Analysis of a Tunable Optical Fiber Coupler Using the Vectorial Finite-Element BPM Scheme

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***Abstract* — An efficient finite-element formulation for dielectric media with transverse anisotropy is applied to the analysis of optical fiber coupler, The two cores are curved in parabolic shapes, and between them there are index-matching liquid layers whose refractive index and the width of the liquid layer are taken into account as variable parameters.**

***Index Terms* — Optical fibers coupler, vector beam propagation method (BPM), index matching liquid layers.**

I. Introduction

The beam propagation method (BPM) is one of the most powerful widely used numerical tools employed in the study of optical devices, mainly due to its numerical speed and simplicity [1]-[2]. Vectorial formulations are essential because they consider the vector nature of the electromagnetic fields. For dielectric media, it is quite clear that high accuracy and flexibility is attained by choosing the magnetic field as the wave equation's unknown, due to its continuity over the dielectric interfaces. This permits the use of nodal elements, which exhibit simpler expressions than the edge ones, especially for high order. For this situation, spurious solutions can be efficiently suppressed by forcing the divergence condition into the formulation, which will allow us, as additional advantage, to eliminate the axial field component [3]. As a consequence, a highly efficient scheme, which solves the magnetic field's transverse components, is obtained. In this paper we introduce the applicability of the present method to simulate the energy transference through the fiber coupler

II. Mathematical Formulation

Starting from Maxwell equations, the double curl Helmholtz equation for the magnetic field, $\vec{H}$, is readily obtained,

$$\nabla\times\left(\bar{\bar{k}}\nabla\times\vec{H}\right)-k_0^2\vec{H}=0 \qquad (1)$$

Where $\bar{\bar{k}}=1/\bar{\bar{\varepsilon}}$, $\bar{\bar{\varepsilon}}$ being the relative permittivity tensor. In addition, k_0 is the free space wavenumber and the operator ∇ is defined as,

$$\nabla=\hat{u}_x\alpha_x\frac{\partial}{\partial x}+\hat{u}_y\alpha_y\frac{\partial}{\partial y}+\hat{u}_z\alpha_z\frac{\partial}{\partial z}=\nabla_T+\hat{u}_z\alpha_z\frac{\partial}{\partial z} \qquad (2)$$

Where, α_x, α_y, and α_z, are parameters linked to the PML or virtual lossy media. Following [4] the PML parameters are specified from the parameter S given by $S=1-j\dfrac{3c}{2\omega_0 nd}\left(\dfrac{\rho}{d}\right)^2\ln\left(\dfrac{1}{R}\right)$, where ω_0 is the angular frequency, d is the thickness of the PML, n is refraction index of the adjacent medium, ρ is the distance from inner PML's interface and R is the reflection coefficient and c is the free-space speed of light. All PML parameters are referenced in [3]. Next, the magnetic field's rapid variation is removed by writing, $\vec{H}(x,y,z)=\vec{h}(x,y,z)e^{-jk_0n_0z}$, where n_0 is the reference effective index and $\vec{h}(x,y,z)=\vec{h}_T(x,y,z)+\vec{h}_z(x,y,z)$ is the magnetic field's envelope or slow variation portion. Here, $\vec{h}_T=h_x\hat{u}_x+h_y\hat{u}_y$ and $\vec{h}_z=h_z\hat{u}_z$ represent the magnetic fields' (slow) transverse and axial components, respectively. Using in addition, the magnetic field divergence condition, $\nabla\cdot\vec{H}=0$, which produces $h_z=(\nabla_T\cdot\vec{h}_T+\partial h_z/\partial z)/\gamma$, here, $\gamma=jk_0n_0$. After some algebraic manipulations, the axial field can be effectively eliminated from (1), obtaining the following vectorial wave equation, in terms of the (slow) transverse component. Here, we may assume that the media and fields vary very slowly along the propagation coordinate [3].

$$\overline{\overline{k}}_a \frac{\partial^2 \vec{h}_T}{\partial z^2} - 2\gamma \overline{\overline{k}}_a \frac{\partial \vec{h}_T}{\partial z} - \overline{\overline{k}}_b \nabla_T \left(\nabla_T \cdot \vec{h}_T\right) - \nabla_T \times k_{zz} \nabla_T \times \vec{h}_T + \left(\overline{\overline{k}}_c + \gamma^2 \overline{\overline{k}}_a\right) \vec{h}_T = 0 \tag{3}$$

