Position Controllers With Bounded Actions For Robot Manipulators

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Abstract: This paper addresses the position control problem for robot manipulators. We present three new controllers with bounded actions supported by a rigorous stability analysis including the robot dynamics in the closed–loop using Lyapunov's direct method and the LaSalle's invariance principle. Besides the theoretical results, the performance of proposed controllers is illustrated by experimental results on a two degrees of freedom direct drive robot manipulator.

Key-Words: - Position Control, Robot Manipulators, Bounded Actions, $\mathcal{L}_2$ Norm, Lyapunov function, Asymptotic Stability.

1 Introduction

Most of actual robot manipulators use simple Proportional and Derivative (PD) control or Proportional-Integral-Derivative (PID) control into their feedback loops in order to reach a desired configuration. Although the PID control is a popular strategy, it lacks of a global asymptotic stability proof [1, 2, 3]. In contrast, the PD controller with gravity compensation introduced by Takegaki and Arimoto [3] holds a globally asymptotically stable closed-loop system. However, the design of new control schemes with Lyapunov stability for the closed-loop system still is interesting for robotic international community.

The position control problem for robot manipulators consists in to carry the extreme of the robot to a desired position independently from initial position[1, 4]. In general the control objective is that error tend to zero in asymptotically form; this is:

$$\lim_{t\to\infty} \begin{bmatrix} \tilde{q}(t) \\ \dot{q}(t) \end{bmatrix} \to 0.$$

Our motivation occurs by the theoretical-practice interest of to design control schemes with asymptotic convergence of the position error signal in sense global of Lyapunov stability within physical limits of the actuators. The methodology of energy shaping is used for the design of controllers [5].

In this paper we present three new controllers for robot manipulators with bounded actions supported by a rigorous stability analysis including the robot dynamics in the closed–loop using Lyapunov's direct method and the LaSalle's invariance principle. The controllers performance is illustrated by experimental results on a two degrees of freedom direct drive robot manipulator.

This paper is organized as follows: Section 2 presents a brief exposition of the robot dynamics and its useful properties. In Section 3, the new controllers are presented. The stability analysis is also presented in Section 3. Section 4 summarizes the main components of the experimental set up. The experimental results on a direct drive arm are in section 5. Finally, a conclusion is offer in Section 6.

2 Robot Dynamics

The dynamics of a serial n-link rigid robot can be written as [6, 7]:

$$\tau = M(\mathbf{q})\ddot{\mathbf{q}} + C(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathcal{F}(\tau,\dot{\mathbf{q}}) + g(\mathbf{q}) \qquad (1)$$

where $\tau \in \mathbb{R}^n$ is the vector of gravitational torques, $\dot{\mathbf{q}} \in \mathbb{R}^n$ is a vector that represent to articular velocity, $\ddot{\mathbf{q}} \in \mathbb{R}^n$ is a vector that represent to articular acceleration, $q_d \in \mathbb{R}^n$ is a vector that represent to desired positions, $\tilde{\mathbf{q}} = [\mathbf{q_d} - \mathbf{q}] \in \mathbb{R}^n$ is the position errors vector, $M(\mathbf{q}) \in \mathbb{R}^{n\times n}$ is the inercial matrix, $C(\mathbf{q},\dot{\mathbf{q}}) \in \mathbb{R}^{n\times n}$ is the matrix of centripetal and Coriolis torques, $F(\tau,\dot{\mathbf{q}}) \in \mathbb{R}^n$ is the vector that represent the friction phenomenon, and $g(\mathbf{q}) \in \mathbb{R}^n$ is the vector of gravitational torques.

Is assumed that the robot links are joined together with revolute joints. Although the equation of motion (1) is complex. It has several fundamental properties which can be exploited to facilitate the design of control systems.

