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Hybrid modelling of dynamic systems by hybrid automatons and analysis of reachability of states in view of diagnosis

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Abstract: - The strong interaction between the discrete dynamic and the one continue is now seriously taken into account in the model. These models newborn so-called hybrid models describe at one time the two dynamics and are already used as well in control systems as in analysis. The methods and present tools of the dependability of systems are either of quantitative order or qualitative one. The avenue of the hybrid models obliges the scientific community to compose with this new modelling. The scientific objective of the present article is the presentation of a methodology of research on diagnosis of the hybrid dynamic systems. These are modellised by linear hybrid automatons. The analysis of reachability of the states described by these automatons is it a means of the dependability in the hybrid dynamic systems?

Key-words: - Dependability, hybrid dynamic Systems, hybrid automatons, linear hybrid automatons, forward analysis, backward analysis, undesirable state, diagnosis.

1 Introduction

The time and the relation of order between the occurrences of the events are no without least importance. They are now taken into account to provide the new means of the dependability. The present qualitative assessment of the dependability proceeds neither the time nor the order of arrival of the events. The quantitative assessment requires the knowledge of two models, nominal and of failings, in order to generate residual signals. These residual signals translate the presence of shortcoming as soon as they move away of zero. The recent works permitted a better understanding of the modelling of the hybrid dynamic systems. These dynamic systems present continuous and discrete aspects at one time and in interaction.

2 Notions of the hybrid dynamic systems (HDS)

2.1 Introduction

The hybrid dynamic systems make intervene explicitly and simultaneously the phenomena describing continuous and discrete dynamic behaviours.

An academic example extensively used to describe a HDS is the thermostat.

The temperature of a piece is controlled by a thermostat that measures in a continuous way and switch on or switch off the heating. When the heating is extinguished, the temperature decreases with time while following the exponential law:

$$\Phi_1 (\Theta, t) = \Theta.\ \exp(-Kt)$$

K is a constant determined by the features of the piece.

When the heating is swiched on, the temperature goes up while following the exponential law:

$$\Phi_2 (\Theta, t) = \Theta.\ \exp(-Kt)+h.(1-\exp-Kt)$$

h is a constant that depends on the heating power.

The device must assure that the temperature of the piece remains between a minimal value Θ_m and a maximal value Θ_M.

For this, it is necessary to light or to extinguish the heating according to the needs. The state of the system is defined by the temperature and the state of the heating is either on, either off.

The change of discrete state is done when the temperature reaches in a continuous way the values Θ_m and Θ_M.

2.2 Modelling of the HDS

Lately, this domain kept the attention of the researchers and several formalisms have been proposed in order to establish a homogeneous model permitting to reconcile between the continuous part and the discrete one. The approaches of modelling of the HDS are classified in three main categories, a) the discrete approach consists to approximate the continuous dynamics in order to look like a system discrete events, b) on the contrary the continuous approach consists to approximate the discrete dynamics, that permits to use the theory of the continuous dynamic systems, c) and at last, the approach that considers the continuous and discrete behaviours at a time in a homogeneous model. The interest of this approach resides in the fact that it doesn't make abstraction of information concerning the system to modellise. In this modelling, some formalism have been used, transformed and enriched to take into account the hybrid phenomena. These formalisms come from the theory of the systems of discrete events as the Petri Nets and the automatons.

3 Hybrid automaton

3.1 Definition

The considered model is a state-transition extended graph with a set of variables. A global state of the HDS consists in a summit and a state of variables. The change of discrete state takes place while executing a transition labelled by a condition and an affectation. A transition is executed only if the condition is satisfied. The affectation permits to modify the state of the variables after the execution of the transition. The continuous evolution of the variables in a summit is described by the function of evolution associated to the summit. A predicate, named invariant, specifies the maximum stay duration in a summit.

Formally a hybrid automaton is described by:

$A=<X, S, \psi, Inv, A, C, A_f>$ as:

1) X is a set of finished variables {xi}. One notes x the vector composed of the values of the variables to given at an instant, named state of the variables. The set of all states is noted V.

2) S is set of finished summits

3) ψ is a function that associates to every summit s $\in$ S a ψ_s function describing the continuous evolution of the variables according to the time.

4) Inv is a function that associates to every summit s a predicate on the variables of this summit.

5) A is set of finished transitions. Every transition a=(s,s') identifies a summit s of departure and a summit s of arrival.

6) C is a function that associates to every transition a $\in$ A, a predicate C_a called guard condition. A transition a can be executed that if the condition C_a is verified.

7) A_f is a function that associates to every transition a, an A_{fa} relation named affectation.

If one takes the example of the thermostat the hybrid automaton describing its behaviour is as follows:

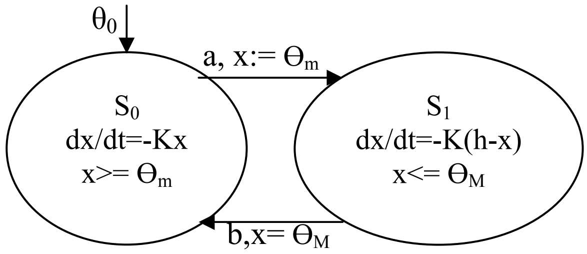

Fig.1: Hybrid model automaton of the thermostat

With:

$X=\{x\}$; $S=\{s_0,s_1\}$; $s_0(x,t)=-Kx$;
$s_1(x,t) = -K(h-x)$; $Inv(s_0)=\{x/x>= \Theta_m\}$;
$Inv(s_1)=\{x/x <= \Theta_M\}$; $A=\{a,b\}$; $a = (s_0,s_1)$;
$b = (s_1,s_0)$; $C=\{C_a,C_b\}$; $Af=\{A_{fa},A_{fb}\}$.

