Three Layers Slot Resonator with PBG

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***Abstract* — This paper shows the study and results of the three layers slot resonator with Photonic Band Gap (PBG) substrate. The full wave Transverse Transmission Line (TTL) method is used to calculate the complex resonant frequency, using double spectral variables. This method presents efficient and concise calculus in the determination of the structure parameters. Numerical results for the resonant frequency of three Layer's substrate slot antenna, for s and p polarization of PBG material, for different thickness substrate and resonator length are presents.**

Key Worlds — **TTL Method, PBG-Photonic Band Gap, Multilayer, Slot antenna.**

I. INTRODUCTION

Multilayer microwave resonator shows complexity when more accuracy methods are applied [1]–[2]. In this paper the concise full wave Transverse Transmission Line (TTL) method is used to calculate the complex resonant frequency of three Layer's substrate slot antenna with Photonic Band Gap (PBG) substrate.

Photonic band gap material exhibits energy band gap. In a photonic crystal if a photon has energy in the band gap it not propagates through of the material in the light direction. This PBG has periodic array of cylinder air with diameters and spacing less than one light wavelength [3]-[6]. This substrate can improve the bandwidth and eliminate propagation of undesirable modes and increases the antenna efficiency.

The choice of the structure has objective expand application the method in multilayer resonators because the increase of bandwidth for 4%[7] if maintain a fine substrate ($h/\lambda_0<0.01$) and low permittivity. This paper presents the comportment of the microwave resonator in function aperture of slot.

Fig. 1 shows one three Layer's substrate slot resonator. The thickness of the patch resonator and the ground plane are perfect conductors

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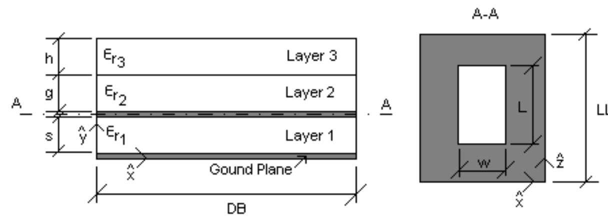

(a) (b)

Fig. 1 – (a) Cross section of the three Layer's substrate slot resonator, (b) Cut of slot resonator region.

II. TTL METHOD

The general equations of the fields in the TTL method are obtained after using the Maxwell's equations,as:

$$\begin{Bmatrix} \vec{E}_{Ti} \\ \vec{H}_{Ti} \end{Bmatrix} = \frac{1}{k_i^2+\gamma_i^2}\left[j\omega\nabla_T \times \begin{Bmatrix} -\mu\vec{H}_{yi} \\ \varepsilon_1\vec{E}_{yi} \end{Bmatrix} + \frac{\partial}{\partial y}\nabla_T \begin{Bmatrix} \vec{E}_{yi} \\ \vec{H}_{yi} \end{Bmatrix} \right] \tag{1}$$

were the index 'T' shows the transversal components directions (x, z):

$$\vec{E}_{Ti} = \vec{E}_{xi}\hat{x} + \vec{E}_{zi}\hat{z} \tag{2.1}$$

$$\vec{H}_{Ti} = \vec{H}_{xi}\hat{x} + \vec{H}_{zi}\hat{z} \tag{2.2}$$

$$\nabla_T = \nabla_x\hat{x} + \nabla_z\hat{z} = \frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial z}\hat{z} \tag{2.3}$$

The equations are used for the analysis in the spectral domain, in the " x " and " z "directions.

Therefore it should be applied to the field equations of double Fourier transform defined as:

$$\tilde{f}(\alpha_n, y, \beta_k) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y,z)\cdot e^{j\alpha_n x}\cdot e^{j\beta_k z}\,dxdz \tag{3}$$

The electromagnetic fields are obtained:

$$\tilde{E}_{xi} = \frac{1}{\gamma_i^2 + k_i^2}\left[-j\alpha_n \frac{\partial}{\partial y}\tilde{E}_{yi} + \omega\mu\beta_k\tilde{H}_{yi} \right] \tag{4.1}$$

$$\tilde{E}_{zi} = \frac{1}{\gamma_i^2 + k_i^2}\left[-j\beta_k \frac{\partial}{\partial y}\tilde{E}_{yi} - \omega\mu\alpha_n\tilde{H}_{yi} \right] \tag{4.2}$$

