

# PBG Substrate in Triangularized Planar Antenna

Késia Cristiane dos Santos and Humberto César Chaves Fernandes

Department of Electrical Engineering - Federal University of Rio Grande do Norte,  
Natal/RN-59.072-970, P. O. Box 1583 - Tel/Fax: +55 84 2153732 - Brazil

**Abstract** — The triangular microstrip antenna is analyzed through of a step discretization using the full wave Transverse Transmission Line method (TTL) in combination with the Moment method. The PBG (Photonic Band Gap) structure is analyzed using the homogenization theory, and the objective is to obtain the equivalent permittivity of the structure. Numerical results of the imaginary and real resonant frequency are shown.

**Index Terms** — Microstrip Antenna, Photonic Band Gap, Triangular Patch, Transverse Transmission Line method.

## I. INTRODUCTION

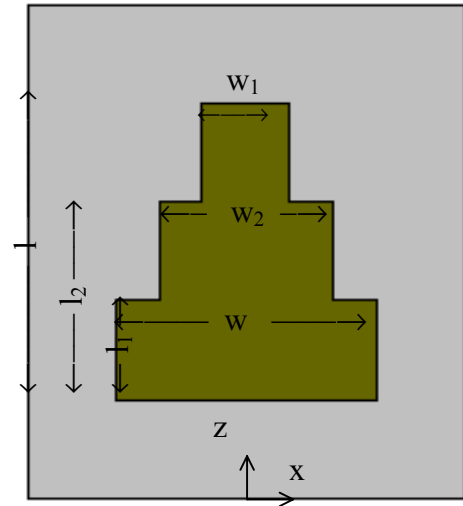
A microstrip patch antenna is used because of its light weight and low cost. The patch of the microstrip antennas can have different geometries such as rectangular, circular, elliptic, squares and triangular. The present paper is about a triangular microstrip antenna with step discretization that is a modification of the rectangular antenna. The complex resonant frequency is calculated for triangular antenna with steps discretization.

The study consists of the analysis of one structure with two dielectric substrates and a rectangular patch that is changed in its width of  $n$  steps. Thus with a great number of steps going to infinite, the patch takes a triangular form.

The triangular microstrip antenna with step discretization is analyzed using the full wave efficient and concise Transverse Transmission Line (TTL) method in the Fourier Transformed Domain (FTD), in combination with the Moment method [1]-[8]. The generalisation study of the structure to  $n$ -steps, makes the analysis more complete. The Fig. 1, shows a superior view of a two steps antenna ( $n = 2$ ).

Analysing the microstrip we conclude that if  $n$  goes very big, but no infinite, we will have a triangular discretization patch, that is a good approximation of a triangular continue one and has a simpler analyse.

Using the PBG as dielectric material in the antenna, the propagation of some frequency are not permitted, producing band gap.



## II. THEORY

### A. TTL Method

Applying the TTL method the equations that represents the electromagnetic fields in the  $x$  and  $z$  directions as function of the electric and magnetic fields in the  $y$  direction, for the microstrip patch antenna are obtained. Starting from the Maxwell's equations, after various algebraic manipulations the general equations for the structure in the FTD are obtained. For the  $x$  direction the tangential ( $t = x$  and  $z$ ) components:

$$\tilde{E}_{xi} = \frac{1}{\gamma_i^2 + k_i^2} \left[ -j\alpha_n \frac{\partial}{\partial y} \tilde{E}_{yi} + \omega\mu\beta_k \tilde{H}_{yi} \right] \quad (1)$$

$$\tilde{H}_{xi} = \frac{1}{\gamma_i^2 + k_i^2} \left[ -j\alpha_n \frac{\partial}{\partial y} \tilde{H}_{yi} - \omega\epsilon\beta_k \tilde{E}_{yi} \right] \quad (2)$$

and for the  $z$  direction:

