Comments on 'Improving the performance of the robust controller for a robot arm'

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Abstract: - The aim of this work is to advance in the idea of adaptive robust controllers for articulated robotic systems presented in a previous work of the same authors. This work explores the performance of classical robust controllers and shows the improvements of the adaptive scheme. Moreover, the adaptation law for the robust design parameter of the robust action is improved by adding a new term what was avoided in the previous work. Some new comparisons between the classical scheme and this new strategy, applied on a two-link robot manipulator, are presented.

Key-Words: - robust control, robot arms, constraints, adaptive control.

1 Introduction

This work is based on the work of Torres et al [1] which presents a new adaptive robust strategy to solve the tracking problem of robot manipulators in case of uncertainties. In earlier literature, adaptive and robust controllers have been extendedly applied to solve this problem, but they present some problems in some circumstances. First of them is the change on the dynamics when an unknown payload mass is taken. Another circumstance is when the same controller is applied to another manipulator, with the same structure but with different dynamics parameters.

In [1] is shown a strategy that solves these problems. The standard robust controller based on the application of the Lyapunov's direct method over an stabilizing control law [2],[3], is improved by adding an adaptation law to the robust design parameter (referred in some works as uncertainty bound). The work shows some results that prove the satisfactory performance of this strategy in the circumstances when the classical strategies failed.

Earlier works related with robust control schemes have not adopted a solution in this way. One of the most used solutions for the control of manipulators is to employ an adequate linear controller to the linearised system resulting of the application of the feedback linearisation scheme [4] to the robot system. This technique is based on a perfect knowledge of the robot model and its dynamics parameters. The problem to solve is the imperfect cancellation of the nonlinear dynamics due to the presence of uncertainties. Lot of works related with adaptive control schemes [5],[6], robust control schemes [2],[8],[9] and even hybrid control schemes [10],[11] have been proposed to deal with these uncertainties. Concretely, most of robust controllers are based on the Lyapunov's direct method. These schemes add a robust term to the control input which tries to compensate the discrepancies between the estimated model and the real model of the system.

This robust term present a satisfactory response when the robot dynamics do not vary or vary very little, even in the presence of uncertainties. In other case the robust action is not effective and has to be revised. This is the reason to add an adaptive scheme for the robust action, as presented in [1].

In this work, the adaptive robust strategy is revised and improved. Some new simulation results prove the satisfactory performance of this controller in the different situations. The paper is presented as follows: section 2 revises the adaptation law previously proposed, section 3 compares this strategy with those proposed in the earlier literature and section 4 present some new simulation results and comparisons with the classical strategies. Finally, some conclusions are presented.

2 Revision of the adaptive robust controller

The control technique used in [1] is applied on the linearised robot system resulting on applying the feedback linearization scheme on the nonlinear system. Due to the presence of uncertainties, the cancellation of the nonlinearities is imperfect. The proposed control law is formed by three terms: a feedforward term which gives the desired (or planned) accelerations of the system, a stabilizing PD law which corrects the deviations from the planned trajectory, and a robust term to cancel the errors due to the imperfect cancellation of the nonlinear terms by the feedback linearisation scheme. The resultant control law can be written as follows:

$$y_k = \ddot{\theta}_{k,d} + K_D \dot{\tilde{\theta}}_{k,d} + K_p \tilde{\theta}_{k,d} + y_{k,r} \qquad (1)$$

where y_k is the input of the linearised system (*k* indicates the instant of time), $\ddot{\theta}_{k,d}$ is the vector of desired accelerations in the *k* instant, $\tilde{\theta}_{k,d}$ is the vector of position errors and $\dot{\tilde{\theta}}_{k,d}$ the vector of velocities errors, K_p and K_D are definite positive matrices for the PD action and $y_{k,r}$ is the robust term, given by:

$$y_{k,r} = \begin{cases} \dfrac{\rho}{\left\| D^t Q \xi_k \right\|} D^t Q \xi_k & , \ if \ \left\| D^t Q \xi_k \right\| \geq \varepsilon \\ \dfrac{\rho}{\varepsilon} D^t Q \xi_k & , \ if \ \left\| D^t Q \xi_k \right\| < \varepsilon \end{cases} \qquad (2)$$

with

$$D_{2n \ x \ n} = \begin{bmatrix} 0_{n \ x \ n} \\ I_{n \ x \ n} \end{bmatrix}, \quad \xi_{2n \ x \ 1} = \begin{bmatrix} \tilde{\theta}_{n \ x \ 1} \\ \dot{\tilde{\theta}}_{n \ x \ 1} \end{bmatrix} \qquad (3)$$

$Q_{2n \ x \ 2n}$ is a positive definite matrix and ρ is the robust design parameter.

