Proceedings of the 5th WSEAS Int. Conf. on SIMULATION, MODELING AND OPTIMIZATION, Corfu, Greece, August 17-19, 2005 (pp140-145)

Adaptive Control: The Issue of Short Sampling Period

PETR PIVOŇKA, KAMIL ŠVANCARA
Department of Control and Instrumentation,
Brno University of Technology, Faculty of Electrical Engineering and Communication
Kolejní 4, 612 00 Brno
CZECH REPUBLIC
http://www.uamt.feec.vutbr.cz/rizeni/index.html.en

Abstract: - The use of short sampling period in adaptive control has not been described properly when controlling the real process by adaptive controller. On one hand faster disturbance rejection due to short sampling period can be an advantage but on the other hand it brings us some practical problems. Particularly, quantization error and finite numerical precision of industrial controller must be considered in the real process control. Presented paper shows a comparison of two real-time identification methods with improved adaptive linear optimal controller.

Key-Words: - Adaptive control, A/D converters, Delta models, Identification algorithms, Linear quadratic regulators, Neural networks, Quantization errors, Recursive algorithm, Sampling period

1 Introduction

Presented paper is motivated by problems which arise from differences between "pure" simulation results and real results after implementation. The same control algorithm is implemented both into simulation environment (MATLAB/Simulink) and into industrial controller (PLC B&R). The reduction of negative influence of quantization effect and objective reasons for short sampling period have lead to the aims of the paper.

Firstly, the quantization effect is explained. The general view of identification methods which may be used in adaptive controller is presented afterwards. Next, improved adaptive linear optimal controller is briefly shown. Lastly, two chosen identification methods are compared.

2 Quantization Effect

The quantization effect is more known for example in instrumentation theory or signal processing theory than in control theory. Furthermore, in control theory the phenomenon has been usually disregarded. It is due to the fact that the conditions used in process control allow the quantization effect to be ignored. Nowadays, when the sampling period is demanded to be very short and the requirements for the control precision are higher then before, the quantization effect plays considerable role in the practical control.

2.1 Quantization Error

The process control of continuous time system and the control of sampled continuous time system are two different fields. It happens that the controller design is created without precise knowledge of sampling, shaping and quantization effect.

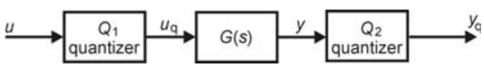

Fig.1 The real model with A/D and D/A converters represented as quantizer.

The A/D and D/A converters are necessary parts of each real-time system [4]. The basic feature of the converters is to convert continues signal to discrete values and back (see Fig.1).

$$y = G(s)u_q \quad y_q = Q_2(y) \quad u_q = Q_1(u)$$
$$y_q = Q_2\{G(s)Q_1(u)\} \qquad (1)$$

The quantization error e is limited to quantization band ≡ 1 LSB. The quantization range Q_{RANGE} and the quantization resolution Q_{RES} are basic parameters for definition of the quantization band. For example $2^{Q_{\mathrm{RES}}} = 2^8 = 256$ number of codes is given for Q_{RES}. Next, for bipolar converters ±10 V the quantization band is $Q_{\mathrm{BAND}} = 10/256 =$ $= 39.1$ mV ≈ 0.04 V. Therefore the value in finite word-length precision is numerically rounding off to the three valid places divisible by ≈ 0.04 V.

The quantization error may be modelled as deterministic or stochastic signal in linear analysis. In deterministic model, the error is modelled as constant having the size of quantization errors and with the resolution in the arithmetic calculation. In the stochastic model, the error introduced by rounding or quantization is then described as additive white noise with rectangular distribution [1]. Next paper [12] deals with quantization analysis and shows cases where after linearization the round off quantization error is uncorrelated with quantizer input.

Simple results where previous mentioned conclusions are not applicable [9] are presented in this section. Let us consider the modelling of quantizer. The model can be built from quantization effect description to show the disturbance properties of quantization effect. The model can be seen in Fig.2 where the linear part of value u_L is disturbed by non-linear part represented as quantization error e. This point of view is very simple, given from description of quantization effect and it gives us the beginning point for explanation of quantization effect.