$$\tilde{E}_{zi} = \frac{1}{\gamma_i^2 + k_i^2} \left[ -j\beta_k \frac{\partial}{\partial y} \tilde{E}_{yi} - \omega\mu\alpha_n \tilde{H}_{yi} \right] \quad (3)$$

$$\tilde{H}_{zi} = \frac{1}{\gamma_i^2 + k_i^2} \left[ -j\beta_k \frac{\partial}{\partial y} \tilde{H}_{yi} + \omega\epsilon\alpha_n \tilde{E}_{yi} \right] \quad (4)$$

where  $i = 1, 2$  are the dielectric regions,  $\gamma_i^2 = \alpha_n^2 + \beta_k^2 - k_i^2$  is the propagation constant in  $y$  direction,  $\alpha_n$  is the spectral variable in  $x$  direction,  $\beta_k$  is the spectral variable in  $z$  direction,  $k_i^2 = \omega^2\mu\epsilon = k_0^2\epsilon_n^*$  is the wave number of the  $i$ th dielectric region and  $\epsilon_n^* = \epsilon_n - j\frac{\sigma_n}{\omega\epsilon_0}$  is the relative dielectric permittivity

of the material,  $k$  is the wave number. After the application of the boundary conditions, the Moment method is used to eliminate the electric fields and to obtain the homogeneous matrix equation for the calculation of the complex resonant frequency. The roots of this matrix are the real and imaginary resonant frequencies.

### B. PBG Structure

One of the problems that appear when working with photonic material it is the determination of the effective dielectric constant. For a non-homogeneous structures submitted the incident sign goes at the process of multiple spread. A solution can be obtained through of numerical process called of homogenization [9].

The process is based in the theory related the diffraction of an incident electromagnetic plane wave imposed by the presence of cylinders of air immersed in a homogeneous material [10].

Chosen a Cartesian coordinates system of axes ( $O, x, y, z$ ), Figura 2. Consider firstly a cylinder with relative permittivity  $\epsilon_1$ , with traverse section in the plane  $xy$ , embedded in a medium of permittivity  $\epsilon_2$ . For this process the two-dimensional structure is sliced in layers whose thickness is equal at the cylinder diameter. In each slice is realized the homogenization process.

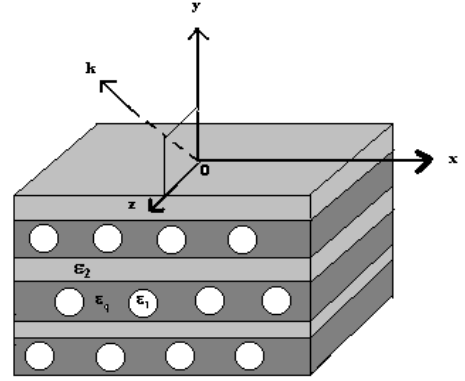


Fig. 2. Homogenized bidimensional crystal.

According to homogenization theory the effective permittivity depends on the polarization [11]. For  $s$  and  $p$  polarization, respectively, are:

$$\epsilon_{eq} = \beta(\epsilon_1 - \epsilon_2) + \epsilon_2 \quad (5)$$

$$\frac{1}{\epsilon_{eq}} = \frac{1}{\epsilon_1} \left\{ 1 - \frac{3\beta}{A_1 + \beta - A_2\beta^{10/3} + O(\beta^{14/3})} \right\} \quad (6)$$

where

$$A_1 = \frac{2/\epsilon_1 + 1/\epsilon_2}{1/\epsilon_1 - 1/\epsilon_2} \quad (7)$$

$$A_2 = \frac{\alpha(1/\epsilon_1 - 1/\epsilon_2)}{4/3\epsilon_1 + 1/\epsilon_2} \quad (8)$$

where  $\beta$  is defined as the ratio of the area of the cylinders over the area of the cells and  $\alpha$  is an independent parameter whose value is equal to 0.523. The  $A_1$  and  $A_2$  variables in (7) and (8) were included only for simplify (6).

