Proceedings of the 5th WSEAS Int. Conf. on SIGNAL, SPEECH and IMAGE PROCESSING, Corfu, Greece, August 17-19, 2005 (pp163-166)

Side lobe minimization of the emitted radiation pattern of a phased array antenna using gradient methods

A. KOUMASIS, C.T. ANGELIS
Department of Communications Informatics and Management
TEI of EPIRUS
Kostakioi, 47100, Arta
GREECE
http://www.teleinfom.teiep.gr

Abstract: - In this paper, we demonstrate three minimization methods whose purpose is to suppress the side lobes of the radiation pattern of a linear phased array antenna (PAA) during emission. The purpose of this suppression is to increase the directivity of the antenna. These methods are: the Gradient Method (GM) or LMS method, the Conjugate Gradient Method (CGM) and the Lagrange Multipliers Method (LMM). Simulation results were taken and the three methods were compared. The two last methods are consistent with what it was expected.

Key-Words: - Phased array Antenna, Optimization, Gradient Methods

1 Introduction

Satellite, aviation modern communications and radar systems require high performance transmitting antennas. Phased array antennas have been mostly used which have the advantage of two dimensional scanning without moving mechanical parts. Unfortunately with the main transmitting lobe, a number of side lobes are developed which are undesired since they contain an appreciable amount of radiating power. As a result of this is the difficulty of detecting targets, while the hardware and software for doing this has a substantial volume. Various methods have been used and demonstrated, such as, the binomial array and the Dolph-Tschebyscheff array [1]. These methods change the array factor (AF) of the PAA to those needed for each method and they are shown to be lengthy in computations and tedious in work. In this work we do not change the initial AF while the computations are straight forward.

2 Theory

The minimization methods, mentioned above, can apply either to electronics or to photonics driving systems. The configuration of a photonic system is shown in Fig. 1 [3, 4], (OSP stands for optical signal processor). In this system the last stages before the antenna elements are the RF amplifiers.

The idea is to design an optimum set of gains of these amplifiers such that to suppress the side lobes of the radiation pattern to a desired point, leaving at the same time the main lobe untouched. Using a PAA this is easy to be done. The AF of such an N-element antenna is given [1] in normalized form as

$$AF = \frac{1}{N}\sum_{n=1}^{N} u_n \exp j(n-1)\psi$$

where $\psi = -kd\cos\theta + b$,

k is the wave number ($2\pi/\lambda$), d the distance between the elements of the array antenna, b the phase shift of the driving RF-signals between adjacent elements of the PAA and un the excitation or gain vector to be optimized.

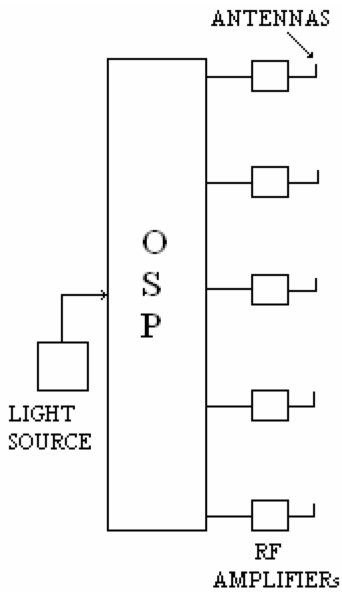

Fig. 1 Photonics driven PAA system

We represent the system under consideration as shown in Fig. 2. The input vector u represents the gains of the N RF-amplifiers, each driving an antenna. The system matrix A represents the characteristics or dynamics of the side lobes, while the output y represents the values of the side lobes.

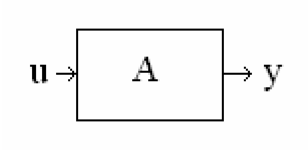

Fig. 2 The system representation

According to this configuration we have [5]:

$$\mathbf{y = Au} \tag{1}$$

where y, u are (nx1) the output and the input vectors respectively, and A is a (nxn) matrix.
If we consider a desired output vector d then we can write the (nx1) error vector equal to

$$\mathbf{e = d - y} \tag{2}$$

and therefore we can form the functional or cost function

$$J = ½(e^T e) \tag{3}$$

This has to be minimized by designing an optimum input vector u. The algorithms of the three minimization methods are described below.

Gradient Method.