It can be written that

$$u_q = u_L + e \quad u_q = f(u) \tag{2}$$

where $f(\cdot)$ is exact non-linear function. The idea to derive presented equation explains answer to the question how the quantization error arises. It is shown that quantization error is dependent on quantizer input signal.

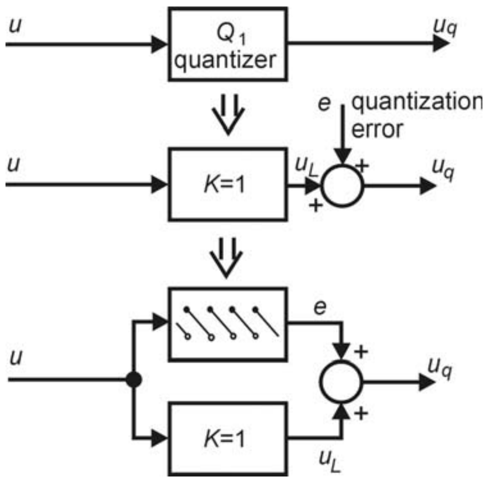

Fig.2 Principal model of quantization effect.

This dependence is negligible as long as the sampling period is not too short and the numerical precision of quantizer error added to output is insignificant. In our case where the process control needs short sampling period, it is clearly shown that quantization error e is not independent from quantizer input u and hence cannot be treated as the independent additive noise [9]. Next, the quantization error cannot be treated as the Gaussian or even white noise because it is directly derived from quantizer input. It means that the noise is deterministic and it can be predicted. For example the quantization error is bigger when the amplitude of quantizer input is smaller.

2.2 Amplitude Shape

The amplitude shape of transformation from continuous time into discrete time could be described mathematically [1]

$$u^*(t) = u(t)m(t) = u(t)\sum_{k=-\infty}^{\infty}\delta(t - kT_S) \tag{3}$$

where $m(t)$ is modulation function of Dirac impulses $\delta(\cdot)$. Sampler is usually followed by shape filter, very often represented by Zero-Order-Hold (ZOH) filter. The sampler could be written in Fourier series (4).

$$m(t) = \frac{1}{T_S}\left(1 + 2\sum_{k=1}^{\infty}\cos(k\omega_S t)\right) \tag{4}$$

The amplitude and phase changes due to ZOH filter are important fact which can be easily forgotten. For tested process with transfer function

$$G(s) = \frac{1}{(10s+1)(s+1)^2} \tag{5}$$

the Bode diagram (see Fig.3) is solved before and after conversion to discrete domain. The final transfer function after conversion from continuous time domain is

$$G_0(j\omega) = \frac{1}{T_S}\frac{1 - e^{-j\omega T_s}}{j\omega(10j\omega+1)(j\omega+1)^2} \tag{6}$$

Fig.3 shows the Bode diagram of ZOH filter and sampled system if the sampling period has been set to $T_S = 0.1$ s. The vertical line represents the Nyquist frequency $\omega_N = \frac{2\pi}{2T_S}$=31.42 rad/s.

The change of phase is -90° at this frequency and the change of the amplitude is almost -4 dB. These

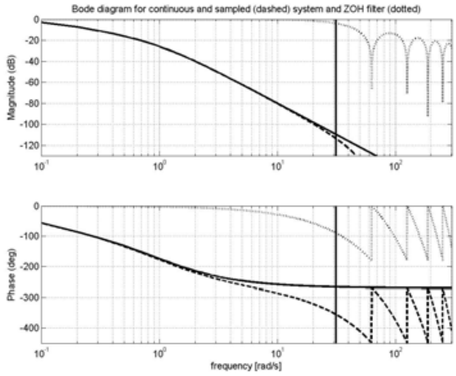

Fig.3 Bode diagram of ZOH filter and continuous and sampled system G(s), $T_S = 0.1$ s.

changes can be explained by the fact that the identified system is different from the real system.