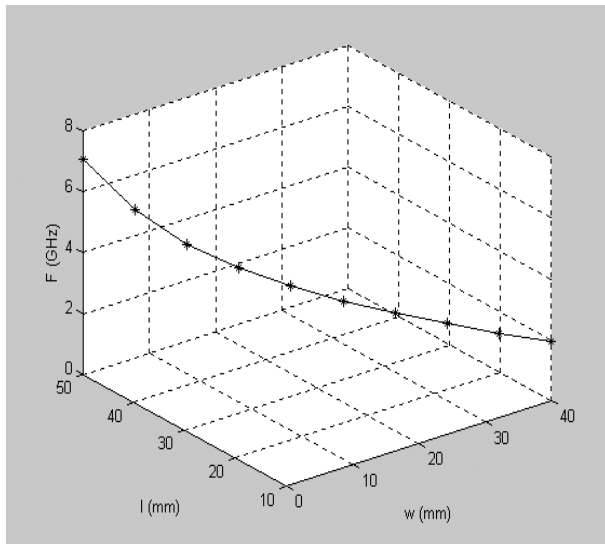
### III. NUMERICAL RESULTS

The computational program used to calculate the resonant frequency of the triangular antenna with step discretization of the dielectric substrate with PBG, was developed in Fortran PowerStation and Matlab 6.0

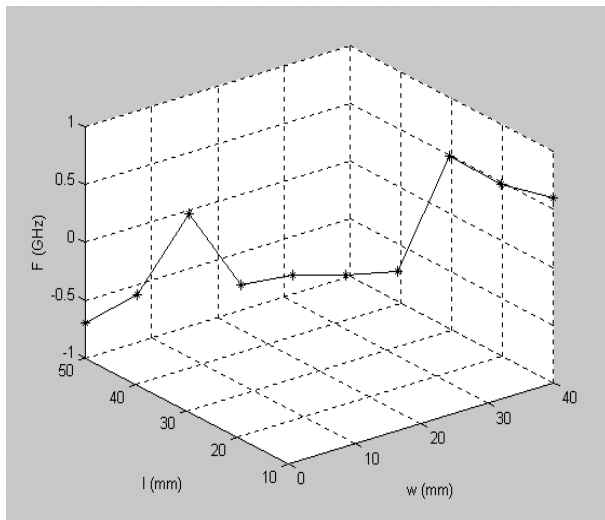
The Fig. 3 and 4 shows the curves of the real and imaginary resonance frequency for the polarization  $s$  and  $p$ , respectively, as function of the length and the width do patch, for the structure with PBG material.

The parameters used to plot the curves are: The permittivity relative is 10.233 for  $s$  polarization and 8.7209 for  $p$  polarization. The substrate thickness is 0.7

mm, the width and length of patch are 14 and 9 mm, respectively, with 15 steps in patch.

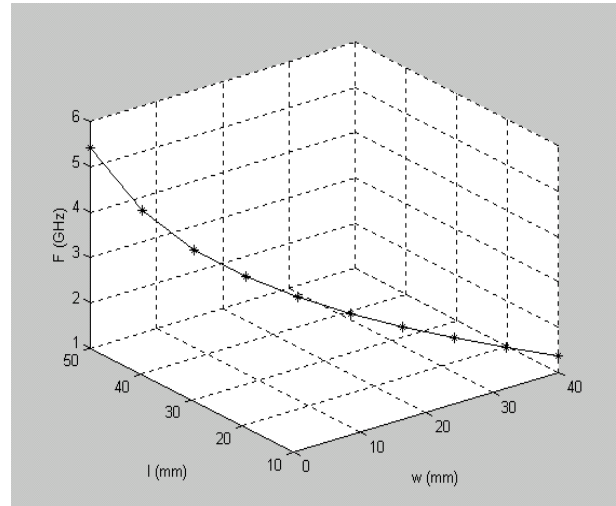


(a)

