The Problem of Boundary Conditions in Seismic Excitation of Inhomogeneous Infinite Waveguides

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Abstract: - The analysis of the wave motion in a segment of the inhomogeneous wave guide excited by a given seismic excitation is presented, as well as some numerical cases of the interaction between the rigid foundation and the layer. The computational procedure yields exact results within the accuracy of the FE modeling, and directly uses the data of the free field seismic wave motion on the fictitious boundary where the computed reflected waves are considered in the boundary conditions. The basic principles of the analysis are explained, and brief outline of the procedure is presented. Numerical examples are for the case of 2D anti-plane shear wave motion in the frequency domain. The results of some simplified solutions are compared and discussed.

Key-Words: - Infinite inhomogeneous waveguide, Seismic excitation, Propagating wave-modes, Fictitious boundary, FEM

1 Introduction

In conceiving a physical model for the interaction between the structure and an infinite layer, excited by seismic waves, we may suppose, that the source of these waves is located in infinite distance. As a consequence, there is no interaction between the layer and structure on one side, and the source of the excitation on the other. The incident seismic wave is reflected from the structure only once and mathematically fulfils Sommerfeld radiating condition. When analyzing the wave field in a segment of an infinite inhomogeneous wave guide modeled by FEM, we have to set finite fictitious boundaries, see Figure 1, and impose correct boundary conditions taking in to the account the above mentioned phenomenon of seismic excitation.

The most common data for the seismic excitation is that the wave motion is given for the "free field wave motion problem". By free field wave motion we mean the waves propagating in the layer, which has no excavation and no structure. It is obvious that the wave motion on the fictitious boundary of the analyzed segment with an irregularity, differs from the free field motion: (1) On the fictitious boundary $\partial\Omega_1$ we have, in addition to the displacements $\mathbf{u}_1$ of the incident seismic wave, also the displacements $\mathbf{u}_3$ of the reflected wave, which fulfils the radiation conditions. (2) On the fictitious boundary $\partial\Omega_2$ are displacements $\mathbf{u}_2$ of the transmitted waves, which are altered because of dissipation of incoming seismic waves on the irregularities. However, we have radiation through both lateral fictitious boundaries.

The problem of seismic excitation was solved by Wolf [1], who employed substructure synthesis. He first computes the seismic wave motion on the surface of the excavation, which represents future contact surface between the structure and the infinite half space or a layer. This approach requires the computation of the stiffness or the flexibility matrix of the half space for the mesh nodes on the mentioned contact surface. However, this is not a simple task. Up to nowadays, boundary conditions for the excitation by an incident wave are considered as suggested by [1], or in even more simplified ways when FEM is applied, see for instance [2-4].

We are presenting a somehow more direct approach to the solution of the seismic excitation, by computing the correct boundary condition on the fictitious boundaries, and then solving the wave motion in the analyzed segment of the inhomogeneous infinite wave guide as an internal problem.

2 The Basics, the Outline of the Computing Procedure and the Key Formulas

The resulting wave motion on the fictitious boundary $\partial\Omega_1$ is the sum of the given incident seismic wave and the computed reflected wave. This resulting wave is uniquely related to the transmitted wave on the fictitious boundary $\partial\Omega_2$ by the formula.

$$\begin{Bmatrix}\mathbf{u}_1\\ \boldsymbol{\tau}_1\end{Bmatrix}+\begin{Bmatrix}\mathbf{u}_3\\ \boldsymbol{\tau}_3\end{Bmatrix}=\mathbf{T}_{1-2}\begin{Bmatrix}\mathbf{u}_2\\ \boldsymbol{\tau}_2\end{Bmatrix} \qquad (1)$$

The above formula is written for the finite element discretised model and has the following meaning. Supposing that there are N mesh nodes on each of the fictitious boundary, then $\mathbf{u}_i$ is the N dimensional column matrix of nodal displacements, and τ_i is N dimensional column matrix of the belonging nodal stresses - in the foregoing text we shall call it simply the displacements or stress "vector", respectively. $\mathbf{T}_{1-2}$ is the 2Nx2N dimensional transfer matrix, which is simple to compute, [5].