Where, transverse tensors, in (3), are defined in [3]. Next, applying the conventional finite element method to the transverse variation of (3), the cross sectional domain Ω, is divided in *Nel* triangles, producing *Np* unknowns over the corresponding nodes. Now, introducing a set of basis functions (Lagrangian polynomials of first or second order), $\{\psi_j\}$, j = 1,..., *Np*; $\vec{h}_T(x,y,z)$ [3], the following differential equation is obtained:

$$[M]\frac{\partial^2\{\vec{h}_T\}}{\partial z^2} - 2\gamma[M]\frac{\partial\{\vec{h}_T\}}{\partial z} + ([K] + \gamma^2[M])\{\vec{h}_T\} = \{0\} \tag{4}$$

Where $\{\vec{h}_T\}$ represents a column vector containing the unknowns h_{xj} and h_{yj}, $\{0\}$ is the null column vector, and $[M]$ and $[K]$ are the so-called global matrices, defined in [3]. Following [5], the Padé (1,1) approximation, can be straightforwardly applied to (4), producing the matrix equation given below,

$$[\tilde{M}]\frac{d\{\vec{h}_T\}}{dz} + [K]\{\vec{h}_T\} = \{0\} \tag{5}$$

with, $[\tilde{M}] = [M] - \frac{1}{4\gamma^2}([K] + \gamma^2[M])$. Finally, the θ - finite difference marching scheme, applied to (5), writes as follows,

$$\begin{aligned}&([\tilde{M}(z)] + \theta\Delta z[K(z)])\{\vec{h}_T(z+\Delta z)\} = \\ &([\tilde{M}(z)] - (1-\theta)\Delta z[K(z)])\{\vec{h}_T(z)\}\end{aligned} \tag{6}$$

Where, Δz is the step's size along the propagation coordinate, and θ $(0 \le \theta \le 1)$ is introduced to control the stability of the method. Extensive tests have shown that stability is ensured when $0.5 \le \theta \le 1$. For $\theta = 0.5$, (6) corresponds to the well-known Crank-Nicholson algorithm. In order to improve the scheme's accuracy, the refractive index is renewed at each propagation step following the prescription given in [6], as follows,

$$n_0^2(z) = \mathrm{Re}\left[\frac{\{\vec{h}_T(z)\}^\dagger [K(z)]\{\vec{h}_T(z)\}}{k_0^2\{\vec{h}_T(z)\}^\dagger [M(z)]\{\vec{h}_T(z)\}}\right] \tag{7}$$

Here, † denotes complex conjugate and transpose.

III. Coupler Description

A schematic of the fiber coupler is shown in Fig 1. The surface of the substrate is first ground, then polished until the desired proximity to the fiber core is obtained [1], [7]. In order to solve the theoretical of a two-waveguide system, an alternate approximate method was developed which uses perturbation formalism. When two dielectric waveguides are placed alongside each other, the introduction of the second guide distorts the field distribution of the guided mode of the first guide [7]. The approximate consist in assuming that each waveguide mode distribution remains unperturbed, and in expressing the field of the two-waveguide structure as a linear combination of the unperturbed field of each waveguide in which the field coefficients depend on the position z along the direction of propagation. This z-dependence provokes the transfer of electromagnetic energy from excited waguide to other waveguide present in the structure. The guided energy is periodically transferred back and forth between the waveguides with a spatial period called the coupling length, equal to $L_c = \pi / 2c$. Where c is the coupling coefficient and is simply give by the spatial overlap of the two interacting waveguides modes [3].

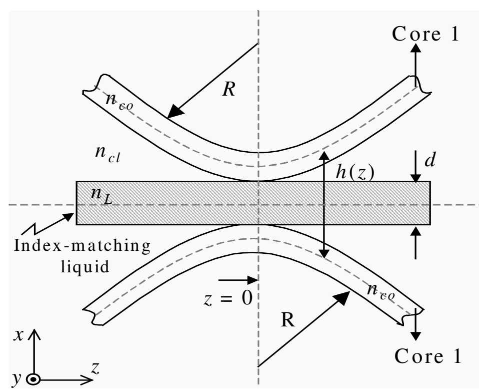

Fig. 1 – Cross section of the side view of the polished-type optical fiber coupler.

