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Reduced-Order Suboptimal Filter for Dynamic Systems with Multi-Sensor Environment

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Abstract:- This paper considers the problem of fusion of local Kalman filters for dynamic systems with multi-sensor environment**.** The filter performance in multi-sensor dynamic system is effected because of communication restriction, data association and estimation errors. A new reduced-order suboptimal filter is proposed for multi-sensor dynamic systems which reduces the computational cost for state estimation. The filtering algorithm includes two stages: the locally optimal Kalman estimates computed at the first stage are linearly fused at the second stage. The proposed filter has parallel structure and is suitable for parallel processing of measurements which can also help to minimize the computation time and produce real time state estimation. Example of systems containing different types of sensors, demonstrating the accuracy of the proposed filter, are given.

Key-Words:- Dynamic system, Kalman filter, Suboptimal filter, Data fusion, Minimum Mean Square Error, Multi-sensor

1. Introduction

Sensors are usually used in a system to acquire maximum information helpful for the proper operation and decision making. To get higher accuracy more and more techniques are being developed like increasing number of measuring devices (sensors), decreasing computational cost and data transmission to improve the system performance. Multi-sensor environment reduces the ambiguities and shortcomings of single sensor environments like uncertainty, less information gathering and location based restrictions. Multi-sensor environment provides us more accurate information about the system at the cost of computation, time, and data redundancy. The problem in multi-sensor environment is data association as redundant, diverse and even conflicting data combining in consistent and unbiased way requires exact data fusion.

The ultimate target of well designed system is large gain of information with reliable and real-time processing. The integration and fusion of information is used in design of high-accuracy control systems. Multiple sensors can be used in a system for different purpose like target tracking, guidance and surveillance, industrial and scientific applications. Sensors some times contain data, not needed, because of errors or limitations which can be controlled by the managed data fusion [1].

In [2], we derived fusion formula (FF) which represents an optimal mean square linear combination of local estimates with weights depending on cross-covariance of estimation errors. In this paper, we propose a new reduced-order suboptimal filter based on the FF. To get more accurate estimates, we fuse local sensor estimates and apply this filter to dynamic systems with multi-sensor environment. This is achieved via the use of a decomposition of the overall measurement vector into a set of subvectors of low

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dimension. The examples demonstrate the high-accuracy of the proposed filter.

This paper is organized as follows, in Section 2, dynamic system with multi-sensor environment is considered. In Section 3, new reduced-order suboptimal filter derived from the FF is proposed. In Section 4, the suboptimal filter is tested numerically and example shows high accuracy of proposed filter. Section 5 contains conclusion of all the discussion.

2. Dynamic Systems with Multi-Sensor Environment

Consider a continuous-time linear dynamic system

$$\dot{x}_t = F_t x_t + G_t v_t, \quad t \geq 0, \quad (1)$$

where $x_t \in \mathbf{R}^n$ is the state vector, $\mathbf{R}^n$ is an n-dimensional Euclidean space, $v_t \sim (0, Q_t)$ is the normal distributed white noise with zero mean and intensity Q_t. Suppose that the measurement system has N sensors,

$$\begin{aligned} &y_t^{(1)} = H_t^{(1)} x_t + w_t^{(1)}, \quad y_t^{(1)} \in \mathbf{R}^{m_1}, \\ &\quad \cdots \quad \cdots \\ &y_t^{(N)} = H_t^{(N)} x_t + w_t^{(N)}, \quad y_t^{(N)} \in \mathbf{R}^{m_N}, \end{aligned} \quad (2)$$

where $w_t^{(i)} \sim \left(0, R_t^{(i)}\right)$. We assume that the initial state $x_0 \sim N(\bar{x}_0, P_0)$, the system noise v_t, and the measurement noises $w_t^{(i)}$, $i = 1, \ldots, N$ are mutually uncorrelated.