As the reflected and the transmitted waves must fulfill the radiation conditions, they are set as a superposition of wave modes $\boldsymbol{\Psi}$ propagating in the appropriate direction with unknown amplitudes $\mathbf{a}$, Equations 2 and 3.

$$\begin{Bmatrix}\mathbf{u}_2\\ \boldsymbol{\tau}_2\end{Bmatrix}=\begin{bmatrix}\boldsymbol{\psi}_{\mathbf{u}}^{+}\\ \boldsymbol{\psi}_{\tau}^{+}\end{bmatrix}\mathbf{a}_2 \qquad (2)$$

$$\begin{Bmatrix}\mathbf{u}_3\\ \boldsymbol{\tau}_3\end{Bmatrix}=\begin{bmatrix}\boldsymbol{\psi}_{\mathbf{u}}^{-}\\ \boldsymbol{\psi}_{\tau}^{-}\end{bmatrix}\mathbf{a}_3 \qquad (3)$$

Each sub-matrix $\boldsymbol{\Psi}$ has the dimension NxN and its computation is presented in [5]. Subscript »u« stands for the displacements wave mode, and subscript »τ« for the stresses of the wave mode, while superscripts designate the direction of wave modes. In our case, positive sign defines the direction of propagation of waves from the fictitious $\partial\Omega_1$ towards the fictitious boundary $\partial\Omega_2$, and the minus sign the opposite direction. Inserting the last two equations into Equation 1, yields the equation 4, which is solved on modal amplitudes $\mathbf{a}_2$ and $\mathbf{a}_3$.

$$\begin{Bmatrix}\mathbf{u}_1\\ \boldsymbol{\tau}_1\end{Bmatrix}+\begin{bmatrix}\boldsymbol{\psi}_{\mathbf{u}}^{-}\\ \boldsymbol{\psi}_{\tau}^{-}\end{bmatrix}\mathbf{a}_3=\begin{bmatrix}\mathbf{T}_{\mathrm{UU}} & \mathbf{T}_{\mathrm{U}\tau}\\ \mathbf{T}_{\tau\mathrm{u}} & \mathbf{T}_{\tau\tau}\end{bmatrix}_{1-2}\begin{bmatrix}\boldsymbol{\psi}_{\mathbf{u}}^{+}\\ \boldsymbol{\psi}_{\tau}^{+}\end{bmatrix}\mathbf{a}_2 . \qquad (4)$$

After computing modal amplitudes, we evaluate the displacements on the fictitious boundaries by Equations 2 and 3. Consequently, the problem becomes an internal one, with the sum of the displacements $\mathbf{u}_1$ and $\mathbf{u}_3$ on fictitious boundary $\partial\Omega_1$, and the displacements of the transmitting wave on the fictitious boundary $\partial\Omega_2$.

3 Numerical Examples

The presented examples are two dimensional cases of anti-plane shear wave motion analyzed in the frequency domain. The wave motion is governed by the Equation:

$$\nabla^2 u + k^2 u = 0 . \qquad (5)$$

The excitation frequency is 0.3 radians per second. The layer includes a buried foundation, with the dimensions 3m x 3m. The layer is otherwise homogenous and infinite. Thus, we are analyzing the case presented in the Figure 1, but without the console on the foundation. The layer is 20 meters high, consisting of ideal linear material with the shear module and density with a unit value. We have chosen such case and the data only to facilitate the computational effort and to facilitate graphical presentations, but otherwise the case contains all the interesting characteristics of the addressed problem of the seismic excitation. The analyzed section is 25 meters long. Mesh domain spans 20m x 25m, and consists of 21 x 26 nodal points of simple linear finite elements.

It is well known that waveguides transmit without decay only some of the first wave modes, others higher modes, separated by cut-off frequency, are rapidly decaying [6]. The discussion and the detailed analytical and numerical results concerning wave modes, even in the layered wave guides, are presented in [7]. For the frequency of the excitation and the characteristics of the waveguide as in our case, only the first two wave modes are non-decaying, see Figures 2a and 2b. Therefore, it is realistic to suppose that an incident wave, whatever it is generated by, would consist of only the first two modes. For our computational example we suppose that an incident wave consists of only the first wave mode with the unit amplitude. For the given displacements of the incident wave, the belonging stresses are directly computed by the formula:

$$\boldsymbol{\tau}_1 = \boldsymbol{\psi}_{\tau}^{+}(\boldsymbol{\psi}_{\mathbf{u}}^{+})^{-1}\mathbf{u}_1 . \qquad (6)$$

