Analytical Model of an Induction Motor Fed from Three-Phase CSI

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Abstract: -This paper deals with the mathematical model of a current source inverter (CSI) feeding an induction motor drive .Neglecting the commutation, we investigate the square waveforms of the input stator phase currents. By means of the mixed p-z approach we can derive from the mathematical model the rotor vector currents .We can also determine the closed-form solution for the magnetizing space-vector current. Finally, from the analytical derived expressions for the currents we can calculate the torque equations.

Key-Words: - Mathematical model, Current source inverter, Electromagnetic torque

1 Introduction

Variable-speed drives, employing an induction motor and a space-vector pulsewidth modulated (SVPWM) voltage source inverter (VSI), are now used for various purposes. The reliability of such adjustable speed AC motor drives is an area of great interest. Although many modern drives offer a high degree of reliability, because of unexpected load conditions or erroneous operations, a three-phase inverter-fed drive can develop various faults. These faults have been analyzed and remedial strategies are suggested in [1].The main problems can arise with the power inverter, because of reduced ability of the power electronics components to sustain overcurrents and overvoltages for a longer time Most commonly, the power inverter failure is caused by the loss of conduction of one leg. Under this condition the motor starts to operate in single phase mode, introducing significant torque ripples and losing the field orientation in FOC drives [3]. In order to reduce inverter faults, a common practice consists of the selection of suitable components as well as the oversizing of the power electronics components. Unfortunately, this solution significantly increases system cost.

Recently, current source inverters (CSI) employing power devices such as GTO and IGBT power devices have been of interest in induction motor drives .This inverter utilizes the line side bridge to form a dc current link rather than a dc voltage link as in the case of the VSI. This inverter also possesses the feature of ruggedness, good starting torque, wide speed range, and protection against overload and open circuits.

As in the case of VSI also the CSI can operate in emergency situations as is the two-phase supply. If one leg of the VSI is broken, then inverter leg is disconnected from the dc link and the center point of the motor is connected with the neutral of supply.

Fig.1 shows the diagram of the proposed system, consisting of the three-phase induction motor and modified power inverter. Under normal operating condition switch F1 is closed, and F2 is open.However, in the event of a fault in the A phase leg F1 opens and F2 closes, connecting the centre point of the motor with the center point of the neutral N.The same is valid for the faults in the phases B or C.The additional switches can be a triac enabling bi-directional current flow. We need additional software routines implements the fault detection, insulation of the faulty leg and recovery [1].

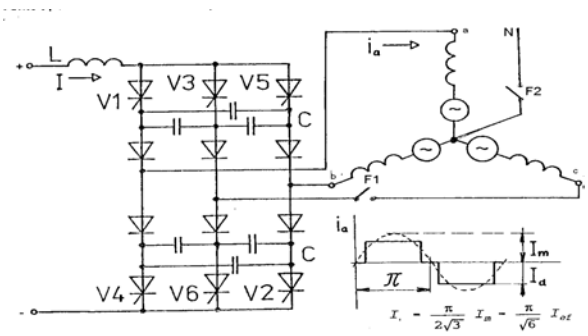

Fig.1 Principal scheme of CSI and phase current

2 Mathematical Model

2.1 Space Vector of Stator Currents

From Fig.1 we can see the current in phase a. The currents in the other two phases are delayed from i_a by 120 and 240 degrees, respectively. The phase currents have square-waves (when neglecting a commutation) with value of I_d ,and with the amplitude of the first harmonics

$$I_{1m} = 2\sqrt{3}I_d / \pi \qquad (1)$$

The first harmonic is shown dotted in Fig.1
From the phase currents i_a ,i_b and i_c we can determine the current space-vector defined as follows:

$$I_S(t) = \frac{1}{3}(i_a + ai_b + a^2 i_c),\ \ a = e^{j2\pi/3} \quad , \qquad (2)$$

For the next calculations we express time in per unit as follow

$$t = (n+\varepsilon)T = (n+\varepsilon)T_1 / 6 \qquad (3)$$

Substituting phase currents in (2) we receive for the stator current vector

$$I_S(n) = \frac{I_d\sqrt{3}}{3} e^{-j\frac{\pi}{2}} e^{jn\frac{\pi}{3}} = I(0)e^{jn\frac{\pi}{3}} \qquad (4)$$

