A Neuro Fuzzy Technique for Modelling Climatic Variations in the Plio-Pleistocene

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Abstract: — In this paper a new approach to model climatic variations in the Plio-Pleistocene is presented. In a recent reference, Rial in [1] introduced the working hypothesis that frequency modulation (FM) of the orbital eccentricity forcing may be an important source of the nonlinearities observed in the $\delta^{18}O$ time series from deep-sea sediment cores. Two models are proposed based on the ANFIS (*Adaptive Neuro Fuzzy Inference System*) structure. The first model uses only past values of the time series under investigation. The second model uses information on the orbital eccentricity forcing and an artificially generated FM which is an extension of the FM signal proposed by Rial. The two models are compared in the light of long term predictions.

Key-Words: Astronomical forcing; Climate variations; Embedding dimension; Neuro-fuzzy; Frequency modulation; Nonlinear models.

1 Introduction

Studying the climate variations in our recent geological past is of fundamental importance to understand the climate phenomena and to help us to predict future global climate variations. The most reliable data showing climatic variations in the Plio-Pleistocene (the last 5.2 million years) is the time series of stable isotope ratios [2], specially, the $\delta^{18}O$ records in marine sediment cores (a proxy for global ice volume).

In 1976 Hays *et al* [3] showed that some frequencies of the $\delta^{18}O$ time series match those of astronomical changes in Earth's insolation. Since then the astronomical theory of the climate [4] has become an useful tool of analysis of the climate variations [5, 6]. This theory holds that variations in insolation caused by changes in earth's orbital eccentricity, obliquity and precession are the cause of the great ice ages of the Pleistocene [7].

However this theory is not without problems [1, 7]. For instance, in the so-called 100 kyr (kiloyears) and 400 kyr problems non-linear alterations in the components of the spectra are not explained by the orbital force alone. In [1] Rial suggested that the Earth's climate system frequency-modulates (FM) the orbital forcing which is similar to electronic modulation of a high-frequency carrier by a low-frequency modulation signal.

The main focus of this paper is to propose a non-linear method to model the $\delta^{18}O$ time series obtained in the ODP (Ocean Drilling Program) site 806 using a classic ANFIS (*Adaptive Neuro Fuzzy Inference System*) structure. To verify the importance of the FM hypothesis two models are proposed, a model using only past values of the output and another model including an artificially generated FM as one of the inputs. Finally both models are compared to show the validity of the proposed input signal.

2 ANFIS Architecture

The Sugeno fuzzy model, known as ANFIS, has a structure that is equivalent to a fuzzy inference system and therefore is based on fuzzy rules and fuzzy reasoning [8]. For instance, assuming the first-order Sugeno fuzzy model with two inputs x and y and one output z a common set of rules with two fuzzy if-then rules is written as follows

Rule 1: If x is A_1 and y is B_1, then
$f_1 = p_1 x + q_1 y + r_1$
Rule 2: If x is A_2 and y is B_2, then
$f_2 = p_2 x + q_2 y + r_2$

In the model architecture shown in Figure 1, the output of the i_{th} node in layer l is represented by $O_{l,i}$. Each layer shown in Figure 1 will be now described.

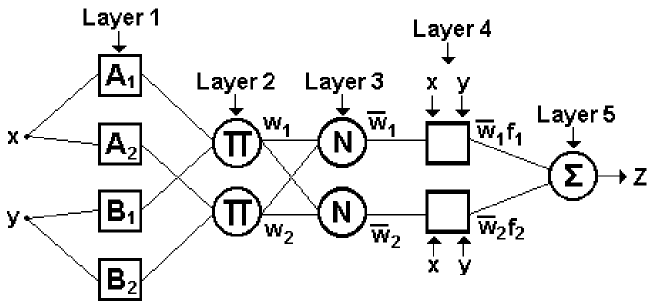

Figure 1. ANFIS architecture

Layer 1 - The output of each node i in Layer 1 is calculated as follows:

$$O_{1,i} = \mu_{A_i}(x), \quad for \quad i = 1,2 \quad or$$
$$O_{1,i} = \mu_{B_{i-2}}(y), \quad for \quad i = 3,4$$

This layer is a measure of how much a given input satisfies the membership function μ_A or μ_B.