Property 1 [1, 7, 8] The matrix $C(\mathbf{q},\dot{\mathbf{q}})$ and the time derivative M(q) of the inertial matrix satisfy:

$$\dot{\mathbf{q}}\left[\frac{1}{2}\dot{M}(\mathbf{q}) - C(\mathbf{q},\dot{\mathbf{q}})\right]\dot{\mathbf{q}} = 0 \;\; \forall\, \mathbf{q},\dot{\mathbf{q}} \in \mathbb{R}^n. \quad (2)$$

3 Problem Formulation of Position Control

In this section are presented three new controllers for robot manipulators to solve the position control problem.

The control problem can be stated by designing a control law such that, the position error $\tilde{\mathbf{q}}(t)$ vanishes asymptotically to zero and keeping in mind the applied torques constrained by the prescribed limits on actuators of the robot.

To solve the control problem, we propose the following proposition.

Considering the robot dynamics together with the following controllers, then the closed–loop system is globally asymptotically stable and the positioning aim is achieved.

$$\tau = K_p \begin{bmatrix} \frac{\sinh(\tilde{q}_1)}{1+\cosh(\tilde{q}_1)} \\ \frac{\sinh(\tilde{q}_2)}{1+\cosh(\tilde{q}_2)} \\ \vdots \\ \frac{\sinh(\tilde{q}_n)}{1+\cosh(\tilde{q}_n)} \end{bmatrix} - K_v \begin{bmatrix} \frac{\sinh(\dot{q}_1)}{1+\cosh(\dot{q}_1)} \\ \frac{\sinh(\dot{q}_2)}{1+\cosh(\dot{q}_2)} \\ \vdots \\ \frac{\sinh(\dot{q}_n)}{1+\cosh(\dot{q}_n)} \end{bmatrix} + F(\tau,\dot{\mathbf{q}}) + g(\mathbf{q}) \qquad (3)$$

$$\tau = K_p diag \left\{ \begin{array}{c} 1-\alpha\cosh(\tilde{q}_1)e^{-\alpha\cosh(\tilde{q}_1)}, \\ 1-\alpha\cosh(\tilde{q}_2)e^{-\alpha\cosh(\tilde{q}_2)}, \\ \vdots \\ 1-\alpha\cosh(\tilde{q}_n)e^{-\alpha\cosh(\tilde{q}_n)}. \end{array} \right\} \begin{bmatrix} \tanh(\tilde{q}_1) \\ \tanh(\tilde{q}_2) \\ \vdots \\ \tanh(\tilde{q}_n) \end{bmatrix} - K_v \begin{bmatrix} \tanh(\dot{q}_1) \\ \tanh(\dot{q}_2) \\ \vdots \\ \tanh(\dot{q}_n) \end{bmatrix} + F(\tau,\dot{\mathbf{q}}) + g(\mathbf{q}) \qquad (4)$$

$$\tau = K_P \begin{bmatrix} \sin(sat(\tilde{q}_1)) \\ \sin(sat(\tilde{q}_2)) \\ \vdots \\ \sin(sat(\tilde{q}_n)) \end{bmatrix} - K_V \begin{bmatrix} \sin(sat(\dot{q}_1)) \\ \sin(sat(\dot{q}_2)) \\ \vdots \\ \sin(sat(\dot{q}_n)) \end{bmatrix} + F(\tau,\dot{\mathbf{q}}) + g(\mathbf{q}) \qquad (5)$$

where $K_p \in \mathbb{R}^{n\times n}$ is a diagonal matrix known as proportional gain, $K_v \in \mathbb{R}^{n\times n}$ is a diagonal matrix known as derivative gain, $\dot{\mathbf{q}} \in \mathbb{R}^n$ is a vector that represent to articular velocity, $\mathbf{q_d} \in \mathbb{R}^n$ is a vector that represent to desired positions, $\tilde{\mathbf{q}} = [\mathbf{q_d} - \mathbf{q}] \in \mathbb{R}^n$ is the position errors vector, $F(\tau,\dot{\mathbf{q}}) \in \mathbb{R}^n$ is the vector that represent the friction phenomenon, $g(\mathbf{q}) \in \mathbb{R}^n$ is the vector of gravitational torques. The *tanh* was proposed by [9]. The model of controller in where is used the Sat(x) function was proposed by [10].