Let us see what happen if the sampling period has been set ten times longer, i.e. $T_S = 1$ s. The results are related to the problem of the choice of the quantization precision for the set of short sampling period. According to theoretical solution of Signal to Noise Ratio (SNR) [4] for A/D converters, it is interesting to compare the SNR for the chosen quantizer resolution with the drop of the amplitude [9]. In our example, the change of sampling period from $T_S = 1$ s to $T_S = 0.1$ s is expressed in amplitude drop -59 dB in Nyquist frequency. Therefore, the precision of A/D converters should follow the amplitude drop to get the same undisturbed results. In [4], it is written that -62 dB is the theoretical SNR value for resolution $Q_{RES} = 1$ bits. For example if 8 bits A/D converter for the sampling period $T_S = 1$ s has been chosen as the minimum appropriate resolution, than after the reduction to $T_S = 0.1$ s, the A/D resolution should be increased too.

3. Unconventional Overview of Identification Methods

Linear and even nonlinear black-box identification can be divided into three elements:

- model structure of identified process,
- item regression vector of observed data,
- and algorithm for minimization.

3.1 Model Structure

The model structure should be chosen according to the observed system. From linear point of view, the structures of model are called: ARX, ARMAX (Auto-Regressive Moving Average model with eXogenous input), OE (Output Error model) etc. All of them are built from generally known formula [7]

$$A(q)y(k) = \frac{B(q)}{F(q)}u(k) + \frac{C(q)}{D(q)}e(k) \qquad (7)$$

The state-space (SS) representation is also taken as a different structure model which is powerful for its general MIMO definition. In nonlinear case, the structures are called NARX, NARMAX or nonlinear SS representation where “N” generally means nonlinear model. Behind such terms the structure as Wavelet, Neural Networks, Fuzzy models etc. are presented.

The new group of above mentioned structures represents the improved structure (the numerical precision). It is always given by new operator which comprises the linear combination of previous operators. The best example is given by [8] where the commonly used q time-shift operator and Z-transform operator z are linearly combined into new δ and γ operators in δ-model domain in way $\delta = \frac{q-1}{T_S}, \quad \gamma = \frac{z-1}{T_S}$.

3.2 Regression vector

The regression vector φ is inseparable part of the model structure but it can be treated as a new part which brings us a possibility of choice. For example the difference between ARX and OE model is just in the difference treatment of data representation. The ARX model uses past measured output data y^k while the OE model uses estimated output data $\hat{y}^k$. Generally speaking, the data can be treated with the purpose to build another model that is not named yet. For example the past output and input can be filtered as in the -model domain. Next example is given in CLOE (Closed Loop Output Error) identification method where both estimated inputs $\hat{u}^k$ and outputs $\hat{y}^k$ are used and the criterion minimizes squared error between measured $y(k+1)$ and estimated $\hat{y}(k+1)$ output in closed loop.

3.3 Algorithm

The algorithm is used to minimize the criterion. It is the last option. The generalization of every minimization algorithm is given in next iterative equation which is basically suggested in [3]

$$w(k+1) = w(k) + \eta(k)d(k+1) \qquad (8)$$

where new updated vector of parameters $w(k+1)$ in step $(k+1)$ are influenced by past parameters $w(k)$ in previous step of iteration and the direction vector of minimization $d(k+1)$ in length given by its learning rate $\eta(k)$.

LD-FIL Matrix decomposition as a robust algorithm could be used mainly for following attributes: numerically stable algorithm and easy implementation for real-time. LD-FIL (lower-diagonal-upper) decomposition algorithm [2] could be used. Standard covariance matrix P is given by

$$P(k+1) = [\Phi^{\mathrm{T}}(k)\Phi(k) + \varphi(k+1)\varphi^{\mathrm{T}}(k+1)]^{-1} = \\ = G(k+1)D(k+1)G(k+1)^{\mathrm{T}} \quad (9)$$

where G denotes lower triangular matrix, G^{T} denotes upper triangular matrix and D denotes diagonal matrix. Parameters on the main diagonal mainly influence identification. Well-known LD-FIL matrix decomposition is derived by lemma for matrix inversion

$$(A + BCD)^{-1} = A^{-1} - A^{-1}B\left(C^{-1} + DA^{-1}B\right)^{-1}DA^{-1} \quad (10)$$

and then [10]