The Kalman filter (KF) gives optimal, in mean square sense, estimate of the state x_t based on the *overall* measurements

$$Y_t = \left[y_t^{(1)^T} \quad \ldots \quad y_t^{(N)^T} \right]^T,$$

$$Y_t = \begin{bmatrix} H_t^{(1)} \\ \vdots \\ H_t^{(N)} \end{bmatrix} x_t + \begin{bmatrix} w_t^{(1)} \\ \vdots \\ w_t^{(N)} \end{bmatrix}. \quad (3)$$

However in case of limited computing and communication resources, the KF can not produce well-timed results as KF requires the overall measurements Y_t at each time instant **t** to calculate the current estimate. Therefore, in the case of limited computing and communication resources, the KF cannot produce well-timed results, especially for large dimensions of the overall measurement vector , $\dim(Y_t) = m_1 + \cdots + m_N$.

Next we show that the FF [2] may serves as an alternative to solve this filtering problem. The derivation of new suboptimal reduced-order filter is based on idea that the individual sensor measurements $y_t^{(1)}, \ldots, y_t^{(N)}$ can be processed simultaneously.

3. Reduced-Order Suboptimal Filter

3.1. Fusion Formula

Suppose we have $\mathbf{N}$ local estimates of a state vector x_t,

$$\hat{x}_t^{(1)}, \ldots, \hat{x}_t^{(N)}, \quad (4)$$

with associated local error covariance

$$P_t^{(ij)} = \mathrm{cov}\left(e_t^{(i)}, e_t^{(j)}\right), \quad e_t^{(i)} = x_t - \hat{x}_t^{(i)}, \quad i, j = 1, \ldots, N. \quad (5)$$

It is desired to find the *fusion* linear estimate of x_t,

$$\hat{x}_t^{FF} = \sum_{i=1}^{N} c_t^{(i)} \hat{x}_t^{(i)}, \quad \sum_{i=1}^{N} c_t^{(i)} = I_n, \quad (6)$$

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where I_n is the $n \times n$ unit matrix, and $c_t^{(1)}, \ldots, c_t^{(N)}$ are $n \times n$ weight matrices determined from the mean square criterion

$$E\left\|x_t - \hat{x}_t^{FF}\right\|^2 = E\left(\left\|x_t - \sum_{i=1}^{N} c_t^{(i)} \hat{x}_t^{(i)}\right\|^2\right) \to \min_{c_t^{(i)}} . \quad (7)$$

The following theorem completely defines the estimate $\hat{x}_t^{FF}$ and its error covariance

$$P_t^{FF} = \mathrm{cov}\left(e_t^{FF}, e_t^{FF}\right), \; e_t^{FF} = x_t - \hat{x}_t^{FF}. \quad (8)$$

Theorem[2]. *Let* $\hat{x}_t^{(1)}, \ldots, \hat{x}_t^{(N)}$ *be the local estimates (4) of an unknown state* x_t *. Then the weight matrices* $c_t^{(1)}, \ldots, c_t^{(N)}$ *are given by*

$$\sum_{i=1}^{N} c_t^{(i)} \left[P_t^{(ij)} - P_t^{(iN)}\right] = 0, \quad \sum_{i=1}^{N} c_t^{(i)} = I_n ,$$
$$j = 1, \ldots, N-1, \quad \sum_{i=1}^{N} c_t^{(i)} = I_n . \quad (9)$$

Corollary 1. If $\hat{x}_t^{(1)}, \ldots, \hat{x}_t^{(N)}$ *are unbiased estimates then the fusion estimate* $\hat{x}_t^{FF}$ *in (6) is unbiased.*
Corollary 2. The fusion error covariance P_t^{FF} *is given by*

$$P_t^{FF} = \sum_{i,j=1}^{N} c_t^{(i)} P_t^{(ij)} \left(c_t^{(j)}\right)^T . \quad (10)$$