Next, we present the stability proof for each proposed controller.

3.1 Design of the Position Control 1

Using the position control proposed in the equation (3) and with the equation (1) of the dynamic model of robot manipulator, it is formed the closed–loop equation:

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix}\tilde{\mathbf{q}}\\ \dot{\mathbf{q}}\end{bmatrix}=\begin{bmatrix}-\dot{\mathbf{q}}\\ M(\mathbf{q})^{-1}\left[K_p\begin{bmatrix}\frac{\sinh(\tilde{q}_1)}{1+\cosh(\tilde{q}_1)}\\ \frac{\sinh(\tilde{q}_2)}{1+\cosh(\tilde{q}_2)}\\ \vdots\\ \frac{\sinh(\tilde{q}_n)}{1+\cosh(\tilde{q}_n)}\end{bmatrix}-...\right.\\ \left....-K_v\begin{bmatrix}\frac{\sinh(\dot{q}_1)}{1+\cosh(\dot{q}_1)}\\ \frac{\sinh(\dot{q}_2)}{1+\cosh(\dot{q}_2)}\\ \vdots\\ \frac{\sinh(\dot{q}_n)}{1+\cosh(\dot{q}_n)}\end{bmatrix}-C(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}\right]\end{bmatrix}. \tag{6}$$

The closed–loop equation is an autonomous differential equation and from (6) can be observed that the first component must fulfill $-\dot{\mathbf{q}}=-I\dot{\mathbf{q}}=0 \iff \dot{\mathbf{q}}=0$. And the second component: $M(\mathbf{q})>0 \Longrightarrow \exists M^{-1}(\mathbf{q})>0$ furthermore $C(\mathbf{q},0)=0\in\mathbb{R}^{nxn}$ also $\begin{bmatrix}\frac{\sinh(\dot{q}_1)}{1+\cosh(\dot{q}_1)}\\ \frac{\sinh(\dot{q}_2)}{1+\cosh(\dot{q}_2)}\\ \vdots\\ \frac{\sinh(\dot{q}_n)}{1+\cosh(\dot{q}_n)}\end{bmatrix}=0$; therefore, since K_p is a diagonal matrix $\Longrightarrow \begin{bmatrix}\frac{\sinh(\tilde{q}_1)}{1+\cosh(\tilde{q}_1)}\\ \frac{\sinh(\tilde{q}_2)}{1+\cosh(\tilde{q}_2)}\\ \vdots\\ \frac{\sinh(\tilde{q}_n)}{1+\cosh(\tilde{q}_n)}\end{bmatrix}=0 \iff \tilde{\mathbf{q}}=0\in\mathbb{R}^n$. Then it is had $\{\dot{\mathbf{q}},\tilde{\mathbf{q}}\}=\{0,0\}$ is the unique point of equilibrium in this system.

To proof stability in the equilibrium point in the Lyapunov sense, the following candidate's function is proposed:

$$V(\tilde{\mathbf{q}},\dot{\mathbf{q}})=\tfrac{1}{2}\dot{\mathbf{q}}^T M(\mathbf{q})\dot{\mathbf{q}}+ \begin{bmatrix}\sqrt{\frac{\ln(1+\cosh(\tilde{q}_1))}{2}}\\ \sqrt{\frac{\ln(1+\cosh(\tilde{q}_2))}{2}}\\ \vdots\\ \sqrt{\frac{\ln(1+\cosh(\tilde{q}_n))}{2}}\end{bmatrix} K_p \begin{bmatrix}\sqrt{\frac{\ln(1+\cosh(\tilde{q}_1))}{2}}\\ \sqrt{\frac{\ln(1+\cosh(\tilde{q}_2))}{2}}\\ \vdots\\ \sqrt{\frac{\ln(1+\cosh(\tilde{q}_n))}{2}}\end{bmatrix}. \tag{7}$$

It is possible to be observed of (7) that $V(\tilde{\mathbf{q}},\dot{\mathbf{q}})$ is a defined function positive. From where the first term of (7) is a defined function positive with respect to $\dot{\mathbf{q}}$ because $M(\mathbf{q})$ is a diagonal matrix, and the second part of (7) is a defined function positive with respect to the position error since K_p by definition is a diagonal matrix.

