Proceedings of the 5th WSEAS Int. Conf. on SIMULATION, MODELING AND OPTIMIZATION, Corfu, Greece, August 17-19, 2005 (pp384-390)

OPTIMAL DESIGN OF IRRIGATION NETWORKS USING THE DYNAMIC METHOD AND A SIMPLIFIED NONLINEAR METHOD

M. THEOCHARIS[1] , C. TZIMOPOULOS [2] , S. YANNOPOULOS [2] AND M. SAKELLARIOU - MAKRANTONAKI [3]

[1] Department of Crop Production
Technical Educational Institution of Epirus
47100 Arta, GREECE
http://www.teiep.gr

[2] Department of Rural and Surveying Engineers
Aristotle University of Thessaloniki
54006 Thessaloniki, GREECE

[3] Department of Agricultural Crop Production and Rural Environment
University of Thessaly
38334 Volos, GREECE

Abstract: - The designating factors in the design of branched irrigation networks are the cost of pipes and the cost of pumping. They both depend directly on the hydraulic head of the pump station. It is mandatory for this reason to calculate the optimal head of the pump station as well as the corresponded optimal pipe diameters, in order to derive the minimal total cost of the irrigation network**.** The mathematical research of the problem, using the classical optimization techniques, which have been proposed until now, is very complex and the numerical solution calls for a lot of calculations, especially in the case of a network with many branches. For this reason, many researchers have developed simplified calculation methods with satisfactory results and with less calculation time needed. In this paper a comparative calculation of the pump station optimal head as well as the corresponded economic pipe diameters, using the dynamic programming method and a proposed simplified nonlinear programming method, is presented. Comparative evaluation in a particular irrigation network is also developed.

Key-Words: - Irrigation, head, pump station, network, cost, optimization, simplified method, dynamic.

1 Introduction

The problem of selecting the best arrangement for the pipe diameters and the optimal pumping head so as the minimal total cost to be produced, has received considerable attention many years ago by the engineers who study hydraulic works. The knowledge of the calculating procedure in order the least cost to be obtained, is a significant factor in the design of the irrigation networks and, in general, in the management of the water resources of a region. The classical optimization techniques, which have been proposed so long, are the following: a) The linear programming method, b) the nonlinear programming method [2,3,4], c) the dynamic programming method [2,4,5,6,7], and d) the Labye's method [1,2,4].

The common characteristic of all the above techniques is an objective function, which includes the total cost of the network pipes, and which is optimized according to specific constraints. In this study, a systematic calculation procedure of the optimal pump station head using the dynamic programming method and a new simplified nonlinear programming method is presented, which can replace the existing methods with the best results. Application and comparative evaluation in a particular irrigation network is also developed.

2 METHODS

2.1 The dynamic programming method

According to this method the search for optimal solutions of hydraulic networks is carried out considering that the pipe diameters can only be chosen in a discrete set of values corresponding to the standard ones considered. The least cost of the pipe network is obtained from the minimal value of the objective function, meeting the specific functional and non-negativity constraints.

2.1.1 The objective function

The objective function is expressed by [2,4]:

$$F_i^* = \min\left\{C_{ij} + F_{(i+1)}^*\right\} \tag{1}$$

where F_i^* is the optimal total cost of the network downstream the node i, and C_{ij} is the cost of each pipe i under a given diameter j.

The decision variables, D_{ij}, are the possible values of each pipe accepted diameters.

2.1.2 The functional constraints

The functional constraints are specific functional constraints and non-negativity constraints [2,4].
The specific functional constraints for every complete route of the network, are expressed by:

$$H_{\pi_j} + \sum_{k=i}^{\pi_j} \Delta h_k \geq H_i \tag{2}$$

where H_{π_j} is the required minimal piezometric head in the end π_j, H_i is the required minimal piezometric head in the node i, and the sum $\sum_{k=i}^{\pi_j} \Delta h_k$ is the total friction losses from the node i until the end π_j of every complete route of the network.
The non negativity constraints are expressed by:

$$\Delta h_i > 0 \tag{3}$$

where i = 1... n, and n is the total number of the pipes in the network.