$$\tilde{H}_{xi} = \frac{1}{\gamma_i^2 + k_i^2}\left[-j\alpha_n \frac{\partial}{\partial y}\tilde{H}_{yi} - \omega\varepsilon\beta_k\tilde{E}_{yi} \right] \tag{4.3}$$

$$\widetilde{H}_{zi} = \frac{1}{\gamma_i^2 + k_i^2}\left[- j\beta_k \frac{\partial}{\partial y} \widetilde{H}_{yi} + \omega\varepsilon\alpha_n \widetilde{E}_{yi} \right] \quad (4.4)$$

where:
i = 1, 2, 3, represent three dielectric regions of the structure;

$$\gamma_i^2 = \alpha_n^2 + \beta_k^2 - k_i^2 \quad (4.5)$$

is the propagation constant in y direction; α_n is the spectral variable in "x" direction and β_k is the spectral variable in "z" direction.

$k_i^2 = \omega^2 \mu\varepsilon = k_0^2 \varepsilon_{ri}^*$ is the wave number of i^{th} dielectric region;

$\varepsilon_{ri}^* = \varepsilon_{ri} - j\dfrac{\sigma_i}{\omega\varepsilon_0}$ is the relative dielectric constant of the material with losses;

$\omega = \omega_r + j\omega_i$ is the complex angular frequency;

$\varepsilon_i = \varepsilon_{ri}^* \cdot \varepsilon_0$ is the dielectric constant of the material;

III. THE ADMITANCE MATRIX

The equations above are applied to the resonator, being calculated the E_y and H_y fields through the solution of the Helmoltz equations in the spectral domain [8]-[9]:

$$\left(\frac{\partial^2}{\partial y^2} - \gamma^2 \right) \widetilde{E}_y = 0 \quad (5.1)$$

$$\left(\frac{\partial^2}{\partial y^2} - \gamma^2 \right) \widetilde{H}_y = 0 \quad (5.2)$$

The solutions of these equations for the three regions of the structure are applied and using the boundary conditions, the general equations of the electromagnetic fields are then obtained. The following equations relate the linear current densities in the sheets ($\widetilde{J}_{xg}$ and $\widetilde{J}_{zg}$) and the magnetic fields in the interface y = s (Fig.1 a):

$$\widetilde{H}_{x2} - \widetilde{H}_{x3} = \widetilde{J}_{zg} \quad (6.1)$$

$$\widetilde{H}_{z2} - \widetilde{H}_{z3} = -\widetilde{J}_{xg} \quad (6.2)$$

The substitutions of the magnetic fields [10], after various calculus gives,

$$Y_{xx}\widetilde{E}_{xg} + Y_{xz}\widetilde{E}_{zg} = \widetilde{J}_{zg} \quad (7.1)$$

$$Y_{zx}\widetilde{E}_{xg} + Y_{zz}\widetilde{E}_{zg} = \widetilde{J}_{xg} \quad (7.2)$$

that in matrix's form is:

$$\begin{bmatrix} Y_{xx} & Y_{xz} \\ Y_{zx} & Y_{zz} \end{bmatrix} \begin{bmatrix} \widetilde{E}_{xg} \\ \widetilde{E}_{zg} \end{bmatrix} = \begin{bmatrix} \widetilde{J}_{zg} \\ \widetilde{J}_{xg} \end{bmatrix} \quad (8)$$

The " Y " matrix is the dyadic Green admittance function of the slot antenna[11].

IV. RESONANT FREQUENCY

The electric fields in the slot interface are expanded in terms of known base functions, as [12]-[16]:

$$\widetilde{E}_{xg} = \sum_{i=1}^{n} a_{xi} \cdot \widetilde{f}_{xi}(\alpha_n, \beta_k) \quad (9.1)$$

$$\widetilde{E}_{zg} = \sum_{j=1}^{m} a_{zj} \cdot \widetilde{f}_{zj}(\alpha_n, \beta_k) \quad (9.2)$$

where a_{xi} and a_{zj} are unknown constant and the n and m terms are positive and integer numbers. Using one base function for each component :

$$\widetilde{E}_{xg} = a_x \cdot \widetilde{f}_x(\alpha_n, \beta_k) \quad (10.1)$$

$$\widetilde{E}_{zg} = a_z \cdot \widetilde{f}_z(\alpha_n, \beta_k) \quad (10.2)$$