According to this method we want the gradient of the functional J w.r.t. the u to become zero, i.e.

$$g = \partial J / \partial u = 0 \tag{4}$$

This gives us, after substituting eqs (1), (2) and (3) into eq. (4),

$$g = - A^T d + A^T A u \tag{5}$$

and the algorithm goes as follows:
1. We set an initial value of **u**,
2. Compute the gradient g from eq. (5) and the next value of **u** is calculated from
3. $u_{i+1} = u_i - K_i g_i$

We repeat the process from the second step until no significant change occurs of the gradient g. K is a positive scalar less than unity, and in this work is taken as a function of the iteration index, and is chosen with respect to such factors as convergence, rate of descent, etc [2].

Conjugate Gradient Method.

This method (Fletcher and Powell, 1963) generates a conjugate direction vector s, which is conjugate or orthogonal with respect to the second derivative of the functional J w.r.t. u and the algorithm goes as follows:
1. Set an initial value of **u**
2. Compute g from eq. (5)
3. Set $s_i = -g_i$ and the next value of **u** is
4. $u_{i+1} = u_i + \alpha s_i$
5. Compute g_{i+1} from eq. (5)
6. $\beta = (g^T{}_{i+1}\, g_{i+1}) / (g^T{}_i\, g_i)$
7. $s_{i+1} = - g_{i+1} + \beta s_i$

We repeat the process from the fourth step until no significant change occurs of the gradient g. The factor α, is a positive scalar less than unity and is determined by trying several candidate values and select one which yields a minimum of eq (5).

C. Lagrange multiplier Method.

This method utilizes the Lagrange multipliers to tackle the problem. According to this we have to minimize the functional J, subject to the equation (1)
We form the Hamiltonian:
$H = J + \lambda^T(y\text{-}Au)$
where λ is the Lagrange multiplier vector.
Substituting eqs (1), (2) and (3) into the Hamiltonian we have:
$H = ½(d\text{-}Au)^T(d\text{-}Au) + \lambda^T(y\text{-}Au)$
and taking the partial derivative of H w.r.t. u we have the gradient which should become zero at the optimum point of u. Thus we get:

$$\frac{\partial H}{\partial u} = - A^T d + A^T A u\, A^T \lambda \tag{6}$$

Also rewriting the Hamiltonian as
$H = ½(d\text{-}y)^T(d\text{-}y) + \lambda^T(y\text{-}Au)$
and taking the partial derivative of H w.r.t. y and equating to zero we get the Lagrange variable

$$\lambda = d - y \tag{7}$$

So the algorithm goes as follows:
1. Set an initial value of u
2. Compute the output y from eq. (1)
3. Compute the variable λ from (7)
4. Compute the slope from (6) and
5. $u_{i+1} = u_i - K_i \frac{\partial H}{\partial u_i}$

We repeat the process from the second step until no significant change occurs of the slope determined by eq (6). The value of K has the same meaning mentioned earlier.

3 Simulation results and discussions

Attention was given to the selection of the entries of the matrix A. This matrix contains the characteristics of the side lobes. The first rows were filled with the maximum values of the side lobes while the rest were filled with the nulls of the radiation pattern according to the following description. The entry of the ith row and the nth column was filled by the value defined by
$a_{in} = \cos(n\text{-}1)\,\psi_i$, n=1,2…N
where N is the number of the antennas in the array, and
$\psi_i = - kd\cos\theta_i + b$ where θ_i

a) for the maximum of the side lobes:
θ_i=acos[b/π ± (2i+1)/N], i=1,2…(N-1)/2
b) for the nulls:
θ_i=acos[b/π ± 2i/N], i=1,2…(N+1)/2
and kd = π,
b is the phase shift of the RF signal between adjacent elements of the array antenna [1].
It was not necessary to write down all the maximum values of the side lobes and the nulls of the pattern but only those specified by the equations of θi. The others are symmetrical quantities of these. The plus or minus sign of these equations is taken whether the value of b is negative or positive respectively. The scalar value K was taken as a function of iteration, as mentioned above, and equal to 1/(k+1), where k is the current iteration value. For the CGM the best value of α was fount to be 0.2. Figs 3, 4, and 5 show the radiation patterns of the GM, CGM, and LMM respectively, while Tab 1 shows the values of the vectors at the reached point. The fifth column of Tab 1 shows the optimum values of the vectors u for the three methods and as mentioned earlier, these are the amplifier gains of the N=5 elements of the PAA. Observing the Figs 3, 4, and 5 we see that the side lobes are suppressed to the desired level while the maximum of the main lobe remained unchanged but its width became wider, which was expected. In these Figs the dotted lines show the transmission pattern without minimization, while the continuous lines show the optimized pattern. Of course the width of the main lobe can become narrower by adding more elements to the array.