Layer 2 - The output of this layer is the product of all incoming signals:

$$O_{2,i} = w_i = \mu_{A_i}(x)\mu_{B_i}(y), \qquad i = 1,2.$$

where $O_{2,i}$ is the output of a set of rules.

Layer 3 - The output of each node i is calculated as follows

$$O_{3,i} = \bar{w}_i = \frac{w_i}{w_1 + w_2}, \qquad i = 1,2.$$

In this particular case, the output of each node is called normalized firing strengths.

Layer 4 - The following function is evaluated in every node i of this layer:

$$O_{4,i} = \bar{w}_i f_i = \bar{w}_i(p_i x + q_i y + r_i). \tag{1}$$

The parameters p_i, q_i and r_i are called consequent parameters.

Layer 5 - In this Layer, the summation of all incoming signals are calculated as follows

$$O_{5,1} = \sum_i \bar{w}_i f_i = \frac{\sum_i w_i f_i}{\sum_i w_i} \tag{2}$$

where $O_{5,1} = z$ is the output of the Sugeno fuzzy model.

2.1 Hybrid Learning

In order to fit an ANFIS, two sets of different values need to be estimated: a set of premise parameters (nonlinear) and a set of consequent parameters (linear). To estimate the linear parameters, the least-squares estimator can be used. For instance, consider the previous example where an ANFIS structure with two rules and two inputs was described. Substituting Equation 1 in Equation 2 yields

$$\begin{aligned} O_{5,1} &= \textstyle\sum_i O_{4,i} \\ O_{5,1} &= \bar{w}_i(p_i x + q_i y + r_i), \qquad i = 1,2. \\ O_{5,1} &= \bar{w}_1 p_1 x + \bar{w}_1 q_1 y + \bar{w}_1 r_1 + \bar{w}_2 p_2 x \\ &\quad + \bar{w}_2 q_2 y + \bar{w}_2 r_2 \end{aligned}$$

The above set of Equations can be written in matrix notation as follows

$$\underbrace{\begin{bmatrix} O_{5,1} \end{bmatrix}}_{O} = \underbrace{\begin{bmatrix} \bar{w}_1 x & \bar{w}_1 y & \bar{w}_1 & \bar{w}_2 x & \bar{w}_2 y & \bar{w}_2 \end{bmatrix}}_{\phi} \underbrace{\begin{bmatrix} p_1 \\ q_1 \\ r_1 \\ p_2 \\ q_2 \\ r_2 \end{bmatrix}}_{\zeta} \tag{3}$$

Note that the matrices O, ϕ and ζ, can be written for any number of inputs and outputs in a similar way as the one presented above. Then, applying the least-squares method yields to

$$\hat{\zeta} = (\phi^T \phi)^{-1} \phi^T O \tag{4}$$

where $(\phi^T \phi)^{-1} \phi^T$ is the pseudo inverse and $\hat{\zeta}$ are the estimated parameters.

The gradient method is used to estimate the set of nonlinear parameters. In this case the quadratic error function error (e) is minimized by consecutive adjustments of a learning rate α of the parameters of the membership function. There are several methods to choose the value of α. In practice, α is made close to zero. In this work the membership function is chosen to be a gaussian function.

3 Model Inputs

Based upon the analysis of the correlation between several different inputs and the output, the relevant inputs are chosen [9]. When there is no correlation between a given input and the output, the input is then discarded. In case there is correlation between two different inputs, only one of them is chosen to be in the final model.

4 Frequency Modulation (FM) Signal

Two of the principal long-standing problems in the astronomical theory of the climate, the 400 kyr and 100 kyr problems, are the absence in the $\delta^{18}O$ signal of spectral amplitude at 413 kyr even though it is the largest component of eccentricity forcing and the shift of glacial cycles durations that do not correspond to the insolation variation in the last 500 kyr [7].