The first analyzed case considers stiff foundation located 15 meters from the excitation boundary. Figure 3 presents the computed displacements on both lateral boundaries, which represent the computed boundary conditions for the internal problem of the analyzed segment of the layer. On the left part of the Figure we can notice the reflected wave and the transmitted wave, altered due to the scattering of waves on the foundation. It is worth mentioning that the reflected waves at the location of the foundation are indeed very spurious, theoretically consisting of more or less of all wave modes. But, while reflected waves travel to the left fictitious boundary, the decaying wave modes practically vanish (see decaying factors in Figure 2). Thus, it is not surprising, that the displacements of the reflected waves on the fictitious boundary consist practically of only the first two modes, yielding simple displacements graph. However, when considering real material with damping, the reflected ~~waves~~ amplitudes on the fictitious boundary would be smaller. Taking into the account some average values for soil damping presented in [8], yields an estimated additional 30% relative reduction of the reflected waves on the boundary, which is four meters away from the foundation. However, the reflected waves in real materials play no role from some distance on. This distinctive distance depends on several factors, but the detailed discussion on this issue exceeds the scope of this paper.

Using the computed boundary displacements consisting of the incident seismic wave and the reflected wave on the left fictitious boundary, and the displacements of the transmitted wave on the right fictitious boundary, yields wave motion field presented in Figure 4.

It is interesting to compare the foundation displacements, caused by seismic excitation, to the displacements of the foundation, due to the boundary conditions, which are simply equal to the displacements of the free field seismic motion on the left fictitious boundary (that is, by neglecting the radiation of the reflected waves). This comparison is presented in the Figure 5 for the foundation, which is located only 4 meters from the left fictitious boundary. We can observe substantial difference between the results. For the case of more distant foundation, specifically 15 meters from the excitation boundary, the difference between the results is much more modest, Figure 6. These results are expected. They suggest well known fact that the reflected waves vanish rapidly with the distance from the foundation, providing that it is small compared to the height of the layer. On the other hand, the displacements of the foundation do not depend on the relative distance between the foundation and the fictitious boundary, when the foundation is excited by seismic waves. Thus, we can locate the fictitious boundaries as near as we want, to reduce computational effort, provided we consider the radiation of the reflected waves.

4 Conclusion

As there are no approximations of the computed wave field in the presented procedure, except for the FE modeling, it yields accurate results. The resulting wave motion does not depend on the location of fictitious boundaries. Consequently, the computational segment of the layer may be reduced substantially, almost only to include the foundation, but, clearly, not exactly on the foundation boundaries. Two advantages are worth pointing out: there is no need to compute the dynamic stiffness matrix for the infinite space (in our case an infinite layer), and the consideration of the seismic excitation is based only on the data of the free field wave motion on the fictitious boundary. The results also suggest that by foundations, which are small compared to the height of layer and at the same time more distant from the fictitious boundary, we can neglect the reflected waves. Consequently, we may in adequate cases employ simpler computation, which considers boundary conditions on the fictitious boundary of the incident waves being simply equal to the displacements of the free field wave motion. However, when this is exactly acceptable is subject to an extensive study, which should consider all the characteristics of the real problems.

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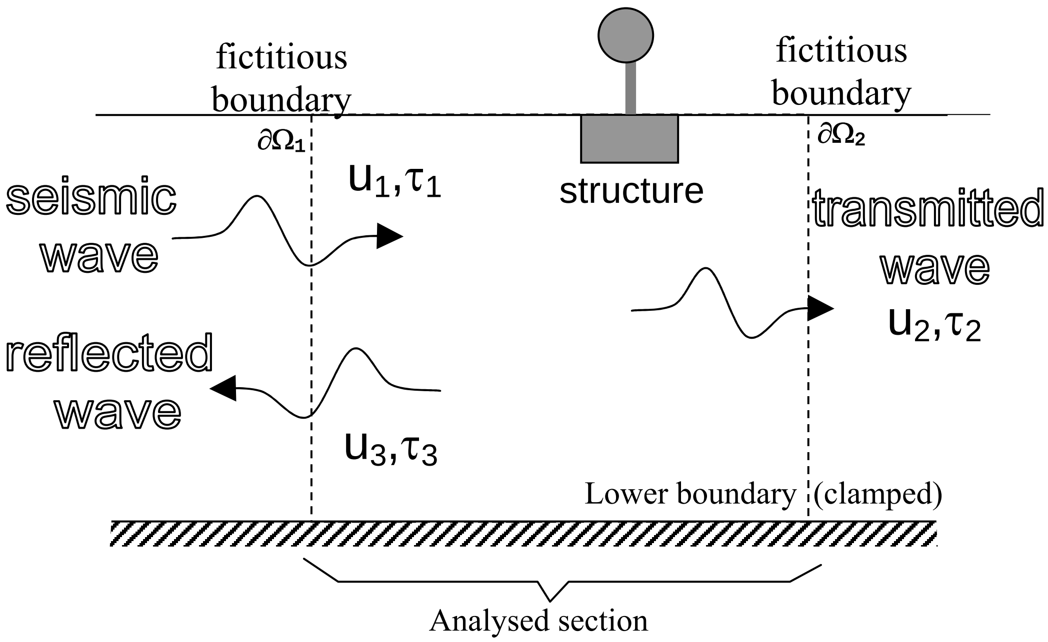