Deriving with respect to the time, substituting $\ddot{\mathbf{q}}$ from the expression (6) and using property 1, it is obtained:

$$\dot{V}(\tilde{\mathbf{q}},\dot{\mathbf{q}})=-\dot{\mathbf{q}}^T K_v\begin{bmatrix}\frac{\sinh(\dot{q}_1)}{1+\cosh(\dot{q}_1)}\\ \frac{\sinh(\dot{q}_2)}{1+\cosh(\dot{q}_2)}\\ \vdots\\ \frac{\sinh(\dot{q}_n)}{1+\cosh(\dot{q}_n)}\end{bmatrix}\leq 0. \tag{8}$$

Therefore, one concludes that the system is stable in the sense of Lyapunov. To ensure asymptotic stability, LaSalle's theorem[4] is applied:

$$\Omega=\{\begin{bmatrix}\tilde{\mathbf{q}}\\ \dot{\mathbf{q}}\end{bmatrix}\in\mathbb{R}^{2n}/\dot{V}(\tilde{\mathbf{q}},\dot{\mathbf{q}})\equiv 0 \iff \dot{\mathbf{q}}=0\wedge\tilde{\mathbf{q}}=0\in\mathbb{R}^n\}.$$

Therefore the equilibrium point ($\dot{\mathbf{q}}=0,\tilde{\mathbf{q}}=0$) will convergence globally asymptotic to Ω as $t\longrightarrow\infty$.

3.2 Design of the Position Control 2

Consider the dynamic equation of a robot (1), which in combination with controller 2 (4) can be written in the following form:

$$\frac{d}{dt}\begin{bmatrix}\tilde{\mathbf{q}}\\ \dot{\mathbf{q}}\end{bmatrix}= \begin{bmatrix}-\dot{\mathbf{q}}\\ M^{-1}(\mathbf{q})\left[K_p diag\begin{Bmatrix}1-\alpha\cosh(\tilde{q}_1)e^{-\alpha\cosh(\tilde{q}_1)},\\ 1-\alpha\cosh(\tilde{q}_2)e^{-\alpha\cosh(\tilde{q}_2)},\\ \vdots\\ 1-\alpha\cosh(\tilde{q}_n)e^{-\alpha\cosh(\tilde{q}_n)}.\end{Bmatrix}\begin{bmatrix}\tanh(\tilde{q}_1)\\ \tanh(\tilde{q}_2)\\ \vdots\\ \tanh(\tilde{q}_n)\end{bmatrix}\right.\\ \left.-K_v\begin{bmatrix}\tanh(\dot{q}_1)\\ \tanh(\dot{q}_2)\\ \vdots\\ \tanh(\dot{q}_n)\end{bmatrix}+C(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}\right]\end{bmatrix} \tag{9}$$

that it is an autonomous differential equation, and in space of states the origin is the only equilibrium point. For the analysis of stability of the equation (9), it is proposed the following function candidate of Lyapunov:

$$V(\tilde{\mathbf{q}},\dot{\mathbf{q}}) = \tfrac{1}{2}\dot{\mathbf{q}}^T M(\mathbf{q})\dot{\mathbf{q}} + \begin{bmatrix} \sqrt{\begin{matrix}\ln(\cosh(\tilde{q}_1)) \\ +e^{-\alpha\cosh(\tilde{q}_1)}-1\end{matrix}} \\ \sqrt{\begin{matrix}\ln(\cosh(\tilde{q}_2)) \\ +e^{-\alpha\cosh(\tilde{q}_2)}-1\end{matrix}} \\ \vdots \\ \sqrt{\begin{matrix}\ln(\cosh(\tilde{q}_n)) \\ +e^{-\alpha\cosh(\tilde{q}_n)}-1\end{matrix}} \end{bmatrix}^T K_P \begin{bmatrix} \sqrt{\begin{matrix}\ln(\cosh(\tilde{q}_1)) \\ +e^{-\alpha\cosh(\tilde{q}_1)}-1\end{matrix}} \\ \sqrt{\begin{matrix}\ln(\cosh(\tilde{q}_2)) \\ +e^{-\alpha\cosh(\tilde{q}_2)}-1\end{matrix}} \\ \vdots \\ \sqrt{\begin{matrix}\ln(\cosh(\tilde{q}_n)) \\ +e^{-\alpha\cosh(\tilde{q}_n)}-1\end{matrix}} \end{bmatrix} \tag{10}$$