Where

$$I(0) = \frac{I_d\sqrt{3}}{3} e^{-j\frac{\pi}{2}}$$

n is number of a sector, $0 \le \varepsilon \le 1$, ε is per unit time inside of the sector T, and T_1 is the output period.
I(0) is an initial value of the stator current vector

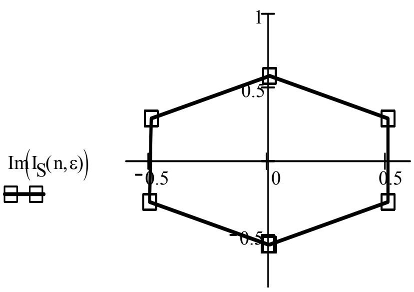

Fig.2 Trajectory of the stator current vector in $\alpha\beta$ complex plane

2.2 Space Vector of Rotor Currents

From the motor equations in the stator co-ordinate system we can write the following relationships in the Laplace transform:

$$I_R(p) = -I_S(p)\frac{L_m}{L_R}\frac{(p-j\nu)}{(p+k_R-j\nu)} \qquad (5)$$

where:

$$k_R = \frac{L_R}{R_R}$$

R_R is the rotor phase resistance,L_R and L_m are the armature rotor and mutual inductance, respectively, and ν is the electrical angular velocity
The Laplace transform of the stator vector current can be calculated as in [2.
To derive the Laplace transform of (4) we can use the relation between the Laplace and modified Z-transform [2

$$I_S(p) = T\int_0^1 I_S(e^{pT}).e^{-pT\varepsilon}.d\varepsilon. \qquad (6)$$

By substituting (4) into (6) and using z-transform of the exponential function

$$Z\left\{e^{nT}\right\} = \frac{z}{z-e^T} \quad , \quad z = e^{pT} \qquad (7)$$

we obtain for the Laplace transform of the stator current space-vector:

$$I_S(p) = I(0)\frac{(e^{pT}-1)}{p(e^{pT}-D)} \ , D = e^{j\frac{\pi}{3}} \qquad (8)$$

Substituting (8) into (5) we get the final equation in the Laplace transform for the rotor current space vector as follows:

$$I_R(p) = -I(0)\frac{(e^{pT}-1)}{p(e^{pT}-D)}\frac{L_m}{L_R}\frac{(p-j\nu)}{(p+k_R-j\nu)} \qquad (9)$$

2.3 Time-Domain Analysis

To find the inverse Laplace transform of (9) new can use the mixed p-z approach [2].
In the first step we transverse (9) into the modified Z-transform

$$I_R(z,\varepsilon) = -I(0)\frac{L_m}{L_R}\frac{z-1}{z-D} Z_m\left\{\frac{p-j\nu}{p(p+k_R-j\nu)}\right\} \quad (10)$$

where Z_m denotes the modified Z-transform [2].Using the inverse theorem in the Z-transform we get

$$Z_m\left\{\frac{p-j\nu}{p(p+k_R-j\nu)}\right\} = \frac{-j\nu}{(k_R-j\nu)}\frac{z}{z-1} + \frac{k_R}{(k_R-j\nu)^2}\frac{z}{z-e^{p_1T}}e^{p_1T\varepsilon} \quad (11)$$

where $p_1 = j\nu - k_R$,is the root of the characteristic equation (5)

The inverse modified Z-transform of (11) can be calculated by means of the residua theorem

$$f(n,\varepsilon) = \frac{1}{2\pi j}\oint f(z,\varepsilon) z^{n-1} dz \quad (12)$$

Using (12), we obtain the original (time dependency) of (10) as follows:

$$I_{RC}(n,\varepsilon) = -\frac{I(0)}{(k_R-j\nu)}\frac{L_m}{L_R}\begin{bmatrix} -j\nu D^n + \frac{(D-1)}{(D-e^{pT})}k_R e^{p_1T\varepsilon} + \\ (\frac{1-e^{p_1T}}{D-e^{p_1T}})k_R e^{p_1T(n+\varepsilon)} \end{bmatrix} \quad (13)$$

As the third term in (13) containing $e^{p_1T(n+\varepsilon)}$, is vanishing for $n \to \infty$,forming the transient part of the solution.
The steady-state solution for the rotor space vector current is given by the following analytical expression:

$$I_R(n,\varepsilon) = -\frac{I(0)}{(k_R-j\nu)}\frac{L_m}{L_R}\begin{bmatrix} -j\nu D^n + \frac{(D-1)}{(D-e^{pT})}k_R e^{p_1T\varepsilon} \\ \end{bmatrix} \quad (14)$$