The parameter ε is determined in the final proofs with the system.

As it is proved in [1], the value of the design parameter ρ is important in order to have a good performance of the closed-loop system. A small value of ρ gives poor tracking results over the desired trajectories for the joints, while a very great value of ρ leads to saturation of the inputs and consequently an unsatisfactory behaviour of the closed-loop system. To have a satisfactory performance in all cases, the following adaptation law is proposed for the parameter:

$$\rho_k = \rho_{k-1} = -\gamma \frac{\partial J_k}{\partial \rho_{k-1}} \qquad (4)$$

A difference with the previous work is presented here. The cost function J_k is chosen as:

$$J_k(\rho_{k-1}) = \frac{1}{2} \xi_k{}^T Q_{ad} \xi_k + \frac{1}{2} y_{k-1}{}^T R_{ad} y_{k-1} \qquad (5)$$

where the 2n x 2n matrix Q_{ad} weighs the state error and the n x n matrix R_{ad} weighs the influence of the inputs to the linearised system. This choice gives the following adaptation law:

$$\rho_k = \rho_{k-1} - \\ -\gamma \left[\xi_k{}^T Q_{ad} \frac{\partial \xi_k}{\partial \rho_{k-1}} + y_{k-1}{}^T R_{ad} \frac{\partial y_{k-1}}{\partial \rho_{k-1}} \right] \qquad (6)$$

The computation of the derivatives is made taking into account the same approximations that in [1]. The correct expressions, after correcting some errors, are the following:

$$\frac{\partial y_{k-1}}{\partial \rho_{k-1}} = M \xi_{k-1} \qquad (7)$$

$$\frac{\partial \xi_k}{\partial \rho_{k-1}} = -\begin{bmatrix} CB_D M \xi_{k-1} \\ \frac{1}{h} CB_D M \xi_{k-1} \end{bmatrix} \qquad (8)$$

where $M = D^T Q / \varepsilon$, C and B_D matrixes are obtained form the linearised state-space model of the robot, and *h* is the sampling time.

3 Comparison with previous robust controller schemes

As it has been said, the value of the design parameter ρ is important in order to have a good performance of the closed-loop system. But this problem is not solved in the earlier literature.

In [3] the same robust action (2) is proposed and ρ is given as a single measure of the uncertainty. It is proved that the closed-loop system with this control law is uniformly ultimately bounded, as defined in [13], but it does not say anything about how to measure the uncertainty.

In [14], the authors propose a parameterization of both inertia and Coriolis and centrifugal matrices, which leads to a robust control law with two terms quite similar to (2), proving that the tracking error is again uniformly ultimately bounded. The robust design parameter for each control law (referred as uncertainty bounds), is chosen measuring the maximum difference between the nonlinear terms of the robot dynamic equations in the case of no load case and maximum load case, but it is a bound expression and it is assumed that the load masses and the dynamic parameters are well-known.

In [15], a comparison between these two controllers and others is proposed, but it is avoided the choice of the parameter ρ in the different cases. In [16], a similar robust law to (2) is proposed, and ρ is set by the measure of the tracking error after a random choice of others constants.

And following the whole literature, the choice of this design parameter is not clear or easy, or it is avoided. In order to solve that, in this work an adaptive law for updating the value of this parameter is proposed.

4 Results

This work applies this controller on a two-link robot manipulator. The parameters of the links are 5.0 kg and 4.5 kg for the masses of the links and 0.43 m for its lengths. In the different simulations, input constraints are considered: $|u_{k,1}| \leq 60 \ \ N \cdot m$, and $|u_{k,2}| \leq 15 \ \ N \cdot m$, where the sub-index 1 and 2 indicates respectively the links 1 and 2. For the stabilizing control law, the following matrixes are used: K_p=diag([155 115]) and K_D=diag([12 15]), being diag(•) the function to define a diagonal matrix.