where the first term of (10) is a defined function positive with respect to $\dot{\mathbf{q}}$ because $M(\mathbf{q})$ is a diagonal matrix, and the second part of (10) is a defined function positive with respect to the position error since K_p by definition is a diagonal matrix.

Derived temporary from the candidate function of Lyapunov throughout the trajectories of the equation in closed–loop and after simplifying algebraically using property 1 this can be written as:

$$\dot{V}(\tilde{\mathbf{q}},\dot{\mathbf{q}}) = -\dot{\mathbf{q}}^T K_v \begin{bmatrix} \tanh(\dot{q}_1) \\ \tanh(\dot{q}_2) \\ \vdots \\ \tanh(\dot{q}_n) \end{bmatrix} \leq 0 \tag{11}$$

which is a semidefined function negative globally with which it concludes that the equilibrium point is stable. In addition, the test of asymptotic stability is possible taking into account the autonomous nature from the equation in closed–loop and applying the LaSalle's theorem.

3.3 Design of the Position Control 3

Considering the following equation in loop-closed formed by the controller (5) and the dynamics robot:

$$\frac{d}{dt}\begin{bmatrix} \tilde{\mathbf{q}} \\ \dot{\mathbf{q}} \end{bmatrix} = \begin{bmatrix} -\dot{\mathbf{q}} \\ M^{-1}(\mathbf{q}) \begin{bmatrix} K_P \sin(Sat(\tilde{\mathbf{q}})) - K_V \sin(Sat(\dot{\mathbf{q}})) \\ -C(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} \end{bmatrix} \end{bmatrix} \tag{12}$$

where the equation (12) is an autonomous differential equation and the origin of the space of states is the only equilibrium point. The stability analysis is made by the direct method of Lyapunov, sets out the following function candidate of Lyapunov:

$$V(\tilde{\mathbf{q}},\dot{\mathbf{q}}) = K(\tilde{\mathbf{q}},\dot{\mathbf{q}}) + U_a(\tilde{\mathbf{q}}) \tag{13}$$

$$K(\tilde{\mathbf{q}},\dot{\mathbf{q}}) = \frac{1}{2}\dot{\mathbf{q}}^T M(\mathbf{q})\dot{\mathbf{q}}$$

$$U_a(\tilde{\mathbf{q}}) = \sum_{i=1}^{n} U_i(\tilde{q}_i)$$

$$U_i(\tilde{q}_i) = \left\{ \begin{array}{ll} K_{Pi}\cdot(1-\cos(\tilde{q}_i)) & if \quad |\tilde{q}_i| < \pi/2. \\ K_{Pi}\cdot|\tilde{q}_i| & if \quad |\tilde{q}_i| \geq \pi/2. \end{array} \right\}$$

From where the firths term of (13) is a defined function positive with respect to $\dot{\mathbf{q}}$ because $M(\mathbf{q})$ is a diagonal matrix, and the second part of (13) is a defined function positive with respect to the position error since K_p by definition is a diagonal matrix.

Derived temporary from the candidate function of Lyapunov throughout the trajectories of the equation in closed–loop and after simplifying algebraically using property 1, this can be written like:

$$\dot{V}(\tilde{q},\dot{q}) = -\dot{\mathbf{q}}^T K_V \sin(Sat(\dot{\mathbf{q}})) \leq 0, \tag{14}$$

which is a semidefined function negative globally with which it concludes that the equilibrium point is stable. In addition, the test of asymptotic stability is possible taking into account the autonomous nature from the equation in loop back and applying the LaSalle's theorem.