Figure 1. Symbolic presentation of a segment of a layered wave guide with a structure confined by the fictitious boundaries, the excitation seismic wave, the reflected and the transmitted wave.

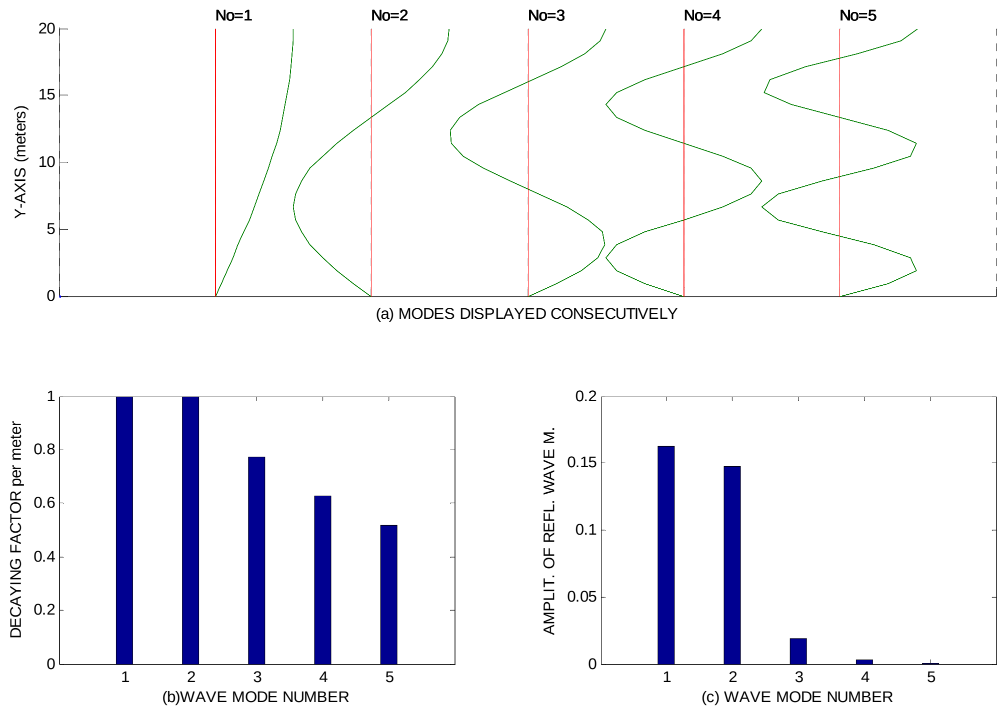

Figure 2. (a) First five wave modes normalized to yield a unit displacement on the surface. So normalized wave modes are used in the presented analysis. (b) The decaying factors for the displayed wave modes, a unit represents no decay. (c) Absolute values of amplitudes of the reflected wave modes on left fictitious boundary.