In the Fig. 1, z = 0 μm represent the center of the coupling region where the spacing between fibers is minimum, L is the effective interaction length of the coupler [1]. In the practice case, the geometry of the fibers coupler present a spacing h (z) between fiber axis in function of both the radius of curvature R of the fiber and the lateral fiber offset y, give by,

$$h = \left[\left(h_0 + \frac{z^2}{R} \right)^2 + y^2 \right]^{1/2} \qquad (8)$$

where a parabolic approximation was used since $z >> R$ and h (0) is the value of the h at the center-coupling region ($z = 0$).

IV. Numerical Results and Comparisons

As first results, we consider a polished-type optical fiber coupler with fibers parallel and superposed [1]. The side view of the polished-type coupler is shown in the Fig. 1. Two cores are curved in parabolic shapes, and between them an index-matching liquid layer (a slab layer) is introduced, Fig. 4a. The refractive indexes of the core and the cladding are $n_{co} = 1.46$ and $n_{cl} = 1.456$, respectively. The core radius is 2 μm, used for both fiber cores, the curvature radius of the curved core is R = 25 cm, the wavelength is λ = 0.6328 μm. The refractive index and the width of the liquid layer are taken to variable parameters. To simulate the coupler performance by launching the fundamental mode in the initial plane of the one core, the structure was excited by the x-polarized fundamental mode, obtained through modal analysis. For this purpose, we used a computational window of 20 μm (x-direction) x 12 μm (y-direction), covered by 3,833 linear elements, and propagation step Δz = 0.1 μm. The table I, shows a comparison between the results obtained using our numerical technical, with results published in [1]. The Fig. 2 shows the contours of the magnetic fields distributions of the x-polarized guide mode of fiber coupled.

TABLE I

Comparison of Power Transfer Ratios for the Polished-Type Couplers Obtained Using our numerical technical (FE-VBPM) and Ref. [1]

Structure	n_L	d (μm)	FE-VBPM	Ref. [1]	Difference
1	1.443	1.1	0.1013	0.1002	0.0011
2	1.447	1.0	0.2900	0.2907	-0.0007
3	1.450	1.0	0.4965	0.4953	0.00012
4	1.453	0.8	0.9851	0.9830	0.0021

The Fig. 3 shows the normalized power along the z-direction for core 1 (simple's line) and core 2 (line with symbol) considering the superposed fibers with y = 0 and d = 0.8 μm for n_L = 1.453. This case present the maximum transfer from core 1 to core 2.