4 Experimental Set-Up

The experimental results were obtained on a two degrees of freedom direct-drive robot arm shown in Figure 1, whose characteristics are:

Figure 1: SPAC system 1.

- Robot Manipulator of 2 DOF[11] of the anthropomorphic type with a space of work of 90cm. shown in the Figure 2.
- Each joint formed by a motor of direct transmission [1] with the characteristics shown in the Table 1, where also is the encoders resolution [2].

Link	Model	Torque	Resolution
Shoulder	DM1150A	150Nm	1,024,000 p/rev
Elbow	DM1015B	15Nm	655,360 p/rev

Table 1: Characteristics of the servo-motors of the robot manipulator.

- It is used for the control of the robot manipulator, a computer Pentium II with compiler of language C.
- Data acquisition board model Mfio-3A from Precision Microdynamics.

[1]Motors of direct transmission without brushes, the Dynaserv series of Parker Compumotor; that they include his power drivers and encoders.
[2]The resolution is given in Pulses by Revolution.

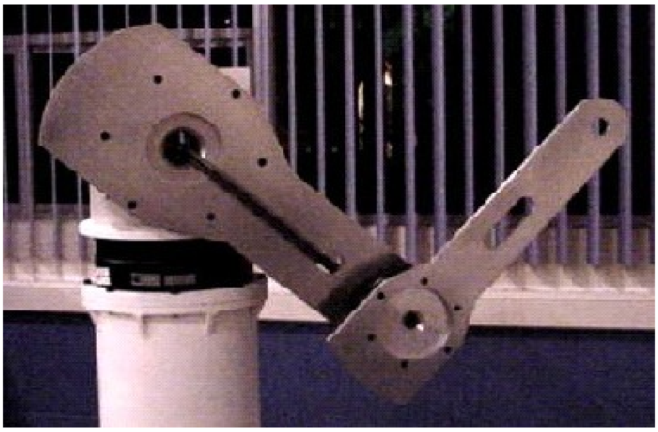

Figure 2: Robot Manipulator of 2 DOF used in the experiments.

5 Experimental Results

The experimental evaluation of the controller must support the theoretical developments. Therefore, an extensive set of experiments were carried out between the position's controllers proposed. During the experimental test, no friction compensation was modeled on the controller.

The desired positions utilized in each case are showed in the Table 2.

Link	Desired Position
Shoulder	45 degrees
Elbow	90 degrees

Table 2: Desired Position.

For the position control 1 was used the gains showed in the Table 3.

Parameter	Value
Kp_1	270.0 Nm
Kv_1	73.0 Nm
Kp_2	22.0 Nm
Kv_2	4.4693 Nm

Table 3: Used gains with the control 1.

The experimental results for the control 1 are showed in the Figures 3-4. The Figure 3 depicts the error position and in the Figure 4 are presented the applied torques.

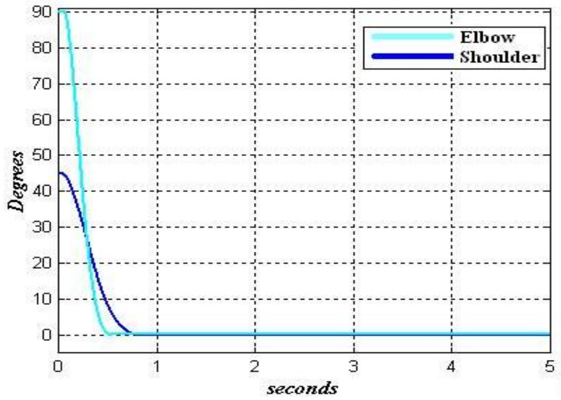

Figure 3: Joint errors of control 1.