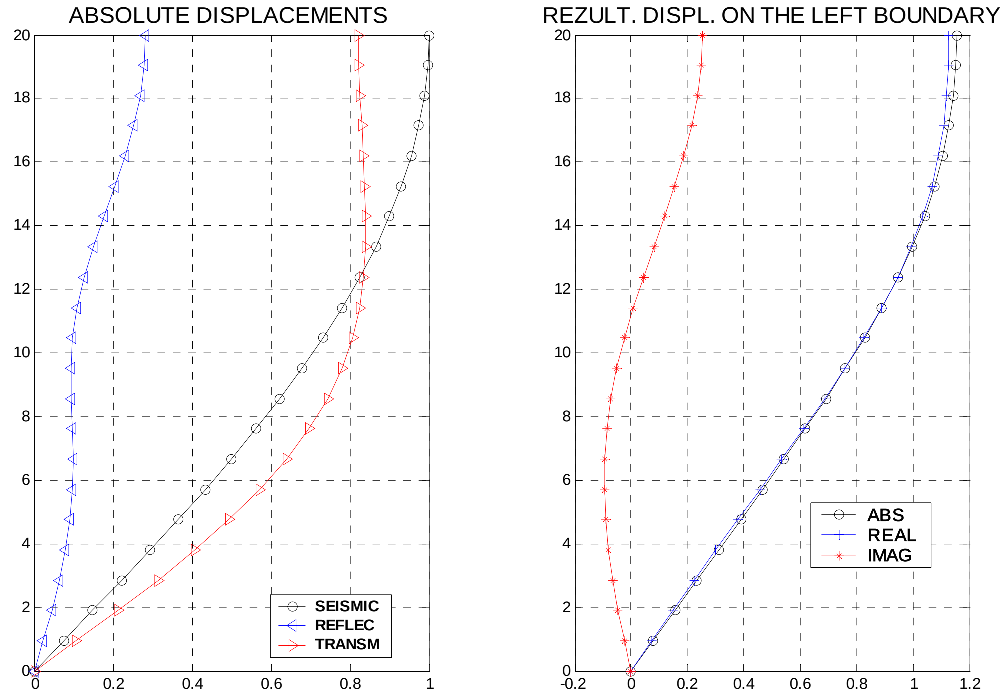

Figure 3. Left figure: absolute values of displacements of the incident seismic wave and the reflected wave on the fictitious boundary $\partial\Omega_1$, and absolute values of the transmitted wave on $\partial\Omega_2$. Right figure: the resulting displacements on the fictitious boundary $\partial\Omega_1$.

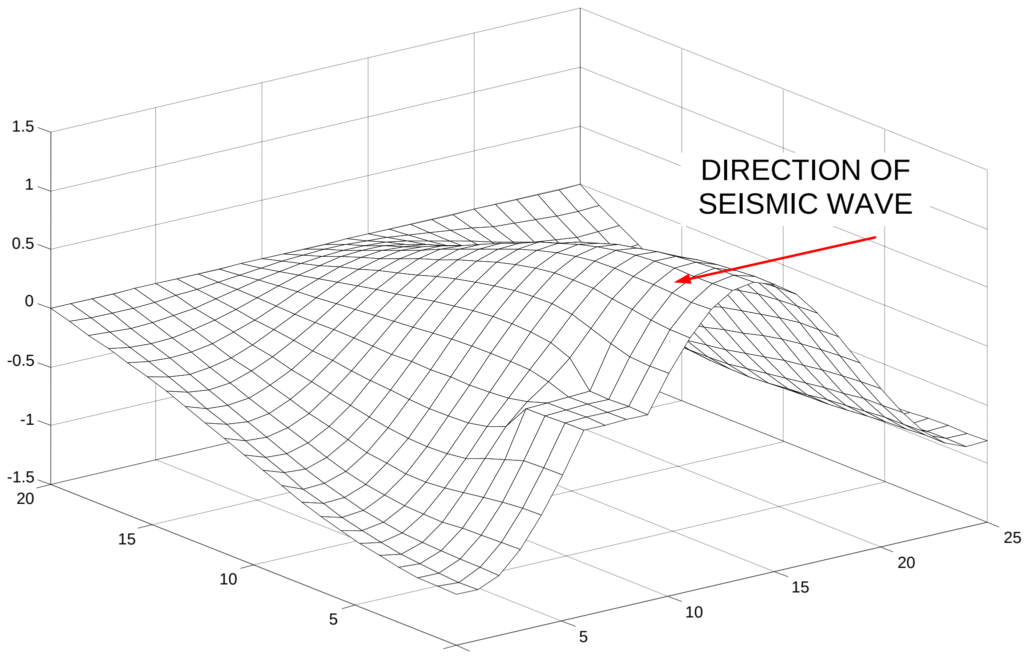

Figure 4. Real part of the displacements of the computed wave field "captured" in an instant.