To solve these problems, Rial in [1] proposes a frequency modulation of a main carrier of period 95 kyr by a 413 kyr modulating signal. In order to build a FM signal, Rial determined empirically that this signal should contain the 95, 125 and 100 kyr signals as carries and the 413 and an 826 kyr subharmonic as modulating signals. The resulting FM signal is

$$\begin{aligned} FM(t) = a sin[\tfrac{2\pi t}{95} + \beta sin(\tfrac{2\pi t}{413}) + \beta' sin(\tfrac{\pi t}{413})] \\ + b sin[\tfrac{2\pi t}{100} + \beta sin(\tfrac{2\pi t}{413})] \\ + c sin[\tfrac{2\pi t}{125} + \beta sin(\tfrac{2\pi t}{413})] \end{aligned} \tag{5}$$

where t is the time in kiloyears, the constants a, b and c are adjusted parameters. β' is the modulation index for the subharmonic part.

In order to obtain ANFIS structures which can reproduce the overall dynamics the original system using only the available time series, some modifications on Rial's FM signal were necessary. These modifications are presented in the next section, and used in the final model, see section 5.2.

4.1 The New FM Signal

The following modifications were introduced to the original FM signal [1]

- inclusion of two functions sine: a_{12} e a_{18};
- inclusion of a constant (phase) in each function sine: a_{14}, a_{15}, a_{16}, a_{17} e a_{19};
- β is not constant for all functions sine: a_8, a_{10}, a_{11}, a_{13} e a_{20};
- inclusion of an extra (subharmonic) frequency in the signal: a_5.

The proposed signal takes the form

$$\begin{aligned} & FM(t) = \\ & a_6 sin[(\tfrac{2\pi t}{a_1} + a_{14}) + a_8 sin(\tfrac{2\pi t}{a_2}) + a_9 sin(\tfrac{2\pi t}{a_3})] \\ & + a_7 sin[(\tfrac{2\pi t}{a_4} + a_{15}) + a_{10} sin(\tfrac{2\pi t}{a_2})] \\ & + a_8 sin[(\tfrac{2\pi t}{a_5} + a_{16}) + a_{11} sin(\tfrac{2\pi t}{a_2})] \\ & + a_{12} sin[(\tfrac{2\pi t}{a_5} + a_{17}) + a_{13} sin(\tfrac{4\pi t}{a_2})] \\ & + a_{18} sin[(\tfrac{4\pi t}{a_5} + a_{19}) + a_{20} sin(\tfrac{8\pi t}{a_2})] \end{aligned} \tag{6}$$

where the parameters were adjusted by the algorithm Quasi-Newton and the initial conditions of the frequency parameters are the same as the ones suggested by Rial [1]. Since the series $\delta^{18}O$ present different characteristics in the first half of data, we considered different parameters for Equation 6. Figure 2 shows the FM signal compared to the $\delta^{18}O$ data. Note that the proposed FM signal does not follow the signal data in the second half of the data, as shown on the second panel of Figure 2.

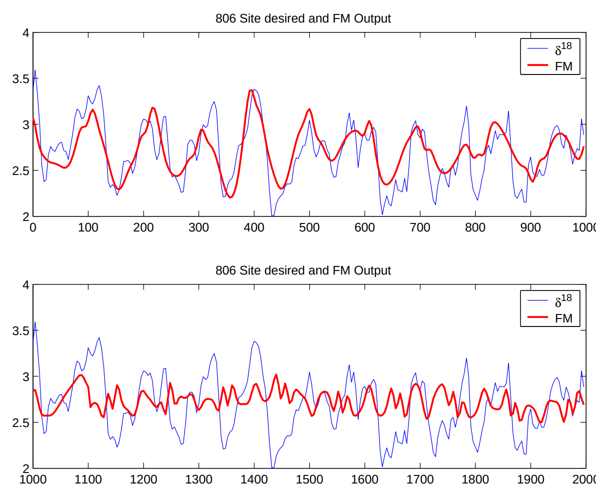

Figure 2. FM and $\delta^{18}O$ for the first half (0...1000 kyr) and second half (1000...2000 kyr) of the data

5 Main Results

In this section two new models are identified and their outputs compared to the original time series. The ANFIS structure in both models has 15 membership functions.