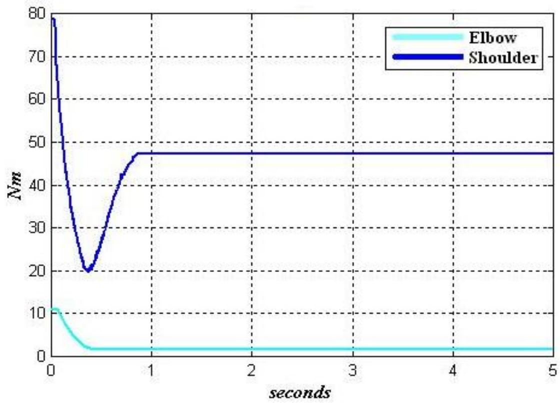

Figure 4: Applied torques of control 1.

For the position control 2 was used the gains showed in the Table 4.

Parameter	Value
Kp_1	70.0 Nm
Kv_1	70.0 Nm
Kp_2	10.0 Nm
Kv_2	7.0 Nm

Table 4: Used gains with the control 2.

The experimental results for the control 2 are showed in the Figures 5-6. The Figure 5 depicts the error position and in the Figure 6 are presented the applied torques.

For the position control 3 was used the gains showed in the Table 5.

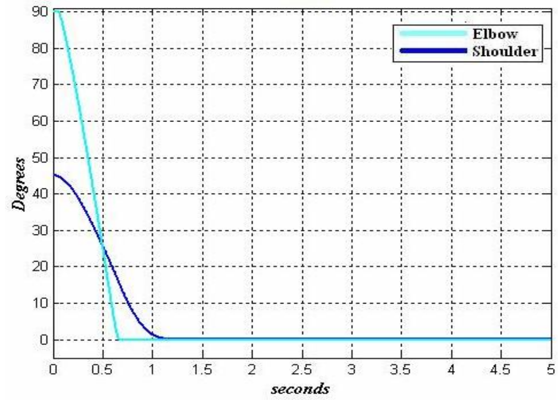

Figure 5: Joint errors of control 2.

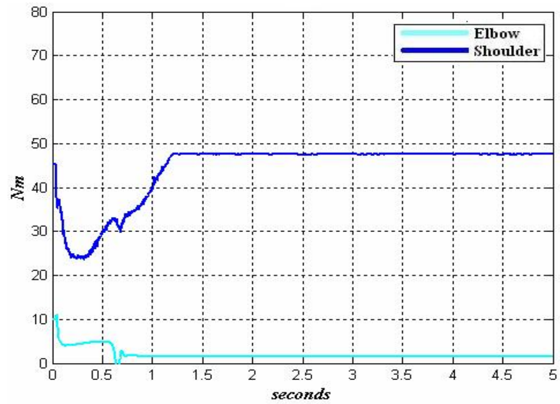

Figure 6: Applied torques of control 2.

The experimental results for the control 3 are showed in the Figures 7-8. The Figure 7 shows the error position and in the Figure 8 are presented the applied torques.

5.1 Performance Indicators

The performance evaluation is solved implementing the scalar value $\mathcal{L}_2|\tilde{\mathbf{q}}|$ marks the compromise between velocity and precision of the movements performed by the robot. The $\mathcal{L}_2|\tilde{\mathbf{q}}|$ norm measures the root-mean-square (RMS) of the position error and it's given by the following equation[4, 12, 13, 14, 15]:

$$\mathcal{L}_2 = \sqrt{\frac{1}{t-t_0}\int_{t_0}^{t} \tilde{\mathbf{q}}^T \tilde{\mathbf{q}} dt}$$

Parameter	Value
Kp_1	150.0 Nm
Kv_1	67.5 Nm
Kp_2	11.0 Nm
Kv_2	6.9 Nm

Table 5: Used gains with the control 3.