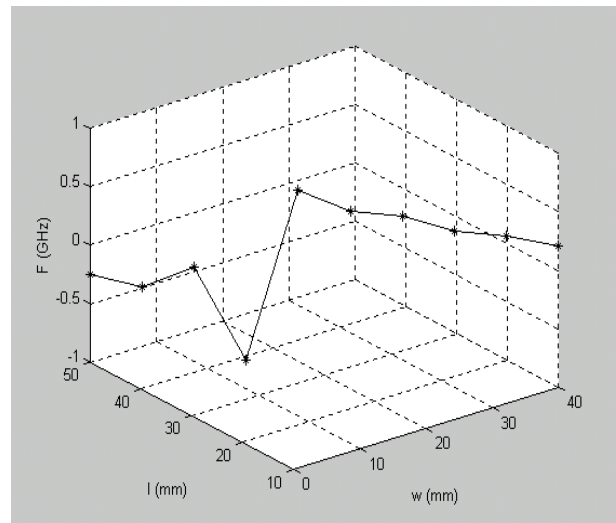


(b)

Fig. 3. Resonant frequency of microstrip patch for s polarization (a) real and (b) imaginary.



(a)

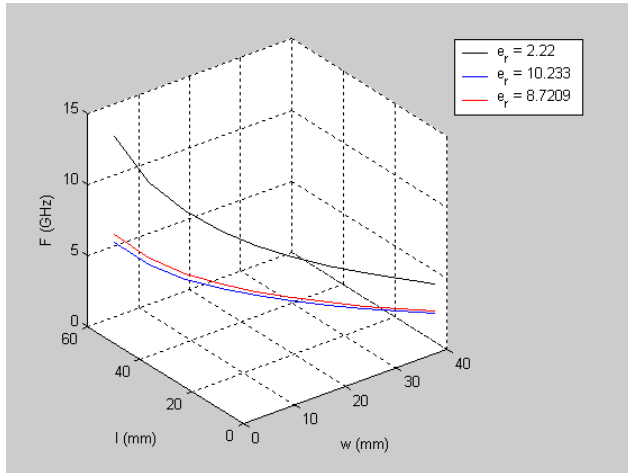


(b)

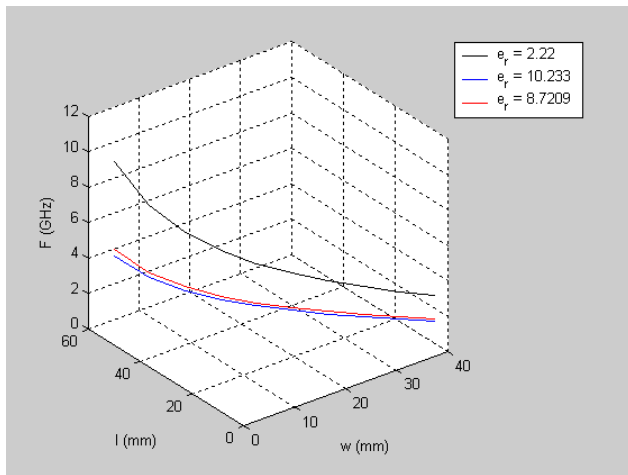
Fig. 4. Resonant frequency of microstrip patch for p polarization (a) real and (b) imaginary.

The Fig. 5 shows the curves of the real resonance frequency for the polarization s and p, respectively, as function of the length and the width of the patch, for the structure with PBG material in comparison with substrate of permittivity relative 2.22 [12].

The parameters used to plot the curves are: The relative permittivity is 10.233 for s polarization and 8.7209 for p polarization. The substrate thickness is 0.7 mm, the width and length of patch are 14 and 9 mm, respectively, with zero and 15 steps in patch.