To define the robust action, the following values are used: ε=0.25 and Q=[0 0 0 0 ; 0 0 0 0 ; 10 0 1 0 ; 0 30 0 5]. To define the cost function employed in the adaptive law, the following matrixes are used: Q_{ad} = diag([100 100 1 1]) and $R_{ad} = O_{2x2}$, which implies not weighting the inputs in the cost function. This can be assumed because in the different simulations the maximum values for the inputs were not reached. To do the different simulations, a sampling time h=0.0001 sec. is used.

4.1 Fixed robust action

The reference trajectory for the two-link manipulator in the different simulations is a smooth trajectory of 8 seconds between 120.3º to 17.2º for the first link and -74.5º to -55.6º for the second link. At the final point, the arm takes a spherical payload mass of 1.5 Kg and 15 cm., returns to the initial point and, finally, to the final point again.

Figures 1 and 2 show the results obtained with a fixed robust action with ρ=17. The real trajectory for the first link is quite similar to the reference trajectory, but the position errors are appreciable for the second link (near 0.03 radians in the worst case). This value of ρ is chosen after several simulation trials. As the value of ρ increases, the position and velocity errors decrease. It can be seen in figures 3 and 4, where the value of the robust design parameter is, consecutively, ρ=5, ρ=12 and ρ=17.

4.2 Self-adaptive robust action

In this case, the adaptive scheme shown in the previous section is used. In the same situation as before, the parameter ρ is initialized to the value ρ=1. The value of the learning rate is settled to γ=20/h. Position and velocities errors are considerable at the beginning of the trajectory, but then they decrease to nearly zero while ρ is

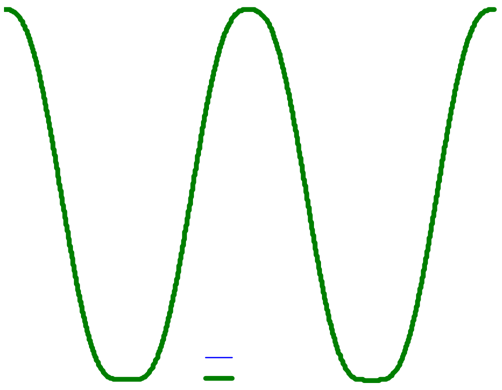

Fig. 1: Desired and simulated trajectory for the first link with a fixed robust action (ρ=17).

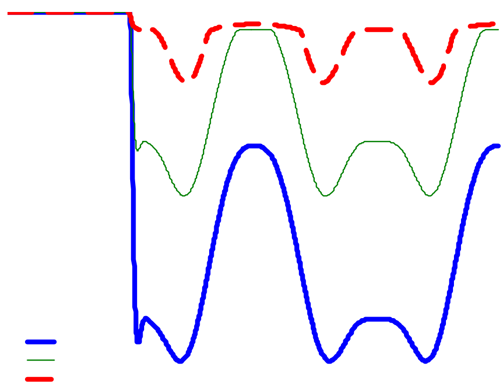

Fig. 4: The same situation as figure 3 for the second link

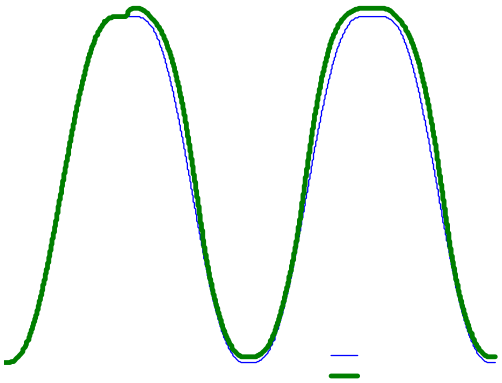

Fig. 2: The same result as figure 1 for the second link.