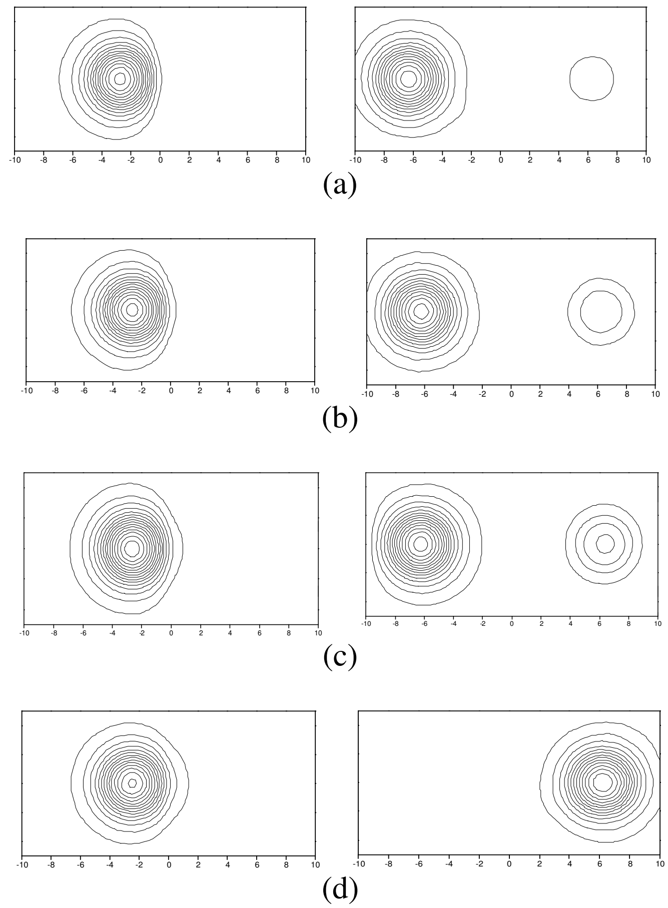

Fig. 2 – Contours of the magnetic fields distributions of the x-polarized guide mode of fiber coupled, z = 0.0 μm (left column) and z = 975.0 μm (right column) for (a) d = 1.1 μm, n_L = 1.443, (b) d = 1.0 μm, n_L = 1.447, (c) d = 1.0 μm, n_L = 1.450 and (d) d = 0.8 μm, n_L = 1.453.

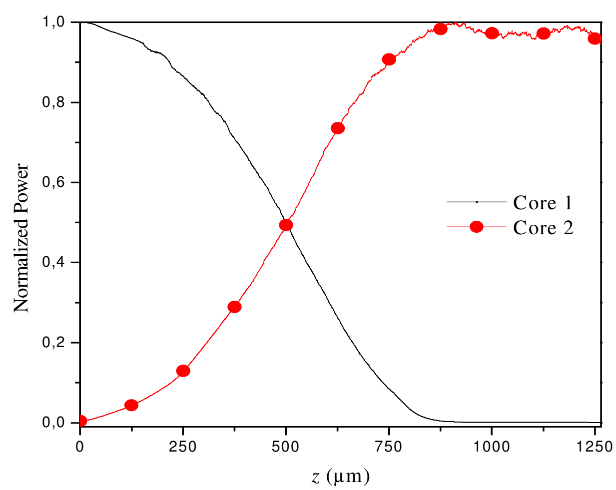

Fig. 3 – Normalized power curves for H_x along the z-direction for core 1 and core 2 considering the superposed fibers with y = 0 for d = 0.8 μm and n_L = 1.453.

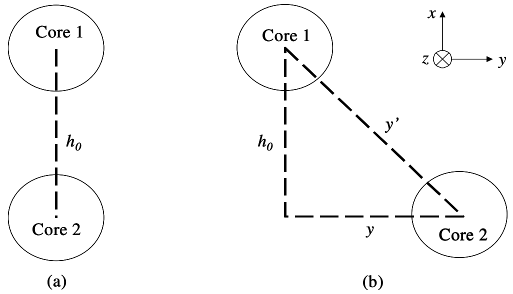

Fig. 4 – Transverse cross section of the relative position between core fibers: (a) Superposed fiber $y = 0$ and (b) Dislocated fiber $y \neq 0$.

Next, we consider a relative shift between the fibers $y \neq 0$, as showed in the Fig. 4b. The parameters of the fiber coupler are the same of the previous example, but considering the relative shift $y = 0.4$ µm. This relative shift enables the confection of couplers with variables coupling ratios [7]. The losses involved in the process can be controlled though the relative shift between fibers. The table II, shows a comparison between the results obtained with $y = 0$ and $y = 0.4$ µm using our numerical technical.

TABLE II

Comparison of Power Transfer Ratios for the Polished-Type Couplers with and without relative shift

Structure	n_L	d (µm)	$y = 0$ µm	$y = 0.4$ µm	Difference
1	1.443	1.1	0.1013	0.1009	0.0004
2	1.447	1.0	0.2900	0.2587	0.0313
3	1.450	1.0	0.4965	0.3001	0.1964
4	1.453	0.8	0.9851	0.4773	0.5078

V. CONCLUSIONS

A vectorial finite-element BPM for transverse anisotropic media was applied successfully to the analysis of a tunable optical fiber coupler. We can see the accuracy of the FE-VBPM schema for this application through the comparisons between our results and the results published in the literature [1]. The results obtained in Table II shows that when the distance between the guides is 0.8 µm and $n_L = 1.453$, the power transfer ratio between the cores of the fibers is significant but when compared with the not shifted case reaching over 50%.

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