Table 1: Results of the three methods

Desired	output	error	Gradient	Gain
0.1000	0.1241	-0.0249	-0.0045	0.9862
0.1000	0.1093	-0.0096	-0.0104	1.6787
0.0010	-0.0204	0.0220	0.0020	1.2613
0.0010	-0.0072	0.0085	0.0231	0.7529
0.0010	-0.0072	0.0085	0.0242	0.3209

Number of iterations =30

Desired	output	error	Conj Grad	Gain
0.100	0.1263	-0.0263	0.0000	1.0010
0.1000	0.1263	-0.0263	-0.0000	1.7074
0.0010	0.0012	-0.0002	0.0000	1.2701
0.0010	0.0013	-0.0003	0.0000	0.7294
0.0010	0.0013	-0.0003	0.0000	0.2921

Number of iterations =20

Desired	output	error	Lagrange	Gain
0.1000	0.1006	-0.0006	0.0000	1.0007
0.1000	0.1002	-0.0002	-0.0006	1.7069
0.0010	0.0005	0.0005	0.0001	1.2699
0.0010	0.0008	0.0002	0.0010	0.7299
0.0010	0.0008	0.0002	0.0011	0.2926

Number of iterations =15

Observing Table 1, we note that with the Gradient method the output shows a significant error with the desired value, the Conjugate Gradient method attempts a good approach of the output to the desired value, while the Lagrange method makes the output almost the same with the desired value performing the minimum number of iterations. Of course the number of iterations depends on the initial value of the vector u, which was taken equal to 1 for all the methods. The computed optimum value of u is independent of the transmission angle (angle at the maximum value of the main lobe). It depends only on the selection of the desired value d. The forth column of the Tab 1 shows the values of the derivative of the function for each method.

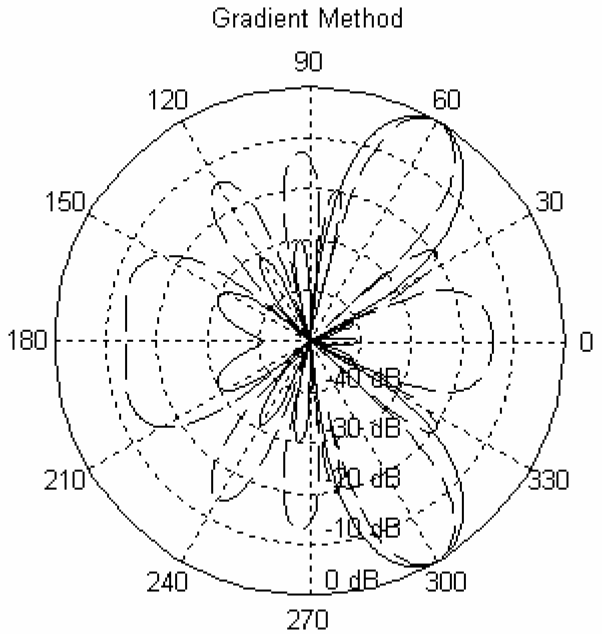

Fig. 3 The attenuation is less than -20dB

The attenuation of the side lobes is equal to 20log (0.1) = -20dB, and with reference to the 0dB point is (-20 -12 = -32) dB, where the -12dB is the initial difference of the maximum value of the side lobe with the 0dB point, while the attenuation of the nulls is 20log (0.001) = - 60dB. Observing Figs 3, 4, and 5 one can see that the result of the minimized side lobes using the GM is not so accurate as it is with the other two methods. The two first values (0.1), of the vector d correspond to the desired attenuation of the side lobes, while the latter ones (0.001), correspond the attenuation of the nulls. The attenuation of the nulls sounds peculiar, but when we excite the antennas of the array nonuniformly, extra side lobes appear at the null positions. To avoid the appearance of these extra side lobes we have to apply attenuation as high as possible, -60 dB in our case.