2.1.3 Calculating the minimal total cost of the network

At first the minimal acceptable piezometric head h_i for every node of the network is determined. If the node has a water intake, then $h_i=Z_i+25$m,otherwise $h_i=Z_i+4$ m, where Z_i is the elevation head at the ith node. Thereinafter the technically acceptable heads at the nodes of the network are obtained from the minimal value of the objective function. The calculations begin from the end pipes and continue sequentially till the head of the network. The algorithm that will be applied is the complete model of dynamic programming with backward movement (Backward Dynamic Programming, Full Discrete Dynamic Programming – BDP, FDDP).

2.1.3.1 A network with pipes in sequence

The required piezometric head at the upstream node of each end pipe is calculated [2], for every possible acceptable commercial diameter, if the total head losses are added to the acceptable piezometric head at the corresponded downstream end.

From the above computed heads, all those that have smaller value than the required minimal head at the upstream node, are rejected (Criterion A). The remaining heads are classified in declining order and the corresponding costs are calculated that should be in ascending order. The solutions that do not meet with this restriction are rejected (Criterion B). The application of the B criterion is easier, if the check begins from the bigger costs. Each next cost should be smaller than every precedent one, otherwise it is rejected. The described procedure is being repeated for the next (upstream) pipes. Further every solution that leads to a diameter smaller than the collateral possible diameter of the downstream pipe is also rejected (Criterion C).

2.1.3.2 Branched networks [2]

From now on, every pipe whose downstream node is a junction node will be referred as a supplying pipe.

Defining of the downstream node characteristics

The heads at the upstream nodes of the contributing pipes are calculated. The solutions, which lead to a head that is not acceptable for the all the contributing pipes, are rejected (Criterion D). The remaining heads, which are the acceptable heads for the branched node, are classified in declining order.
The relevant cost of the downstream part of the network – which is the sum of the contributing branches costs – is calculated for every possible node head.

Defining of the upstream node characteristics

The total dead losses, corresponded to every acceptable diameter value of the supplying pipe, are added to every head value at the downstream node and so the acceptable heads at the upstream node are calculated.
The total cost of the downstream part of the network including the supplying pipe is calculated for all the possible heads at the upstream node.
The acceptable characteristics of the upstream node are resulted using the same procedure with the one of the pipes in sequence (criteria B and C).
The described procedure is continued for the next supplying pipes until the head of the network where the total number of possible heads of the pub station is resulted. For every acceptable head of the pub station, the corresponded optimal total cost, P_N, of the network is calculated.

2.1.4 The total annual cost of the project

For every calculated value of the pub station head, the total annual cost of the project is calculated by [2]:

$$P_{an} = P_{Nan} + P_{Man} + P_{Oan} + E_{an} =$$
$$= 0.0647767P_N + 2388.84QH_{man} \text{ €} \qquad (4)$$

where $P_{Nan.}$= $0.0647767P_N$ is the annual cost of the network materials, $P_{Man.}$= $265.292QH_{man}$ is the annual cost of the mechanical infrastructure, $P_{Oan.}$= $49.445QH_{man}$ is the annual cost of the building infrastructure, and $E_{an.}$= $2074.11QH_{man}$ is the annual operational cost of the pump station

2.1.5 The optimal head of the pump station

From the calculated values of the pump station head, H_A, and the total annual cost, P_{an}, of the project, the graph $P_{an.}$-H_A is constructed and the minimal value of P_{an} is resulted. Then the H_{man}=H_A-Z_A corresponding to minP_{an} is calculated, which is the optimal head of the pump station.

2.1.6 Selecting the economic pipe diameters [2,4]

The head losses correspondents to every acceptable diameter of the first pipe of the network are subtracted from the optimal value of H_A, and so the possible heads at the end of the first pipe are resulted. The bigger of these heads included within the limits determined by the backwards procedure, is selected. The diameter value correspondent to this head is the optimal diameter of the first pipe. This procedure is repeated for every network pipe till the end pipes.

2.2 The simplified nonlinear programming optimization method

2.2.1 Calculating the optimal head losses

Consider a branch irrigation network under its ideal form [2,3](suppose that the network consists only of branches and all the single nodes are neglected) as shown in fig.1.

