + \gamma 1tgh(\gamma 2h2)} + \frac{\gamma 2^2}{\frac{\gamma 2}{tgh(\gamma 2\cdot h2)} + \frac{\gamma 1}{tgh(\gamma 1\cdot h1)}} \quad (9.4.1)$$

$$B = \left(\frac{\varepsilon 1}{\gamma 1\cdot\varepsilon 2tgh(\gamma 1\cdot h1) + \gamma 2\cdot\varepsilon 1tgh(\gamma 2\cdot h2)} + \frac{\gamma 1\cdot\varepsilon 2}{\frac{\gamma 1.\gamma 2.\varepsilon 2}{tgh(\gamma 2.h2)} + \frac{\gamma 2^2.\varepsilon 1}{tgh(\gamma 1.h1)}}\right) \quad (9.4.2)$$

$$C = \left(\frac{\frac{\gamma 3\cdot\varepsilon 4}{\gamma 4\varepsilon 3} + tgh(\gamma 3h3)}{1 + \frac{\gamma 3\varepsilon 4.tgh(\gamma 3h3)}{\gamma 4\varepsilon 3}}\right) \quad (9.4.3)$$

$$D = \left(\frac{\frac{\gamma 4}{\gamma 3} + tgh(\gamma 3h3)}{1 + \frac{\gamma 4.tgh(\gamma 3h3)}{\gamma 3}}\right) \quad (9.4.4)$$

$$E = \left(\frac{\gamma 1 + \gamma 2.tgh(\gamma 1h1).tgh(\gamma 2h2)}{\gamma 2tgh(\gamma 1h1) + \gamma 2.tgh(\gamma 2h2)}\right) \quad (9.4.5)$$

The electric fields tangents in the interface, they are expanded in we have of known base functions through a add, as [3], [5]:

$$\tilde{E}_{xg} = \sum_{i=1}^{n} a_{xi}\cdot\tilde{f}_{xi}(\alpha_n, \beta_k) \quad (10.1)$$

$$\tilde{E}_{zg} = \sum_{j=1}^{m} a_{zj}\cdot\tilde{f}_{zj}(\alpha_n, \beta_k) \quad (10.2)$$

Where a_{xi} and a_{zj} are constant unknown and the terms n and m are numbers integer and positive that can be done equal to 1, as in the equations (12.1) and (12.2) following:

$$\tilde{E}_{xg} = a_x\cdot\tilde{f}_x(\alpha_n, \beta_k) \quad (11.1)$$

$$\tilde{E}_{zg} = a_z\cdot\tilde{f}_z(\alpha_n, \beta_k) \quad (11.2)$$

Where chosen base functions in the space domain expresses for [6]:

$$f_x(x,z) = f_x(x).f_x(z) \qquad (12.1)$$

$$f_x(x) = \frac{1}{\sqrt{\left(\frac{w}{2}\right)^2 - x^2}} \qquad (12.2)$$

$$f_x(z) = \cos\left(\frac{\pi z}{l}\right) \qquad (12.3)$$

Whose transformed of Fourier are:

$$\widetilde{f}_x(\alpha_n) = \pi \cdot Jo\left(\alpha_n \frac{w}{2}\right) \qquad (13.1)$$

$$\widetilde{f}_x(\beta_k) = \frac{2\pi l \cdot \cos\left(\frac{\beta_k l}{2}\right)}{\pi^2 - (\beta_k l)^2} \qquad (13.2)$$

$$\widetilde{f}_x(\alpha_n, \beta_k) = \frac{2\pi^2 l \cdot \cos\left(\frac{\beta_k l}{2}\right)}{\pi^2 - (\beta_k l)^2} \cdot Jo\left(\alpha_n \frac{w}{2}\right) \qquad (13.3)$$

Where J_0 is the function of Bessel of first species and order zero.

