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Application of Safety Index Approach to Reliability Study of External Tank Attach Ring of Space Shuttle

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Abstract

The main aim of this study is to calculate the probabilities of ETA ring falling below the specified values with fracture toughness and ultimate tensile strength as limit states. This is done using the statistical properties of fracture toughness and ultimate strength of the actual ETA ring specimen. The measurements are taken at various points of the ETA ring. The method used for calculation of the probabilities is the FOSM (First Order Second Moment) method. This method is well documented in literature and gives a reasonably good starting probability values. These estimates can be improved by either advanced reliability methods (AFOSM) or using simulation techniques such as Monte Carlo simulation method.

Key Words: Reliability, Tensile Strength, Ring, External Tank, Probability, Fracture, Toughness.

1 Introduction

Lot of data have been collected by USA for fracture toughness (Kc), Rockwell hardness (HRC) and ultimate tensile strength (UTS). The aim of this study is to calculate the probabilities of ETA ring for various limits such as fracture toughness and ultimate tensile strength (UTS). In the case of fracture toughness limit state the acceptable probability is the probability of fracture toughness being above 62.0. Conversely, the probability of fracture toughness value falling below 62.0 can then be calculated. Similarly, in the case of Rockwell hardness limit state the acceptable probability is the probability of Rockwell hardness being above 34.0. Lastly, in the case of ultimate tensile strength limit state, the acceptable probability is the value of ultimate strength being above 174.0. If the specimen has to satisfy all the three limit states then the joint probability can easily be calculated using the laws of probability and statistics. This will depend on whether these failures are considered as series failures or parallel failures. In other words, it has to be decided a priori whether the ETA ring is considered structurally safe if all the probabilities for the limit states of fracture toughness, Rockwell hardness and ultimate strength are above the acceptable ranges or the system is considered safe if one them is in above the acceptable range, for example fracture toughness. In the case of the latter, it will be a parallel system whereas in the case of the former, it will be a series system. There is abundant information in the literature to calculate the reliabilities of the system both for series and parallel system [1]. The basic probabilities themselves of an structural element for any limit state can be calculated using the FOSM (First Order Second Moment Method) which is again well documented in the literature dealing with Reliability. In FOSM, the safety index (β) is used as a measure of safety. This is discussed further in subsequent sections of this paper. A lot of research work has been done in the past several years [see 1, 2, 3, 4, 5, 6, 7, 8, 9, 10].

2 Problem Formulation and Solution

As a first step, the reliability is calculated for a given limit state using the FOSM method. This method is a first order method and uses the first and second moment of the design variable. This procedure will be explained with respect to fracture toughness limit state. The procedure is as follows:

1. Collect all the experimental data for fracture toughness.
2. Calculate the statistical properties (sample mean $\bar{x}$ (first moment) and sample standard deviation (s_x) of the sample. Standard deviation is the square root of the variance (second moment) of the variable. First and Second moment of fracture toughness would be sufficient for FOSM method.
3. Set a minimum acceptable (limiting value) of fracture toughness (x_l) below which, if the fracture toughness falls, it is not acceptable. Alternatively, this is the threshold value and the fracture toughness has to be above this value for the safety of the structure.
4. Calculate $P(x <= x_l)$ or $P(x > x_l)$ for the assumed distribution.
5. For a **normal distribution** this expression is given as:

$$P(x >= x_l) = 1 - \Phi\ [(x_l - \mu_x)/\sigma_x] \qquad (13)$$

where,

μ_x= theoretical mean value of design variable under consideration (fracture toughness in this case)

σ_x = theoretical standard deviation of the design variable. (fracture toughness in this case).

For a Normal Distribution:

$$\mu_x = \bar{x} \qquad (14)$$

$$\sigma_x = s_x \qquad (15)$$

The values of the function Φ can easily be obtained from standard normal distribution tables or by error function calculation using a computer.

For a Uniform Distribution:

$$P(x <= x_l) = (x_l - a) * f_d \qquad (16)$$

f_d represents the density function value corresponding to x_l. a and b are the parameters of the uniform distribution. f_d is given as:

$$f_d = 1/(b - a) \qquad (17)$$

a and b can be solved in terms of the sample mean and standard deviation using the following equations:

$$\bar{x} = (a + b)/2 \qquad (18)$$

$$\sigma_x^2 = (b - a)^2/12 \qquad (19)$$

σ_x^2 can be taken as square of the sample deviation s_x. Solving these two will give a and b, the parameters of the uniform distribution.

For a Log Normal Distribution: the parameters of the distribution are λ and ξ. These can be obtained solving the following two equations.

$$\bar{x} = \exp\left(\lambda + 0.5 * \xi^2\right) \qquad (20)$$

$$\mathrm{Var}(x) = E^2(X)\,[e^{\xi} - 1] \qquad (21)$$

As discussed, these two equations can be solved to find λ and ξ, the parameters of log normal distribution. The details are shown in [10].

Data Analysis has also been performed as part of this research work [10]. The results for reliability analysis are compared for various methods in the detailed report [10]. The results checked very well.

3 Conclusions

It is seen from the results that the probability of exceedance of the limiting value of fracture toughness as well as the ultimate strength (UTS) is quite high for all the three distributions- normal, lognormal and uniform. It is possible that the refined methods such as the AFOSM (advanced First Order Second Moment method) or Monte Carlo simulation might give better results.

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