$$G(k+1)D(k+1)G(k+1)^{\mathrm{T}} = \\ = G\left(D - Df_i f^{\mathrm{T}} D \frac{1}{1 + f^{\mathrm{T}} Df}\right)G^{\mathrm{T}} \quad (11)$$

where an auxiliary vector f is given $f(k) = G(k)^{\mathrm{T}} \varphi(k+1)$

The back-propagation algorithm can be summarized as [11]

$$w(k+1) = w(k) - \eta(k)g(w) + \alpha\big((w(k) - w(k-1)\big) \quad (12)$$

Newton's idea is that the energy function $V(w)$ being minimized is approximated locally by a quadratic function and this approximation function is minimized exactly [3]. The standard form of iterative Newton method also called as

$$w(k+1) = w(k) + \eta\left[\nabla^2 V(w(k))\right]^{-1} \nabla V(w(k)) = \\ = w(k) + \eta H^{-1}(w(k))g(w(k)) \quad (13)$$

4. Integral Action Implementation into Adaptive LQ Controller

Implementation of adaptive LQ controller has been already published in literature. Briefly, quadratic performance is defined by

$$J = x^{\mathrm{T}}(N)Qx(N) + \\ + \sum_{k=0}^{N-1}\left(q_y(w(k) - y(k))^2 + q_u(u(k) - u_0(k))^2\right) \quad (14)$$

where $w(k)$ denotes desired value, $y(k)$ denotes output of the process, $u(k)$ denotes action value, $u_0(k)$ denotes action value for offset elimination and it is equal to desired value. Parameter q_{y} (q_{u}) denotes weight for process output (input), $k = 0$ denotes the first step while the minimization is used and $x^{\mathrm{T}}(N)Qx(N)$ denotes the minimum at the last step N. LQ is solved at the each one step ahead. The quadratic performance can be rewritten into more suitable form

$$J = \sum_{k=0}^{N} z^{\mathrm{T}}(k)Qz(k) \quad (15)$$

where $z^{\mathrm{T}}(k) = S(k)[u(k), \varphi(k-1), w(k), u_0(k)]$ and $z^{\mathrm{T}}(k) = S(k)z(k-1)$. Pseudo-state matrix is $S = [S_u, S_\varphi, S_w, S_{u_0}]$.

It is well-known that integral action is not included in original definition of linear optimal controller. That is why the integral action is basically solved as parallel action to basic action of controller. The solution is given in next equation

$$u_{\mathrm{i}}(k) = e(k) + u_{\mathrm{i}}(k-1) \quad (16)$$

which can be included into the quadratic performance where an integral action is weighted by term $q_{\mathrm{i}} u_{\mathrm{i}}^2(k)$.

An example of universal weight matrix shows equation (17)

$$Q = \begin{bmatrix} q_u & -q_u & 0 & 0 & 0 & 0 & 0 & 0 & -q_u \\ -q_u & q_u & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & q_u & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & q_y & 0 & 0 & 0 & -q_y & 0 \\ 0 & 0 & 0 & 0 & q_y & 0 & 0 & -q_y & 0 \\ 0 & 0 & 0 & 0 & 0 & q_y & 0 & -q_y & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & q_{\mathrm{i}} & 0 & 0 \\ 0 & 0 & 0 & -q_y & -q_y & -q_y & 0 & q_y & 0 \\ -q_u & 0 & 0 & 0 & 0 & 0 & 0 & 0 & q_u \end{bmatrix} \quad (17)$$

In this example, the incremental weighting of the input and output is included as well. This solution leads to smoother reaction of both action value (weighted by q_u) and output error (weighted by term q_y). Finally, the mutual ratio between q_u and q_y decides according to designer demands between fast controller reaction and smooth action value.