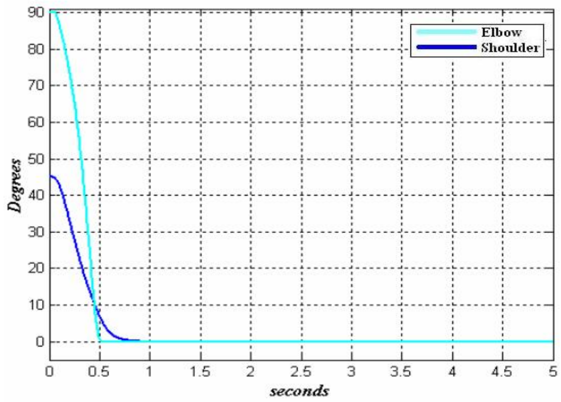

Figure 7: Joint errors of control 3.

where $t_0, t \in \mathbb{R}^+$ are the initial and final time, respectively. A smaller $\mathcal{L}_2|\tilde{\mathbf{q}}|$ represents smaller position error, a fast transient state and a better performance of the evaluated controllers. The comparison graph of the controllers is shown in the Figure 9 and the Figure 10 where it is compared the position controls proposed with the PD control.

As result, the position controls proposed has a better $\mathcal{L}_2$ norm which means a better performance

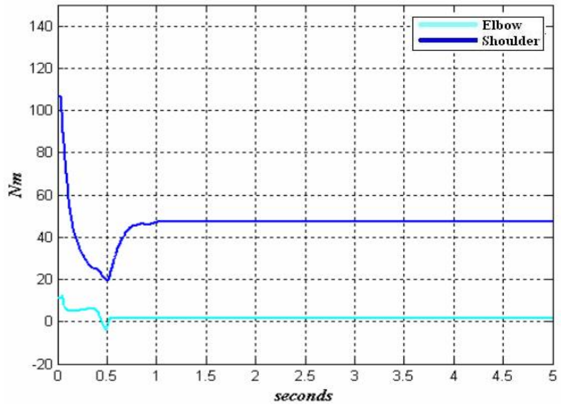

Figure 8: Applied torques of control 3.

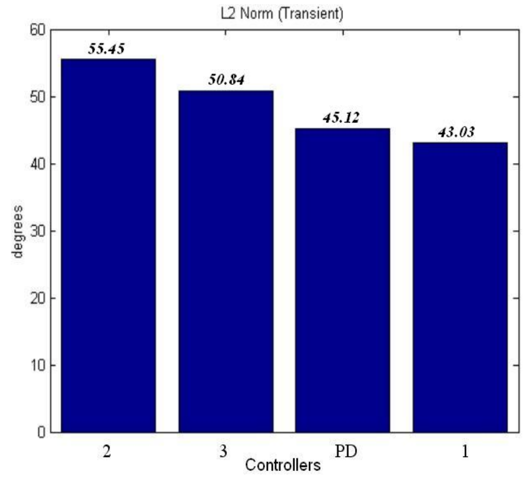

Figure 9: Performance indicators for transient state.

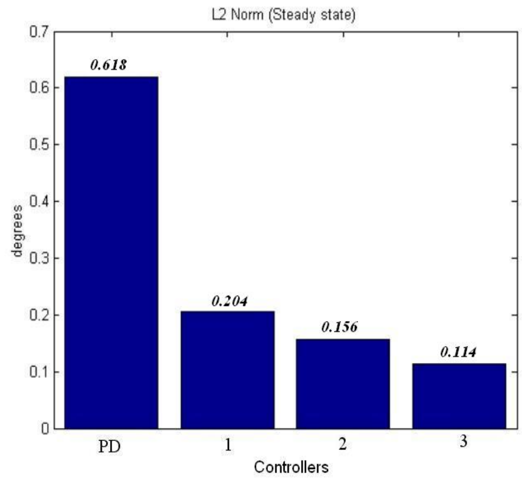

Figure 10: Performance indicators for steady state.

that the PD control; considering that final value $\mathcal{L}_2$ norm is in stationary time for 10 and in transitory time for 9.

6 Conclusions

In this paper has been presented three new position controllers for robot manipulators, with a rigorous stability analysis. Likewise, it is observed that the performance of the controls proposed are better that the PD control. In the experiment, it is possible to be seen that the desired positions are reached quickly and without overshoot.

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