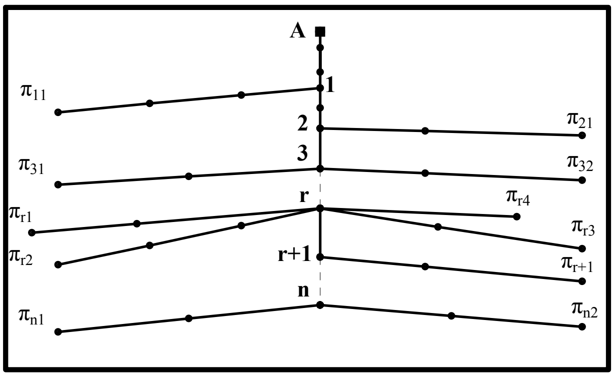

Figure 1. An ideal branched network

Each branch of the network is named from the symbol of the downstream junction point, i or end π_{ij}. Each junction point of the network is named by i and is numbered, from upstream to downstream, with i =1,2,..,r,..,n, where n is the total number of the junction points of the network.

The minimal cost of a branched hydraulic network, that is obtained using the Lagrange multipliers, concludes to the solution of the system [2,3]:

$$\left[\frac{\Phi_i}{\Delta H_i}\right]^{\omega} = \sum_{r=i}^{n}\sum_{j=1}^{p}\left[\frac{\Phi_{\pi rj}}{\Delta H_{\pi rj}}\right]^{\omega} \qquad i=1,2,\ldots,r,\ldots n \quad (5)$$

and

$$\Delta H_{\pi_{rj}} = (H_A - H_{\pi_{rj}}) - \sum_{i=1}^{r}\Delta H_i \qquad (6)$$

where j = 1,2,…,p is the random supplied branch begging from the node r

In the above relations:

$$\omega = 1 + 0{,}2\nu \qquad (7)$$

$$\Phi_i = \sum_{t=1}^{\nu}\varphi_{i} = \sum_{t=1}^{\nu}\left[\frac{A}{1{,}6465^{\nu}} f^{\frac{\nu\omega}{\nu+5}} L_{it} Q_{it}^{0{,}4\nu}\right]^{\frac{1}{\omega}} \qquad (8)$$

$$\Phi_{\pi_{rj}} = \sum_{b=1}^{\tau}\varphi_{rjb} = \sum_{q=1}^{\tau}\left[\frac{A}{1{,}6465^{\nu}} f^{\frac{\nu\omega}{\nu+5}} L_{rjb} Q_{rjb}^{0{,}4\nu}\right]^{\frac{1}{\omega}} \qquad (9)$$

where t =1,2,…,v is the random pipe of the supplier branch i, b =1,2,…,τ is the random pipe of the supplied branch rj, A and ν are fitting coefficients in Mandry's cost function (Mandry 1967) [4] :

$$c_i = AD_i^{\nu} \qquad (10)$$

and f is the friction coefficient calculated from the Colebrook–White.

If a arbitrary complete route of the network is selected and the rest of the network is neglected, the optimal losses head of the supplying branches that belong in this route, $\Delta H_i'$, can be calculated using the following relations [2]:

$$\Delta H_i' = \frac{\Phi_i}{\sum_{i=1}^{r}\Phi_i + \Phi_{\pi_{rj}}}\left(H_A - H_{\pi_{rj}}\right) \qquad (11)$$

It is proved that, if the complete route presenting the minimal average gradient is selected, the $\Delta H_i'$, which have been calculated from eq.11, do not differ considerably from the ΔH_i which have been calculated, according to the general non linear optimization method using Lagrange multipliers (eqs.5 and 6). Using the heads at the junction points, which have been calculated with the above mentioned procedure from the complete route presenting the minimal average gradient as heads of the supplied branches of the network, the frictional head losses, Δh_i , and the diameters of the pipes

Proceedings of the 5th WSEAS Int. Conf. on SIMULATION, MODELING AND OPTIMIZATION, Corfu, Greece, August 17-19, 2005 (pp384-390)

within the other supplied branches can be easily calculated.

2.2.2 The variance of the head of the pump station

The search of the optimal head of the pump station should be realized as following [2]:
If in the complete route presenting the minimal average gradient the smallest allowed pipe diameters are selected, the maximal value of the head of the pump station is resulted, in order to cover the needs of the network. Additionally, if in the complete route presenting the minimal average gradient the biggest allowed pipe diameters are selected, the minimal value of the pump station is resulted, in order to cover the needs of the network. These two extreme values $minH_A$ and $maxH_A$, constitute the lowest and the highest possible value of the pump station head respectively.