And choosing base functions in the space domain expressed by [13]:

$$f_x(x,z) = f_x(x).f_x(z) \quad (11.1)$$

$$f_x(x) = \frac{1}{\sqrt{\left(\frac{w}{2}\right)^2 - x^2}} \quad (11.2)$$

$$f_x(z) = \cos\left(\frac{\pi z}{l}\right) \quad (11.3)$$

Whose Fourier transformed are:

$$\widetilde{f}_x(\alpha_n) = \pi \cdot J_o\left(\alpha_n \frac{w}{2}\right) \quad (12.1)$$

$$\widetilde{f}_x(\beta_k) = \frac{2\pi l \cdot \cos\left(\frac{\beta_k l}{2}\right)}{\pi^2 - (\beta_k l)^2} \quad (12.2)$$

$$\widetilde{f}_x(\alpha_n, \beta_k) = \frac{2\pi^2 l \cdot \cos\left(\frac{\beta_k l}{2}\right)}{\pi^2 - (\beta_k l)^2} \cdot J_o\left(\alpha_n \frac{w}{2}\right) \quad (12.3)$$

where J_0 is the Bessel function of first specie and zero order.

The Gallerkin method and the Parseval theorem [17] are applied to (8), to eliminate the current densities and the new equation in matrix's form is obtained,

$$\begin{bmatrix} K_{xx} & K_{xz} \\ K_{zx} & K_{zz} \end{bmatrix} \cdot \begin{bmatrix} a_x \\ a_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (13)$$

where,

$$K_{xx} = \sum_{-\infty}^{\infty} \tilde{f}x \cdot Yxx \cdot \tilde{f}x^{*} \tag{14.1}$$

$$K_{xz} = \sum_{-\infty}^{\infty} \tilde{f}z \cdot Yxz \cdot \tilde{f}x^{*} \tag{14.2}$$

$$K_{zx} = \sum_{-\infty}^{\infty} \tilde{f}x \cdot Yzx \cdot \tilde{f}z^{*} \tag{14.3}$$

$$K_{zz} = \sum_{-\infty}^{\infty} \tilde{f}z \cdot Yzz \cdot \tilde{f}z^{*} \tag{14.4}$$

The solution of the determinant of the characteristic equation (13), supplies the complex resonant frequency [18].

V. PBG STRUCTURE

One of the problems that appear when working with photonic material is the determination of the effective dielectric constant. For a non-homogeneous structures submited the incident sign goes at the process of multiple spread. A solution can be obtained through of numerical process called of homogenization [19]-[20].

The process is based in the theory related to the diffraction of the incident electromagnetic plane wave imposed by the presence of air cylinders immerged in an homogeneous material [6].

Choosing a Cartesian coordinates system of (*O, x, y, z*) axes shown in the Fig. 2, consider firstly a cylinder with relative permittivity ε_1, with traverse section in the xy plane, embedded in a medium of permittivity ε_2. For this process the two dimensional structure is sliced in layers whose thickness is equal at the cylinder diameter. In each slice is realized the homogenization process.

According to homogenization theory the effective permittivity depends on the polarization [8]. For *s* and *p* polarization, respectively, the effective permittivity are:

$$\varepsilon_{eq} = \beta(\varepsilon_1 - \varepsilon_2) + \varepsilon_2 \tag{15.1}$$

$$\frac{1}{\varepsilon_{eq}} = \frac{1}{\varepsilon_1}\left\{1 - \frac{3\beta}{A_1 + \beta - A_2\beta^{10/3} + O\left(\beta^{14/3}\right)}\right\} \tag{15.2}$$

where

$$A_1 = \frac{2/\varepsilon_1 + 1/\varepsilon_2}{1/\varepsilon_1 - 1/\varepsilon_2} \tag{15.3}$$

$$A_2 = \frac{\alpha(1/\varepsilon_1 - 1/\varepsilon_2)}{4/3\varepsilon_1 + 1/\varepsilon_2} \tag{15.4}$$

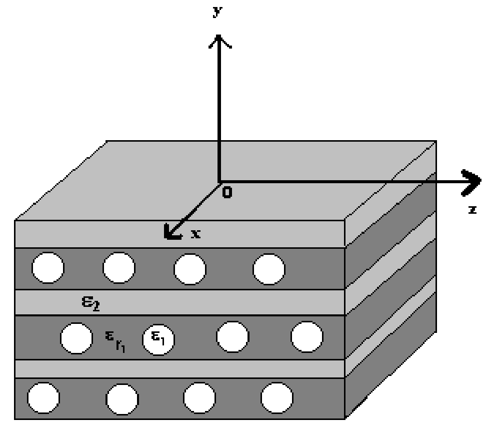

Fig. 2. Homogenized two dimensional PBG crystal.

where β is defined as the ratio of the area of the cylinders over the area of the cells and α is an independent parameter whose value s equal to 0.523. The A_1 and A_2 variables in (15.3) and (15.4) were included for simplify the (15.2) equation.