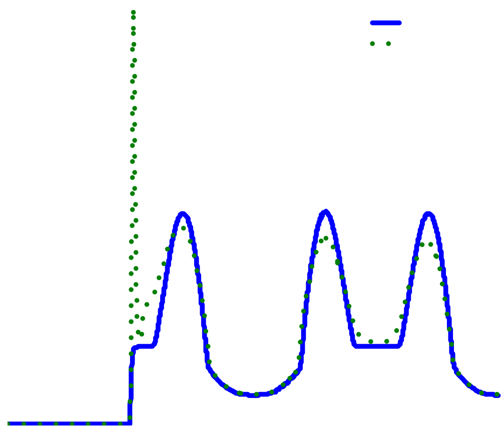

Fig. 5: Comparison of the error in the position of first link whith a fixed robust controller and a self-adaptive robust controller.

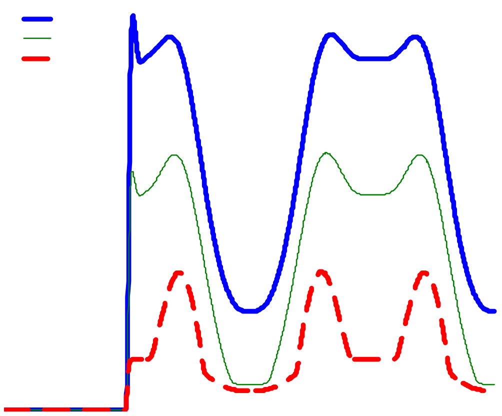

Fig. 3: Comparison of the error in the position of first link when a fixed robust controller is used with three different values.

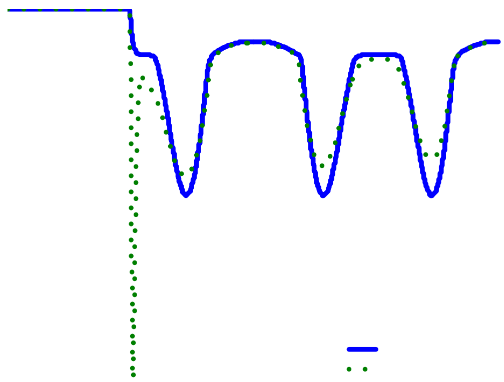

Fig. 6: Comparison of the error in the position of first link whith a fixed robust controller and a self-adaptive robust controller.

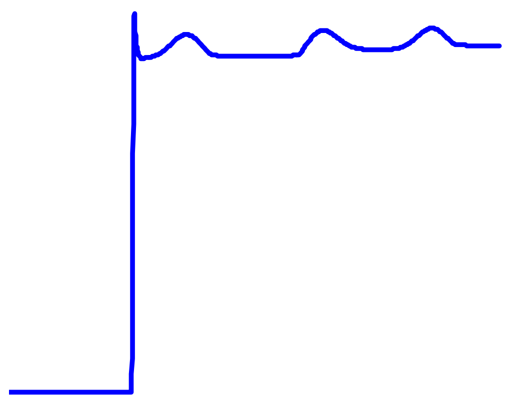

Fig. 7: Evolution of the robust design parameter ρ for the simulations of figures 5 and 6.

settling in its final value. The position errors for both links can be seen in figures 5 and 6, compared with the obtained with a fixed robust action with ρ=17, which is a similar value to the finally obtained with the adaptive scheme.

The error performance is quite similar at the final of the motion. The difference is that the adaptive scheme tries to reduce it (it is achieved after several repetitions of the planned motion) but the fixed robust controller can not do it. In figure 7, the evolution of ρ can be seen. Its value is increased from ρ=1 until a value nearly ρ=17. It presents some oscillations when the position errors are considerable, in order to reduce them.

5 Conclusions

In this work a revision of a previous work of the same authors is made, in order to improve the proposed adaptive robust control scheme applied on robot manipulators. The basics of the controller are the adaptation of the robust action, introduced to avoid the errors due to the imperfect cancellation of the nonlinear dynamics in presence of uncertainties, in case of changing of the dynamics. These changes are produced by taking an unknown payload mass or when the dynamics parameters differ from a manipulator to another one.

The results presented here show the satisfactory behaviour of the closed-loop system when a payload mass is taken at the middle of the trajectory. The self-adaptive law increases the value of the robust design parameter from its initial value to another value, similar to the obtained with a fixed robust controller after several trials.

Then, the proposed scheme reduces the time in designing the robust controller for the manipulator and avoids the tracking errors produced by the changes in the dynamics of the system.

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