(a)



(b)

Fig. 3. Real Resonant frequency of microstrip patch for s and p polarization compared with substrate of  $\epsilon_r = 2.22$  (a) 15 and (b) zero steps.

#### IV. CONCLUSIONS

The Transverse Transmission Line (TTL) method was used for analysis a triangular microstrip antenna with steps discretization, using PBG substrate.

Results for the complex frequency resonance when the number of steps tend the infinite (15 steps) agreed with the encontred in literature, being compared when the number of steps in patch is zero, that is, patch is rectangular.

The results of the complex frequency resonance for the substrate of the permittivity relative  $\epsilon_{r1}=2.22$  are compared with the PBG substrate.

Curves that show the variation of the real and imaginary resonance frequency with relation to the

patch width and length of the triangular microstrip antenna with steps discretization were presented in 3-D.

#### REFERENCES

- [1] Farias, A. R. N. and H. C. C. Fernandes, "Microstrip antenna design using the TTL method", 1997 SBMO/IEEE International Microwave and Optoelectronics Conference, Natal-RN, pp. 291-296, Aug. 1997.
- [2] B. Baht and S. K. Koul, "Analysis, design and applications of finlines", Artech House, 1987.
- [3] Agrawal, A. K. and B. Bhat, "Resonant Characteristics and End Effects of a Slot Resonator in Unilateral Fin Line", Proc. IEEE, Vol. 72, 1416-1418, Oct. 1984.
- [4] H. C. C. Fernandes, G. F. S. Filho, A. R. N. Farias e J. P. Silva, "High Precision Characterization and Analysis of Superconducting Microstrip Patch", 4<sup>th</sup> International Conference on Millimeter and Submillimeter Waves and applications, San Diego-USA, pp. 128-129, Jul. 1998.
- [5] Eritônio F. Silva and Humberto C. C. Fernandes, "Efficient Analysis of Microstrip Rectangular-Patch Antenna and Arrays", AP-2000, Conf. Proc pp.315-319, Davos, Swisse, Ap. 2000.
- [6] Humberto C. C. Fernandes, S. A.P. Silva and L. C. Freitas Júnior, "Computational Program to the finline Coupler in High frequencies", VIII SBMO - Brazilian Microwave and Optoelectronic Symposium, Joinville-SC, pp. 256-259, Jul. 1998.
- [7] Humberto C.C. Fernandes, S.A. Pinto and Eritonio F. Silva, "A New Educational Program for Microstrip Antenna Arrays", ICECE'99- International Conference Eng. Comput. Educational, Rio de Janeiro-RJ, Conf. Proc pp. 519.1-519.4, Aug. 1999.
- [8] H.C.C. Fernandes, J. P. Silva and G.F.S. Filho, "TTL Method Applied on Microstrip Rectangular-Patch Antenna Analysis and Design", XXVIII Moscow International Conference on Antenna Theory and Technology, Moscow-Russia, Conf. Proc. pp. 424-427, Sept. 1998.
- [9] Diederick S. Wiersma, Paolo Bartolini, Ad Lagendijk and Roberto Righini, "Localization of light in a disordered medium", Letters to nature - Vol.390 18/25, Dec. 1997.
- [10] Vesna Radisic, Yongxi Qian, Roberto Coccioli, and Tarsuo Itoh, "Novel 2-D Photonic Bandgap Structure for Micostrip Lines", IEEE Mictrowave and guided wave letters, vol. 8, No. 2 Feb 1998
- [11] E. Centeno and D. Felbacq, "Rigorous vector diffraction of electromagnetic waves by bidimensional photonic crystals", J. Opt. Soc. Am. A/Vol. 17, No.2, pp.320-327, Feb. 2000.
- [12] H.C. C. Fernandes and C. B. Beltrão, "Computational program of triangular microstrip resonators with step discretization", ISSSE-International Symposium on Signals, Systems, and Electronics, Tókió, Japão, pp. 155-158, jul. 2001.