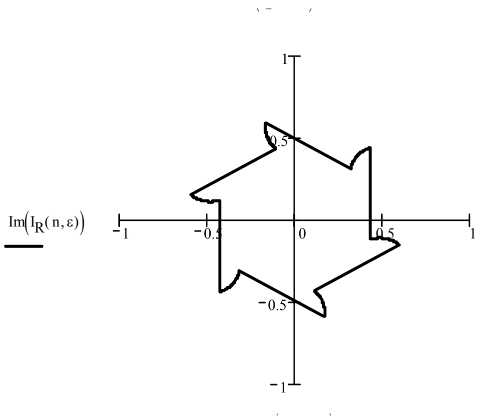

Fig.3 Trajectory of the rotor current in $\alpha\beta$ complex plane

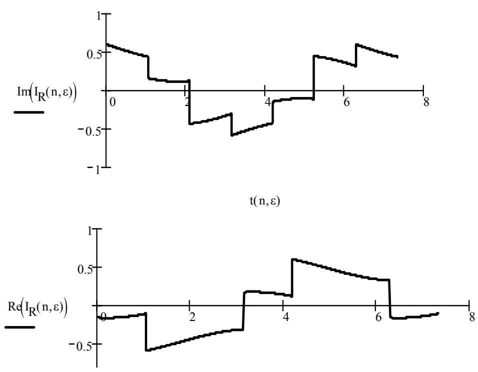

Fig.4 Time dependency of the rotor current. Top:in β axis . Bottom: in α axis .

In Fig.3 we can see the trajectory of the rotor space-vector in the complex plane.
Fig.4 shows the imaginary and thr real part of the rotor current vector as a function of time.

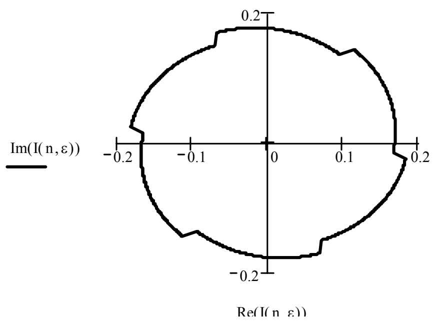

Fig.5 Trajectory of the magnetizing current in $\alpha\beta$ complex plane

From Fig.5 we can see the trajectory of the magnetizing current I_μ again in the $\alpha\beta$ complex plane

The space-vector of the magnetizing current is given as follows:

$$I_\mu(n,\varepsilon) = I_S(n,\varepsilon) + I_R(n,\varepsilon) \tag{15}$$

2.3 Electromagnetic Torque

The electromagnetic torque equation in stator co-ordinate system reads:

$$T_i(n,\varepsilon) = 6p_p L_m \operatorname{Re}\left\{jI^*{}_S(n,\varepsilon) I_R(n,\varepsilon)\right\} \tag{16}$$

where * means conjugate complex.

If we substitute from (4) and (14) into (16) we get analytical equation for the time dependency of the electromagnetic torque.

The electromagnetic torque contains again both the steady state and transient part.

The steady-state solution we again obtain for $t \to \infty$.

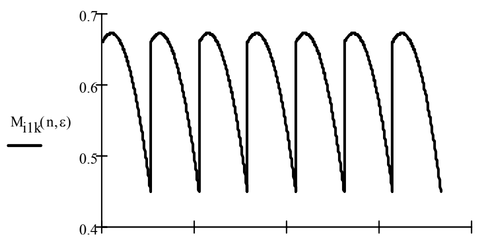

Fig.6 Time dependency of the electromagnetic torque

From Fig.6 we can see the time dependency of the steady-state electromagnetic torque. The torque has mean value about 0.56 p.u. forming an asynchronous torque and the pulsating torque(with the frequency of six-times of f_1) with the ripple value about 0.23 p.u.

3 Conclusion

The mixed p-z approach for the analysis of CSI with per is described. The mathematical model uses the Laplace and modified Z- transforms. The steady state and transient components of the load current are determined in a simple and lucid form that avoids involved matrix inversion as well as exponentiation.

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