5. δ-Model ARX Identification versus NN Identification

The results have been obtained for adaptive control scheme. The process model has been chosen according to equation (5) where the deterministic disturbance enters between process main dynamics $G_1(s) = \frac{1}{10s+1}$ and additional, faster dynamics $G_2(s) = \frac{1}{(s+1)^2}$ The q time-shift transfer function

$$G(q) = \frac{b_1 q^2 + b_2 q + b_3}{q^3 + a_1 q^2 + a_2 q + a_3} \qquad (18)$$

is rewritten for δ-model of the third order ARX structure

$$G(\delta) = \frac{\beta_1 \delta^2 + \beta_2 \delta + \beta_3}{\delta^3 + \alpha_1 \delta^2 + \alpha_2 \delta + \alpha_3} \qquad (19)$$

Transformation formulas for parameter and regression vectors are published in [9]. Fig.4 and Fig.5 shows simulated process model response and disturbance rejection together with controller action value as it is published in [11]. The process output (every upper sub-figures) and input (every lower sub-figures) are shown for desired step set to +2 V at time 20 s. Deterministic disturbance step has been set to +1 V at time 80 s. Quantizers for 10 and 12 bits have been used. Exponential weighting has been se to λ_e = 0.95. T_s is the sampling period. LQ controller has been used with incremental weighting matrix Q where parameters have been set to q_u = 0.005, q_y = 1 and q_i = 0.1.

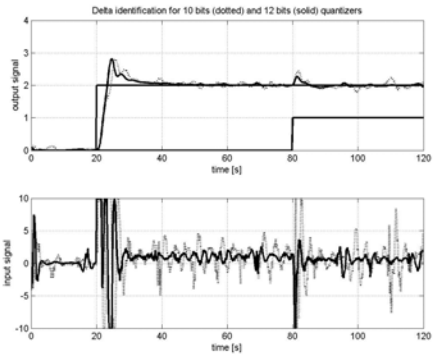

Fig.4 The simulated result of δ-model identification with adaptive LQ controller, T_s = 0.1 s, Q_{RES} = 10 bits (dotted) and 12 bits (solid).

The second possible choice of identification method (in comparison with [10] is the Recursive Least Square method (RLS) applied to δ-model of ARX structure [9]. The simulation results have already justified the idea that the gradient algorithm applied to Nonlinear ARX structure (NARX) of neural net have worked better.

This conclusion means that δ-model is built to overcome finite word-length precision of used variables in controller only [5]. The values with finite mantissa and exponent are easily stored when converging to zero (δ-model domain) than to one (q time-shift domain). The input-output round off error cannot be overcome sufficiently.

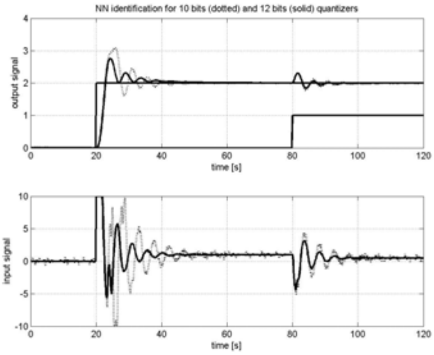

Fig.5 The simulated result of NN identification with adaptive LQ controller, T_s = 0.1 s, Q_{RES} = 10 bits (dotted) and 12 bits (solid).

The real results have been obtained for process (the third order physical model) given by $G_P(s) \approx G(s)$ from eq. (5). Fig.6 and Fig.7 show the real process response and disturbance rejection together with controller action value. The process output (every upper sub-figures) and input (every lower sub-figures) are shown for desired step set to +2 V at time 60 s. Disturbances have disturbed process all the time [9]. 10 bits quantizers have been used and the other parameters have been the same.

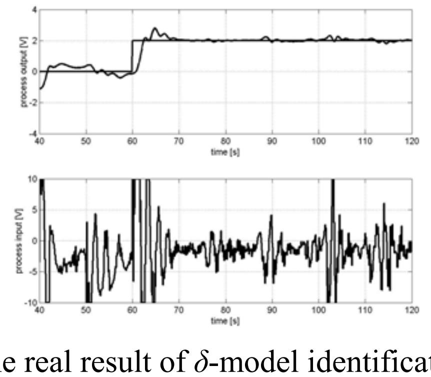

Fig.6 The real result of δ-model identification with adaptive LQ controller, T_s = 0.1 s, Q_{RES} = 10 bits.