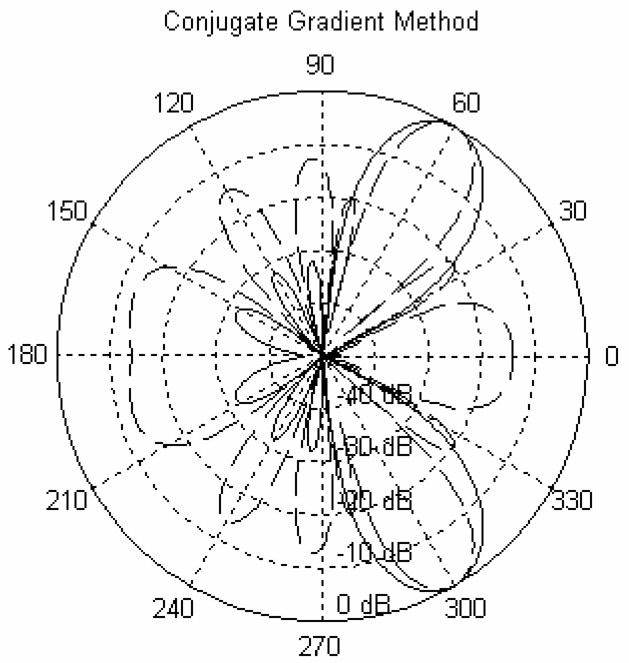

Fig. 4 The attenuation is nearly -20dB

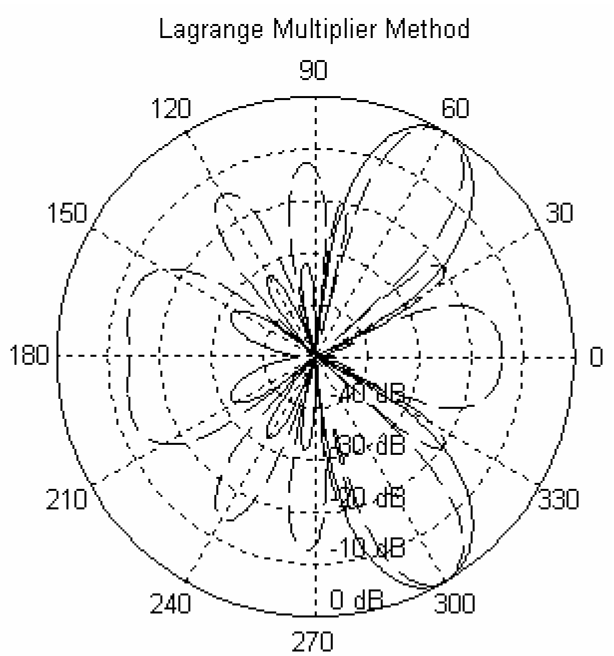

Fig. 5 Shows -20 dB side lobe attenuation

4 Conclusion

We have demonstrated three minimization methods aiming to the suppression of the side lobes of the emitted radiation pattern of a linear phased array antenna. These are the Gradient method, the Conjugate Gradient method and the Lagrange multiplier method. The first method does not approach the desired value as it happens with the others. The Lagrange multiplier method is consistent with the Conjugate Gradient method, but the latter one requires the determination of the value of α. The LMM achieves its goal with the minimum number of iterations. Also the three methods avoid the tedious work of the binomial array and the Dolph-Tschebyscheff array methods.

References:

[1] C.A.Balanis, "Antenna theory: Analysis and Design", *J. Wiley & Sons*, N. York 1997, pp. 288-305.

[2] A.P.Sage, J.L.Melsa, "*Mathematics in Science and Engineering*" Volume 80, Academic Press 1971, pp. 82-110.

[3] Y.Liu, J.Yao, J.Yang, "Wideband true-time-delay unit for phased array beamforming using discrete-chirped fiber grating prism", *Optics Communications* 207 (2002) 177-187.

[4] Y.Liu, J.Yang, J.Yao, "Continuous True-Time-Delay beamforming for phased array antenna using a tunable chirped fiber grating delay line" *IEEE Photon. Techn.*Lett, Vol.14, Aug.2002, 1172-1174.

[5] K. Hooli : Adaptive Filters/LMS algorithm, CWC, University of Oulu.