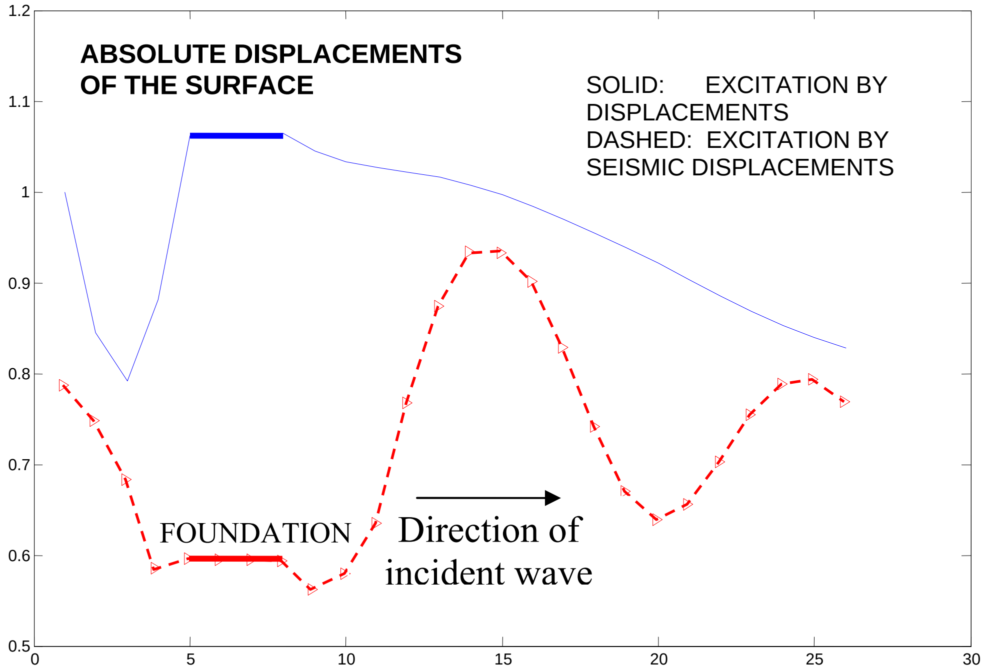

Figure 5. Comparison of the results of seismic excitation and the results of excitation by "rigid"displacements. The foundation is located only 4 meters from the excitation boundary.

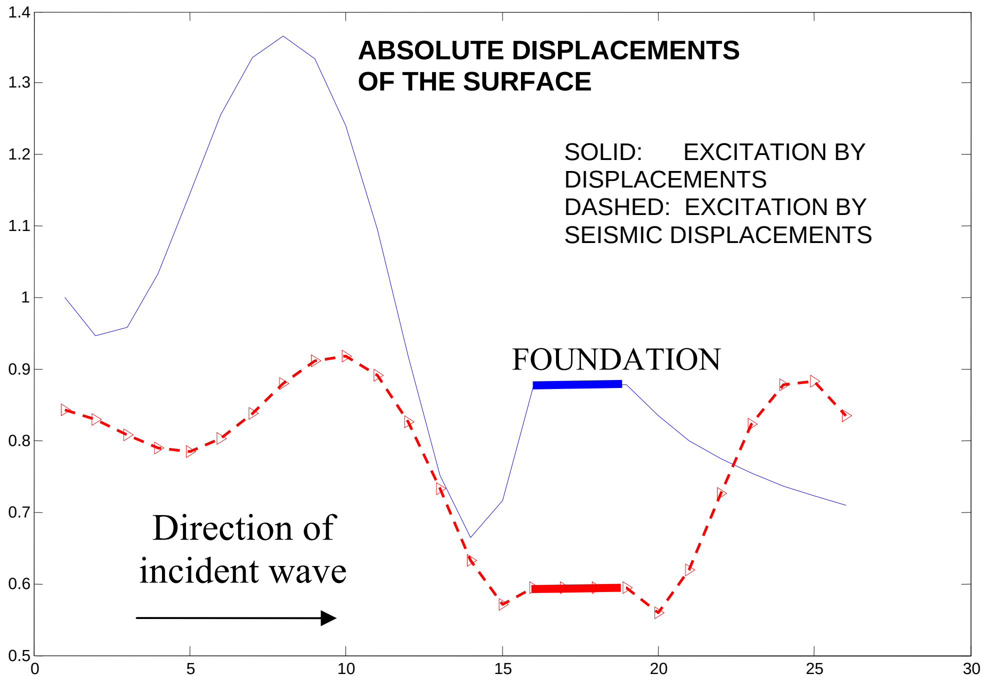

Figure 6. Comparison of the results of seismic excitation and the results of excitation by "rigid" displacements. The foundation is located 15 meters from the excitation boundary.