2.2.3 The least cost of the network, P_N

From the cost function, the cost of every pipe is calculated and then the total minimal cost of the network, P_N, is obtained by [2,3]:

$$P_N = \sum_{i=1}^{n} \left[\frac{\varphi_i^{\omega}}{\Delta h_i^{\omega-1}} \right] \qquad (12)$$

The calculation is carried out for :
$minH_A \leq H_A \leq max\ H_A$ where $minH_A$ and $maxH_A$ are as they were defined above in paragraph 2.2.2.

2.2.4 The optimal head of the pump station

For the variance of the pump station head, P_{an} , is calculated using eq.4 [2,3]. Then the graph P_{an}-H_A is constructed from which the minimal value of P_{an} is resulted. Finally the $H_{man} = H_A - Z_A$ corresponded to $minP_{an}$ is calculated, which is the optimal head of the pump station.

2.2.5 Selecting the economic pipe diameters

The economic diameters of the pipes corresponded to the calculated optimal pump station head, are resulted from [2,3]:

$$D_i = \left[f_i \frac{L_i Q_i^2}{12.10 \Delta h_{f_i}} \right]^{0.2} \qquad (13)$$

and they necessarily must be rounded off to the nearest available higher commercial size.

3. APPLICATION

The optimal pump station head of the irrigation network, which is shown in fig. 2, is calculated. The material of the pipes is PVC 10 atm. The required minimal piezometric head at each node, h_i, is resulted as it is described in paragraph 2.1.3. Figure 2 represents the real network and provide geometric and hydraulic details.

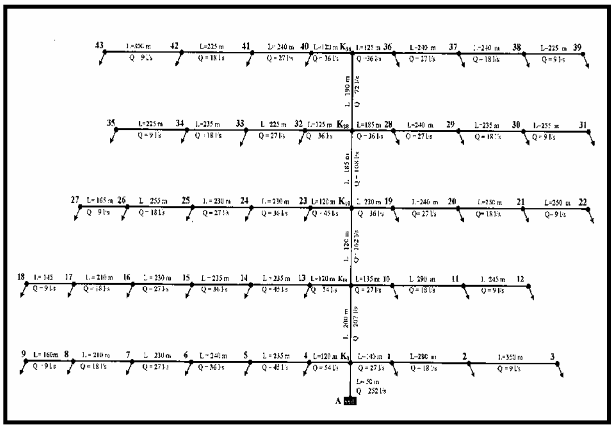

Figure 2. The real under solution network

3.1 Calculation according to the dynamic method

3.1.1 The optimal cost of the network

The complete model of dynamic programming with backward movement (BDP, FDDP) is applied. The results are presented in table 1.

Table 1. The optimal cost of the network, P_N, according to the dynamic programming method

H_A [m]	P_N [€]	H_A [m]	P_N [€]	H_A m]	P_N [€]
83.74	395595	88.56	309127	93.24	275839
84.02	384593	88.79	307246	93.38	274908
84.73	364916	89.24	302955	94.06	271466
85.00	360399	89.50	300660	94.49	269746
85.25	353925	89.71	298800	95.04	267313
85.49	349940	90.02	297106	95.48	264974
85.79	345931	90.25	294676	95.99	261993
86.01	341371	90.50	292930	96.48	261913
86.25	338809	90.77	291689	97.06	261429
86.48	334756	91.04	290119	97.57	260170
86.76	330295	91.24	288481	97.89	258266
87.00	327143	91.48	286134	98.79	256869
87.23	324942	91.75	284619	99.78	256232
87.48	321236	92.01	283533	100.24	255319
87.75	319724	92.23	281717	100.86	254421
88.06	316387	92.49	279850	102.57	253858
88.25	312555	92.82	279005		

3.1.2 The optimal head of the pump station

Using the eq. 4, the total annual cost of the project is calculated for pump heads, H_A, the values given in table 1. The results are presented in table 2. After all, the corresponding graphs $P_{an.}$ - H_A (fig.3) is constructed and the min $P_{an.}$ is calculated. From the min$P_{an.}$ the corresponding $H_{man} = H_A$-Z_A is resulted.