3.2 Linear hybrid automaton

A particular class of the hybrid automatons is the linear hybrid automatons. Their entities are defined as follows:

1. The function of evolution ψ_s in a summit s is limited to a linear function defined by a system of first order differential equations of the form $x=k_s$ when k_s is a constant vector.

2. The invariant Inv_s of the summit as well as the condition of the transition are defined by a system of linear inequalities.

3. The affectation A_{fa} relative to the transition a is defined by the relation $x:= \alpha_x$ when α_x is a linear transformation.

4 The analysis of reachability

It uses two dual methods: The forward analysis consists to calculate the space of all successor states of the initial state space. If the intersection between the space of state Q and the calculated space is not empty, the space of state Q is reachable

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from the initial state space. The backward analysis consists in calculating the space of all predecessor states of the states of the space Q, if the initial space is included in the calculated space, then the space Q is reachable from the space of initial states. The two methods are retailed in [1] and [2].

5 Principle of Diagnosis by the analysis of reachability

The idea is that from a non desirable state is it possible to recover the scenarios (sequences of events) that generated this state?

Let's consider the example of the automaton modelling a tank of water provided with a valve in exit. This valve can be opened or closed. The valve is opened during the open situation (S_1 summit), and therefore the level of water decreases (variable of it). As soon as the reservoir is emptied (y=0), the valve is closed (activation of the closed situation, S_0 summit) and the level of water goes up. The variable x is used as a clock. The invariant of the S_0 summit and the specification of its outgoing transition imply that the length of the closed open cycle is of three units of time.

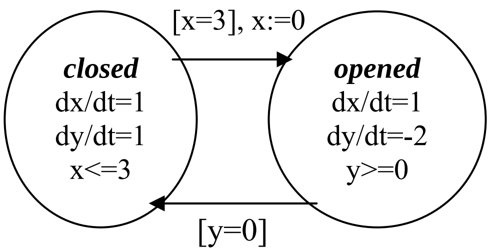

Fig.2: Model of hybrid automaton of a water tank

Let's consider that one wonders about the possibility to reach a state (valve blocked closed) of the domain (closed, 1 <= x <=3^x=y+1) from a state of the S_0 domain = (closed, x=y^x <=3). The calculation of the reachable space is then the next one:

- The initial domain is (closed, I) with I=(x=y^x <=3);
- The extension of I by the dynamics associated to the closed situation is characterized by (x=y), since dx/dt=dy/dt=1, and its intersection with the invariant of the closed situation is therefore I'=(x=y^x <=3);
- The only transition that can be crossed is the one toward the open situation; as its guard condition is [x=3], the I_1 is characterized by I_1=(x=y=3);
- Considering the function of jumping of the transition and the invariant of the open situation, the I_2 and I_3 are given by, I_2 =I_3 = (x=0^y=3);
- The dynamic of the open situation defines I_4, that is then given by I_4=(y+2x=3^y>=0).

The reachable space after this first iteration of calculation is characterized by the set S_1 = S_0 U (opened, y+2x=3^y>=0).

We apply the operator Post to the domain (open, J) with J = (y+2x=3^y>=0) to calculate the sets J_1, J_2, J_3 and J_4 (for example J_4 = x=y+3/2^x <=3) and then start off again from the new domain (closed, J_4) in order to continue the calculation. In this case, the iterative calculation by forward analysis, of the reachable space doesn't converge. On the other hand, from S_{target} = (closed, 1 <= x <=3^x=y+1), an inverse calculation, by backward analysis with equivalent manner, permits to conclude in two iterations that the domains that permit to reach S_{target} are itself and (open, y+2x=2^y>=0). As the S_0 domain has an empty intersection with these two domains, we conclude that it is not possible to reach S_{target} from S_0.

6 Decidability

The problems of calculation of the space of reachable states and the existence of an algorithm doing this calculation are very important to permit an efficient diagnosis. Unfortunately, from theoretical point of view, it is proven, that outside of some particular automatons, the calculation cannot converge and therefore the problem is not decidable [3] and [4].

A temporized automaton can be transformed in an automaton of finished states that are equivalent. The calculation of the reachable space is therefore decidable. In the same way, one can prove that a "multi-rate initialized" automaton can be transformed in an temporized automaton and that the calculation of the reachable space is therefore decidable [5].

7 Conclusions

The interest of the linear hybrid automatons is double, on the one hand, its hybrid character permits to better modelling the system to study and therefore be well suitable to the diagnosis, and on the other hand, they are especially interesting for the characterization of the decidability.

All regions of the continuous state space, defined by linear hybrid automatons, are linear convex and the images of these regions by their functions of continuous and discrete evolutions are also linear regions [6]. The number of the states of a linear hybrid automaton being infinite, their exhaustive enumeration is impossible. They are presented then

in a symbolic manner while using the polyhedrons. Libraries of functions of manipulation of polyhedrons exist [2], [7].
Even though the polyhedrons are the regions that can be to manipulate relatively simply, the calculations become quickly complex. For example, it is not easy to decide if an union of polyhedrons is included in another polyhedron. The operators of approximation of the regions of polyhedrons such" widening, extrapolation" allow to accelerate the convergence with one degree of specification on the result [2].

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