5.1 First Model

The first model is an ANFIS structure using only past values of the output ($\delta^{18}O$ series) as the input vector $[x(t_i), x(t_i + p), ..., x(t_i + (m-1)p)]$, where

m is the embedding dimension and p is the time-delay [10].

The time-delay can be obtained using the autocorrelation of the $\delta^{18}O$ data as shown in Figure 3, while the embedding dimension is obtained by Cao's method [11]. The output of this method is $E1(d)$ that is function of the embedding dimension d. $E1(d)$ reaches a constant value when d is the original embedding dimension of the system under investigation. Figure 4 shows the results for the $\delta^{18}O$ time series.

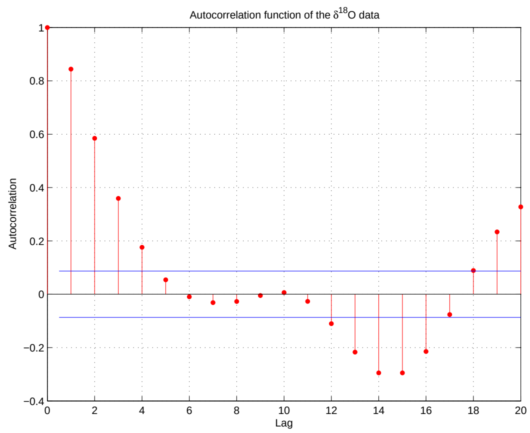

Figure 3. Autocorrelation of the $\delta^{18}O$ data

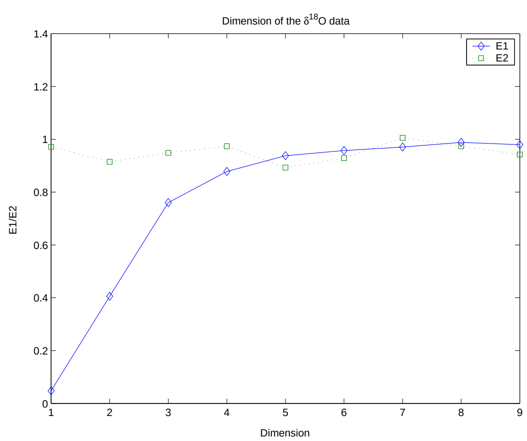

Figure 4. Dimension of the $\delta^{18}O$ data calculated by a time-delay of 3

Figures 5 and 6 show the results of one step prediction and free run of the model using the input (for a m=5 and p=3) vector as defined above. Note that in the free-running the model shows that it can't follow the original data but reaches a steady-state value. This demonstrated that the identified model can not make long term predictions.