Table 2. The total annual cost of the project, $P_{an.}$, according to the dynamic programming method

H_A [m]	P_{an} [€]	H_A [m]	P_{an} [€]	H_A [m]	P_{an} [€]
83.74	44619	88.56	41907	93.24	42550
84.02	44073	88.79	41920	93.38	42571
84.73	43228	89.24	41911	94.06	42759
85.00	43096	89.50	41919	94.49	42905
85.25	42823	89.71	41922	95.04	43073
85.49	42709	90.02	41999	95.48	43187
85.79	42627	90.25	41978	95.99	43299
86.01	42465	90.50	42016	96.48	43583
86.25	42445	90.77	42095	97.06	43900
86.48	42321	91.04	42160	97.57	44126
86.76	42195	91.24	42175	97.89	44197
87.00	42139	91.48	42166	98.79	44640
87.23	42134	91.75	42227	99.78	45192
87.48	42040	92.01	42313	100.24	45412
87.75	42104	92.23	42327	100.86	45723
88.06	42075	92.49	42360	102.57	46712
88.25	41937	92.82	42500		

From table 2 and fig.3 it is concluded that the minimal cost of the network is min$P_{ET.}$ = 41907 €, produced for H_A = 88.56 m with corresponding H_{man} = 88.56 – 52.00 = 36.56 m

3.2. Calculation according to the proposed simplified method

3.2.1 The variance of the head of the pump station

The complete route presenting the minimal average gradient is A-39. The minimal and maximal head losses of all the pipes, which belong to this complete route, are calculated and the results are shown in table 3. From the table is resulted that minH_A=83.57 m and maxH_A=102.59 m

Table 3. The minimal and maximal head losses of all the pipes, which belong to the complete route presenting the minimal average gradient

Pipe	L_i [m]	Minimal losses		Maximal losses	
		maxD_i [mm]	min1.1J_i [%]	minD_i [mm]	Max1.1J_i [%]
39	225	160	0.249	110	1.607
38	240	225	0.167	140	1.774
37	240	280	0.120	160	1.966
36	125	315	0.114	200	1.103
K36	190	450	0.071	250	1.342
K28	185	500	0.090	315	0.909
K19	120	500	0.194	355	1.083
K10	200	500	0.309	400	0.951
K1	50	500	0.450	450	0.766
A	0		0		0

3.2.2 The optimal total cost of the network

The total minimal cost of the network, P_N, is obtained using the eq. 12. The calculation is made for pump heads H_A = 84.00 m till 102.00 m. The results are presented in table 4.

Table 4. The optimal cost of the network, P_N, according to the simplified method

H_A [m]	P_N [€]	H_A [m]	P_N [€]	H_A [m]	P_N [€]
84	373921	91	280821	98	242243
85	351530	92	273923	99	238086
86	334153	93	267585	100	234432
87	320064	94	261545	101	231059
88	308244	95	256300	102	227647
89	297889	96	251200		
90	288850	97	246614		

3.2.3 The optimal head of the pump station

The total annual cost of the project is calculated using the eq. 4. The results are presented in table 5. After that, the corresponding graphs P_{an}-$H_{A,}$ (fig.3) is constructed and the minP_{an} is calculated. From minP_{an} the corresponded H_{man}=H_A-Z_A is resulted.

Table 5. The total annual cost of the project, P_{an}, according to the proposed simplified method

H_A [m]	P_N [€]	H_A [m]	P_N [€]	H_A [m]	P_N [€]
84	43218	91	41465	98	43206
85	42385	92	41625	99	43541
86	41874	93	41821	100	43909
87	41573	94	42035	101	44295
88	41417	95	42301	102	44678
89	41355	96	42576		
90	41378	97	42884		

From table 5 and fig.3 it is resulted that the minimal cost of the network is min$P_{ET.}$ = 41296 €, produced for H_A = 89.05 m with corresponding H_{man} = 89.05 – 52.00 = 37.05 m

Note: The value H_A= 89.05 m have been resulted after the application of the above, for smaller subdivisions of H_A values which are not presented in table 5.

3.3 Selecting the economic pipe diameters

The economic diameters of the pipes are calculated according to the paragraphs 2.1.6 and 2.2.5. The results are presented in table 6.