In the particular case at $N = 2$, the FF (6), (9) reduces to the Bar-Shalom-Campo formula [3]:

$$\hat{x}_t^{FF} = c_t^{(1)} \hat{x}_t^{(1)} + c_t^{(2)} \hat{x}_t^{(2)} ,$$
$$c_t^{(1)} = \left[P_t^{(22)} - P_t^{(21)}\right]\left[P_t^{(11)} + P_t^{(22)} - P_t^{(12)} - P_t^{(21)}\right]^{-1} ,$$
$$c_t^{(2)} = \left[P_t^{(11)} - P_t^{(12)}\right]\left[P_t^{(11)} + P_t^{(22)} - P_t^{(12)} - P_t^{(21)}\right]^{-1} . (11)$$

If these two scalar estimates are uncorrelated, i.e., $P^{(12)} = 0$, then formulas (11) are reduced to the Millman formulas [4]:

$$c_t^{(1)} = P_t^{(22)} \left[P_t^{(11)} + P_t^{(22)}\right]^{-1} ,$$
$$c_t^{(2)} = P_t^{(11)} \left[P_t^{(11)} + P_t^{(22)}\right]^{-1} .$$

3.2. Reduced-Order Suboptimal Filter

According to (1) and (2), we have $\mathbf{N}$ dynamic subsystems ($i = 1, \ldots, N$) with the state vector $x_t \in \mathbf{R}^{\mathbf{n}}$ and the individual sensor $y_t^{(i)} \in \mathbf{R}^{\mathbf{m_i}}$:

$$\dot{x}_t = F_t x_t + G_t v_t , \; y_t^{(i)} = H_t^{(i)} x_t + w_t^{(i)} , \quad (12)$$

where the number of subsystem **i** is fixed.

Next, let us denote the estimate of the state x_t based on the sensor $y_t^{(i)}$ by $\hat{x}_t^{(i)}$. To find $\hat{x}_t^{(i)}$ we can apply the optimal KF to the subsystem (12) [4], [5]. We have

$$\dot{\hat{x}}_t^{(i)} = F_t \hat{x}_t^{(i)} + P_t^{(ii)} H_t^{(i)^T} R_t^{(i)^{-1}} \left[y_t^{(i)} - H_t^{(i)} \hat{x}_t^{(i)}\right],$$
$$\dot{P}_t^{(ii)} = F_t P_t^{(ii)} + P_t^{(ii)} F_t^T - P_t^{(ii)} H_t^{(i)^T} R_t^{(i)^{-1}} H_t^{(i)} P_t^{(ii)} \quad (13)$$
$$+ \; G_t Q_t G_t^T .$$

Thus we have $\mathbf{N}$ local Kalman estimates (LKEs)

$$\hat{x}_t^{(1)}, \quad \ldots, \quad \hat{x}_t^{(N)}$$

based on individual sensors measurements $y_t^{(1)}, \ldots, y_t^{(N)}$, respectively, and corresponding local error covariance (LECs)

$$P_t^{(11)}, \quad \ldots, \quad P_t^{(NN)} .$$

Then the new suboptimal estimate $\hat{x}_t^{sub}$ of the state x_t based on the overall sensors (3) is constructed by using the FF (6), i.e.

$$\hat{x}_t^{sub} = \sum_{i=1}^{N} c_t^{(i)} \hat{x}_t^{(i)}, \quad \sum_{i=1}^{N} c_t^{(i)} = I_n, \tag{14}$$

where the time-varying weight matrices $c_t^{(1)}, \ldots, c_t^{(N)}$ determined by the Eqs. (9), in which the LECs $P_t^{(ii)}$ determined by the KF (13) and the cross-covariances $P_t^{(ij)}$, where $i \neq j$, satisfy to the following differential equation:

$$\begin{aligned} \dot{P}_t^{(ij)} &= \left(F_t - P_t^{(ii)} H_t^{(i)^T} R_t^{(i)^{-1}} H_t^{(i)}\right) P_t^{(ij)} \\ &+ P_t^{(ij)} \left(F_t - P_t^{(jj)} H_t^{(j)^T} R_t^{(j)^T} H_t^{(j)}\right)^T \\ &+ G_t Q_t G_t^T, \quad i,j = 1,\ldots,N; \quad i \neq j. \end{aligned} \tag{15}$$