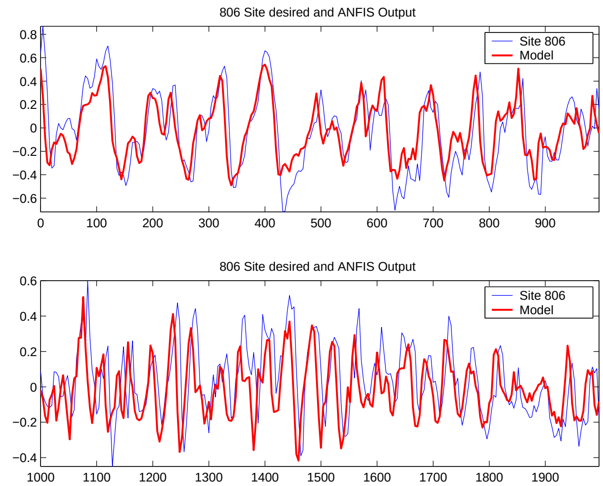

Figure 5. First model one step prediction

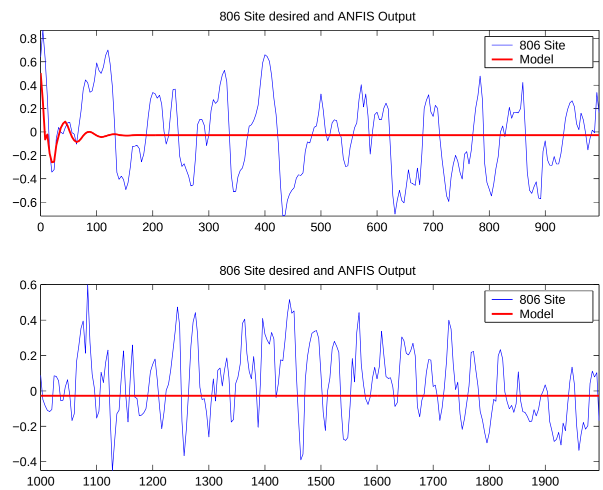

Figure 6. Free running of the first model

5.2 Final Model

In order to improve the model prediction capabilities, two new variables are introduced in the input vector. These variables follow the main ideas shown in the astronomical theory of the climate and the FM theory of Rial [1, 7].

Figures 7 and 8 present the results after the inclusion of the obliquity data and the FM signal proposed in the section 4.1. Note that the identified model using these two new inputs can successfully predict the original time series.

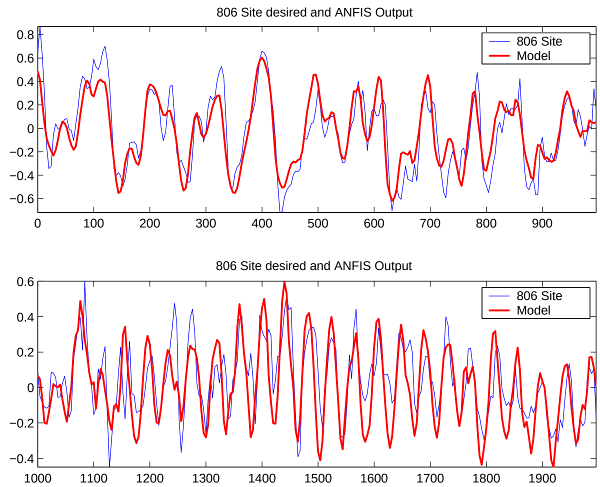

Figure 7. Final model one step prediction

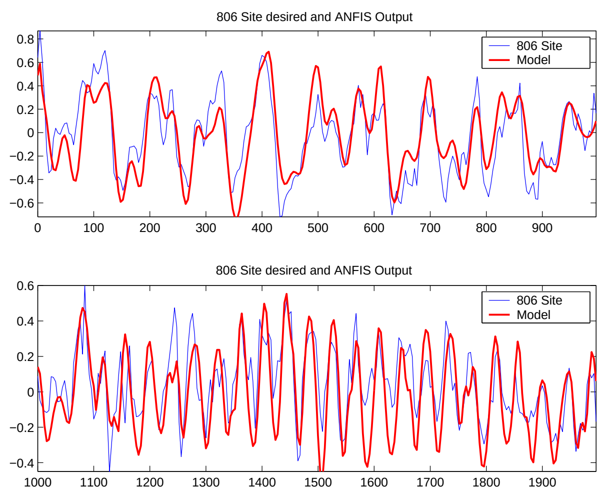

Figure 8. Free running of the final model

6 Conclusion

In this paper two models were proposed to reproduce the climate variations in our recent geological past. It has been shown that the nonlinearities presented in the $\delta^{18}O$ data can not be explained by using only output past value information. To circumvent this problem a new FM signal was proposed.

Yet even with the use of the FM as one of the inputs of the second model the predictions for large time intervals were not so good as for short time intervals. This seems to indicate that the underlying dynamics is time varying which makes the identification process more complex.

The results obtained for the second model show that it is possible to greatly improve the prediction of complex time series when carefully chosen inputs are considered. In this particular case, the chosen input uses frequency information obtained directly from the original time series. The goodness of fit achieved by fitting an ANFIS model was mainly due to the introduction of the proposed FM signal.

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