Table 6. The optimal pipe diameters, D_N , according to the both methods

Pipe	Length [m]	D_N [mm] Dyn. Meth	D_N [mm] Simpl. Meth	Pipe	Length [m]	D_N [mm] Dyn. Meth	D_N [mm] Simpl. Meth
K_1	50	452.2	424	22	250	126.6	122
1	140	144.6	141	23	120	180.8	184
2	280	126.6	126	24	230	180.8	173
3	350	113.0	104	25	230	180.8	160
4	120	203.4	194	26	255	144.6	143
5	235	180.8	184	27	165	99.4	118
6	240	180.8	173	K_{28}	185	321.2	307
7	230	180.8	160	28	185	203.4	187
8	210	126.6	143	29	240	180.8	173
9	160	126.6	118	30	235	144.6	155
K_{10}	200	407.0	393	31	255	126.6	128
10	135	144.6	145	32	125	180.8	170
11	290	144.6	130	33	225	180.8	157
12	245	99.4	107	34	235	144.6	140
13	120	203.4	198	35	225	99.4	116
14	235	180.8	188	K_{36}	190	253.2	263
15	235	180.8	177	36	125	203.4	203
16	230	180.8	163	37	240	203.4	187
17	210	144.6	146	38	240	180.8	167
18	145	126.6	120	39	225	126.6	138
K_{19}	120	361.8	358	40	120	203.4	179
19	230	180.8	180	41	240	180.8	165
20	240	180.8	166	42	225	144.6	147
21	250	144.6	148	43	300	133.0	122

Note: The above calculated diameters from the simplified method necessarily must be rounded off to the nearest available higher commercial size.

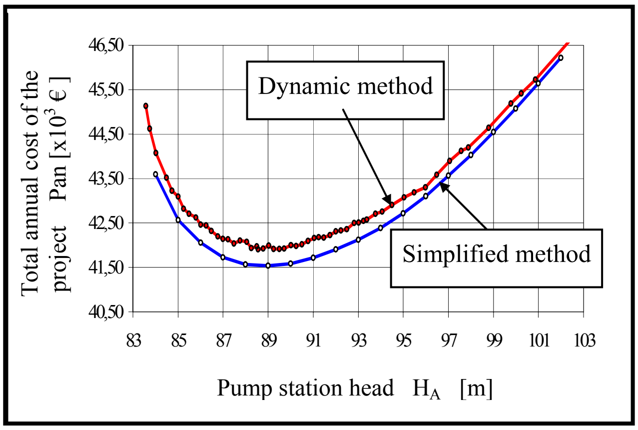

Figure 3. Variance of P_{an} per H_A.

4. CONCLUSIONS

The optimal head of the pump station that results, according to the dynamic programming method is H_{man} = 36.56 m, while according the proposed simplified method is H_{man} = 37.05 m. These two values differ only 1.34% while the corresponding difference in the total annual cost of the project is only 1.48 %.

The two optimization methods in fact conclude to the same result and therefore can be applied with no distinction in the studying of the branched hydraulic networks.

The required calculating procedure for the determination of the available piezometric losses is much shorter when using the simplified method than when using the dynamic programming optimization method. Therefore the proposed simplified method is indeed very simple to handle and for practical uses it requires only a handheld calculator and just a few numerical calculations.

References:

[1] Labye, Y., Etude *des procedés de calcul ayant pour but de rendre minimal le cout d'un reseau de distribution d'eau sous pression*, La Houille Blanche,5: 577-583, 1966.

[2] Theoharis, M.,*"Βελτιστοποίηση των αρδευτικών δικτύων. Επιλογή των οικονομικών διαμέτρων"*, Ph.D. Thesis, Dep. of Rural and Surveying Engin. A.U.TH., Salonika, 2004 (In Greek).

[3] Tzimopoulos, C. *"Γεωργική Υδραυλική"* Vol. II, 51-94, A.U.TH., Salonika, (In Greek),1982.

[4] Vamvakeridou-Liroudia L.,*"Δίκτυα υδρεύσεων - αρδεύσεων υπό πίεση. Επίλυση – βελτιστοποίη-ση"*, Athens, (In Greek),1990.

[5] Yakowitz, S., Dynamic programming applica-tions in water resources, *Journal Water Resources. Research,* Vol. 18, No 4, Aug. 1982, pp. 673- 696.

[6] Walters, G, A., and McKechnie S.J., Determi-ning the least-cost spanning network for a system of pipes by the use of dynamic programming, *Proc. 2nd Int. Conf. on Civil & Structural Eng. Computing,* London, Dec. 1985, pp. 237-243.

[7] Yang K.P., Liang T., and Wu I.P., Design of conduit systems with diverging branches, *Journal. Hydr. Div., ASCE,* Vo1.101, No 1, Jan. 1975, pp. 167-188.