The Galleria's method is applied to (9), the eliminated current densities and the new equation in matrix's form are obtained [5], [7].

$$\begin{bmatrix} K_{xx} & K_{xz} \\ K_{zx} & K_{zz} \end{bmatrix} \cdot \begin{bmatrix} a_x \\ a_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad (14)$$

Where,

$$K_{xx} = \sum_{-\infty}^{\infty} \widetilde{f}x \cdot Yxx \cdot \widetilde{f}x^* \qquad (14.1)$$

$$K_{xz} = \sum_{-\infty}^{\infty} \widetilde{f}z \cdot Yxz \cdot \widetilde{f}x^* \qquad (14.2)$$

$$K_{zx} = \sum_{-\infty}^{\infty} \widetilde{f}x \cdot Yzx \cdot \widetilde{f}z^* \qquad (14.3)$$

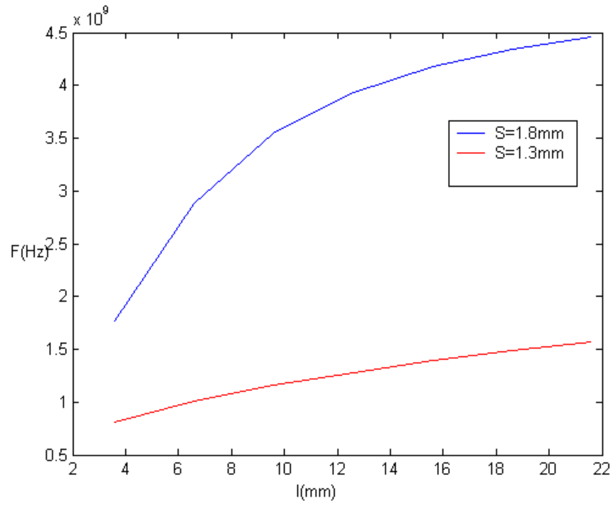
$$K_{zz} = \sum_{-\infty}^{\infty} \widetilde{f}z \cdot Yzz \cdot \widetilde{f}z^* \qquad (14.4)$$

The solution of the characteristic equation of the determinant of (14) it supplies the resonant frequency [8]-[10].

IV. RESULTS

According the graphic of the resonant frequency, it decreases when with decrease of the width or length of the slot. This results showing briefly.

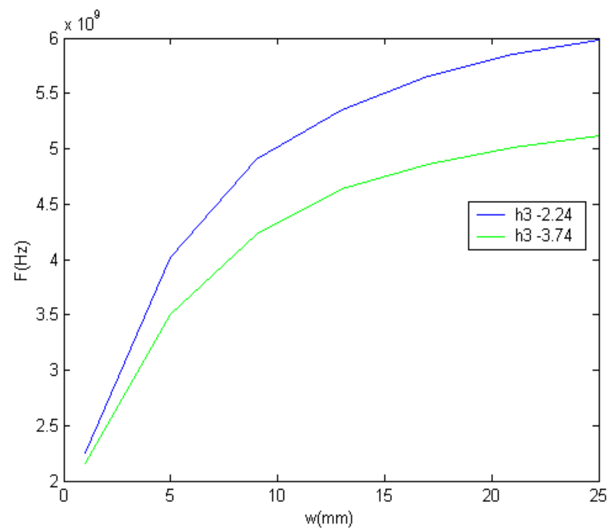
The Figure 2 shows the curve resonant frequency in function of length slot for different measurements thickness substrate at first range



(a)

Fig. 2- (a) Frequency (GHz) in function of length (mm)

The Figure 3 shows the curve resonant frequency in function of width slot for different measurements thickness substrate at third range



(a)

Fig.3. (a)- Frequency (GHz) in function of width (mm)

V. CONCLUSION

The Transverse Transmission Line – TTL method was used, in the analysis to obtain the numeric results of the four layers slot line resonator. According with the concise and effective procedures, was obtained the calculus of the complex resonant frequency with accuracy.

The possibility of the alternate various materials is the greater advantage of multiple layers slot line resonator, that can be used as antenna.

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