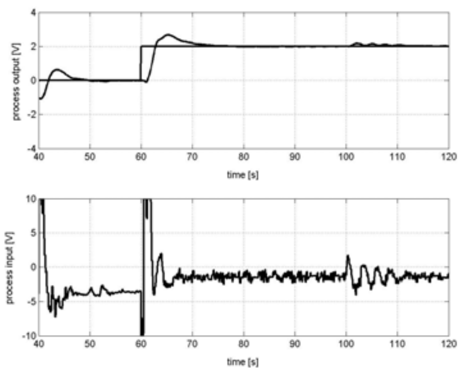

Fig.7 The real result of NN identification with adaptive LQ controller, $T_s = 0.1$ s, $Q_{RES} = 10$ bits.

6. Conclusion

The real results have shown that the gradient algorithm applied to Nonlinear ARX structure (NARX) of neural net have worked better than RLS algorithm applied to δ-model of ARX structure for the same setup of adaptive LQ controller.

Model Structure's Influence on Quantization Effect: The approximation property of nonlinear model based on sigmoid function (NARX) is known. Mentioned approximation property can be used in the real digital process control where quantizers are always inbuilt. In such real case, the quantizers are not ideal. Narrow Code, Missing Code, Wide Code, Integral nonlinearity or Hystereze nonlinearity [4] are included to the previously described “ideal” quantization error. The smooth approximation property of neural networks is advantageously used because of the permanent present of different types of nonlinearities in originally linear processes.

Regression Vector's Influence on Quantization Effect: The second possible choice in each identification method is the regression vector. The results justify the idea that the gradient algorithm applied to NARX structure of neural networks works better than RLS algorithm applied to ARX structure in q time-shift or δ-model domain. The reason can be theoretically explained: δ-model is built to overcome finite word-length precision of used (saved) variables in controller but quantizers decrease the precision more. The input-output round off error (due to A/D and D/A converters) cannot be overcome sufficiently in δ-model domain.

Algorithm's Influence on Quantization Effect: This section explains the difference between performance of two types of iterative minimization algorithms. Generally, two rates of algorithms exist: quadratic rate based on Newton method and gradient based called also steepest descent method.

To sum up, the existing digital control theory does not deal enough with the real process control problems: quantization effect is applied when the sampling period is short or the quantizer resolution are not considered at all [5], [6].

Acknowledgement: This research has been supported by the Czech Ministry of Education in the frame of MSM 0021630503 Research Intention *MICROSYN New Trends in Microelectronic Systems and Nanotechnologies.*

References:

[1] Åström, K.J., Wittenmark B., *Computer-Controlled Systems, Theory and Design*, Prentice Hall, Inc., 1997.

[2] Böhm, J., Survey in Using Square-Roots Factors, Technical Report 1054, Czech Academy of Sciences, UTIA, 1984.

[3] Cichocki, A., Unbehauen R., *Neural Networks for Optimization and Signal Processing*, John Wiley & Sons, Inc., 1993.

[4] Fowler, K., Analog to Digital Conversion in Real-Time Systems, Part 7, *IEEE Instrumentation & Measurement,* 6, 2003, pp. 58–64.

[5] Gessing, R., Whether the Delta Operator Models are Really Better for Small Sampling Periods. In: 15th IFAC Triennial World Congress, 2002.

[6] Guo, L., Tomizuka M., Parameter Identification with Derivative Shift Operator Parametrization. *Automatica* 35, 1999, pp. 1073–1080.

[7] Ljung, L., *System Identification: Theory for the User.* Prentice Hall, Inc., 1987.

[8] Middleton, R.H., G.C. Goodwin G.C., *Digital Control and Estimation, a Unified Approach,* Prentice Hall, Inc., 1990.

[9] Švancara, K., Adaptive Optimal Controller with Identification Based on Neural Network, PhD thesis, Brno University of Technology, 2004.

[10] Švancara, K., Pivoňka P., The Issue of Quantization Effect in Direct Implementation of Adaptive LQ Controller with NN Identification into PLC, In: Proc. of 3rd EUSFLAT Conf., 2003, pp. 361–366.

[11] Švancara, K., Pivoňka P., Quantization Effect Influences Identification in Adaptive LQ Controller, In: Proc. of 11th IEEE Mediterranean Conference on Control and Automation, 2003, pp. 77–82.

[12] Williamson, D., *Digital Control and Implementation*: *Finite Wordlength Considerations*, Prentice Hall, Inc., 1991.