V. NUMERICAL RESULTS

The computational program used to calculate the resonant frequency, with PBG material, was developed in Fortran PowerStation and the Matlab for Windows.

The fig.3 shows the compare of accuracy of the TTL method in the structures when compare with bilateral fin lines resonator in limit case (when the slot aperture L is equal at LL). Three Layer's substrate as function of length of patch at width equal 5 mm. The thickness substrates of one and two regions are 1.4 mm, the third region is the space. The relative permittivity is 10.233 for s polarization without loss.

The Fig. 4 shows the real frequency of resonator rectangular three Layer's substrate as function of length of patch at width equal 15 mm. The thickness substrate of one and two regions is 1.4 mm, the third region is air. The relative permittivity is 10.233 for s polarization and 8.7209 for p polarization without loss. This graphic can observe the non linear variation of resonant frequency for length of slot, caused by insertion of the multiwall and gaps in the PBG substrate application.

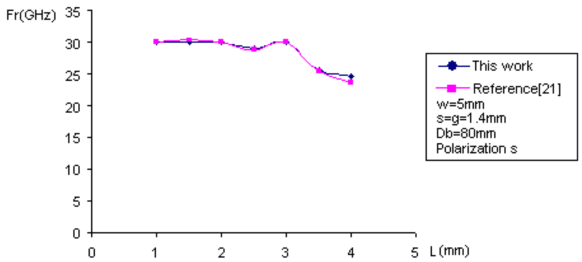

Fig. 3. Resonant Frequency as function of resonator and comparison with bilateral fin-line cavity slot resonator, for s polarization of PBG material

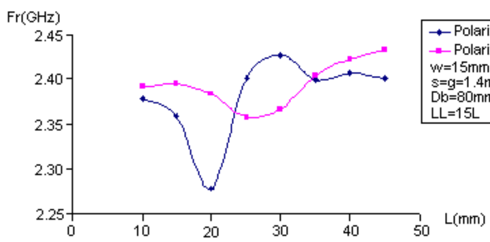

Fig. 4. Resonant frequency of three Layer's substrate antenna, for s and p polarization of PBG material.

Considering the variation of two layer in the Fig. 5 shows the real frequency of resonator as function of thickness of second layer with RT-DUROID material (ε_r =2.22). The patch and width equal 20mm, the first region with 1.4mm of thickness of PBG material for S polarization and third region is air. The relative permittivity is 10.233 for s polarization considering losses of 0.5 S/m. This graphic can observe the non linear variation of resonant frequency for thickness.

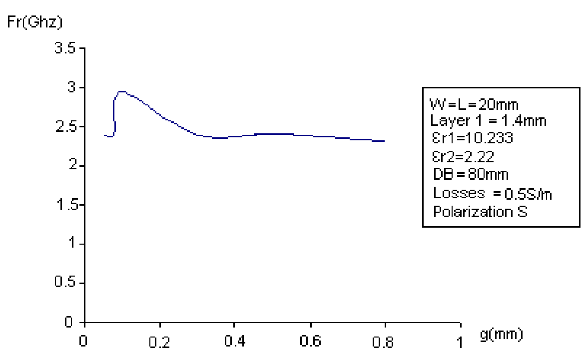

Fig. 5. Resonant frequency of three Layer's substrate antenna, for s polarization of PBG in the layer 1 as function of the thickness of second region with RT-DUROID material.

VI. CONCLUSION

The three Layer's microwave slot patch antenna were analyzed using the full wave Transversal Transmission Line – TTL method. The equations that represent the electromagnetic fields for the layers are obtained, according this concise and effective procedure. Applying the moment method the complex resonant frequency was calculated considering layers with Photonic Band Gap (PBG) material. This resonant frequency was calculated through double spectral variables. This method shows great facility in the structure parameters determination. Numerical results for the resonant frequency, using the Fortran Power Station language with good comparison, were presented.

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