The relations (13)-(15) completely define *the reduced-order suboptimal filter (ROSF).*

Remark 1. The LKEs are separated for different types of sensors, i.e., each local estimate $\hat{x}_t^{(i)}$ is found independently of other estimates. Therefore, the LKEs can be calculated in parallel for various sensors (2). The proposed ROSF is also robust, since it can be corrected even if one of the parallel local estimate $\hat{x}_t^{(i)}$ diverges. In this case, the corresponding weight matrix $c_t^{(i)}$ in (14) will tend to zero, thereby indicating that the diverging local estimate $\hat{x}_t^{(i)}$ will be discarded in the weighting sum.

Remark 2. We may note, that the all covariances $P_t^{(ij)}$, and the weights $c_t^{(i)}$ may be pre-computed, since they do not depend on the measurements Y_t, but only on the noises statistics $Q_t, R_t^{(i)}$, and the system matrices $F_t, G_t, H_t^{(i)}$, which are the part of system and measurement model (1), (2). Thus, once the measurement schedule has been settled, the real-time implementation of the ROSF requires only the computation of the LKEs and the final suboptimal estimate $\hat{x}_t^{sub}$.

Remark 3. In case of one sensor ($N = 1$), the KF and ROSF coincide.

Remark 4. The ROSF can also be used for distributed data fusion system.

4. Example

4.1. Estimation of Damper Harmonic Oscillator Motion from Newton's Law

In this example, we verify the ROSF using the harmonic oscillator motion governed by the Newton's law [4]. Let the system model is

$$\ddot{z} = \frac{u_t}{m}, \tag{16}$$

where z_t be position, m be mass, and u_t be deterministic input (control). In the canonical form we have

$$\dot{x}_t = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x_t + \begin{bmatrix} 0 \\ 1/m \end{bmatrix} u_t + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v_t, \tag{17}$$

where $x_t = [z_t \quad \dot{z}_t]^T$, white noise $v_t \sim (0, q)$ has been added to compensate for modeling errors. Initial condition is $x_0 \sim N(\bar{x}_0, P_0)$, where

$$\bar{x}_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad P_0 = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}.$$

The measurement model containing of two sensors is given by

$$y_t = \begin{bmatrix} y_t^{(1)} \\ y_t^{(2)} \end{bmatrix} = \underbrace{\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}}_{\mathbf{H}} \cdot x_t + \underbrace{\begin{bmatrix} w_t^{(1)} \\ w_t^{(2)} \end{bmatrix}}_{\mathbf{w_t}}, \quad (18)$$

where $w_t = \begin{bmatrix} w_t^{(1)} & w_t^{(2)} \end{bmatrix}^T \sim (0, R_w)$ is normal distributed white noise with intensity matrix

$$R_w = \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix} = \begin{bmatrix} 0.02 & 0 \\ 0 & 0.01 \end{bmatrix}$$

Two filters for the system model (17), (18) are considered:

1. The KF based on the *overall* measurements (18),

$$y_t = Hx_t + w_t,$$

2. The ROSF,

$$\hat{x}_t^{sub} = c_t^{(1)} \hat{x}_t^{(1)} + c_t^{(2)} \hat{x}_t^{(2)},$$

where $\hat{x}_t^{(1)}$ and $\hat{x}_t^{(2)}$ are the LKEs (15) based on the first individual sensor

$$y_t^{(1)} = \begin{bmatrix} h_{11} & h_{12} \end{bmatrix} x_t + w_t^{(1)}$$

and the second one

$$y_t^{(2)} = \begin{bmatrix} h_{21} & h_{22} \end{bmatrix} x_t + w_t^{(2)},$$

respectively.

In this section, three measurement programs are illustrated and compared:

Program 1. Position $x_{1,t}$ is only measured by two different sensors. In this case

$$y_t = \begin{bmatrix} y_t^{(1)} \\ y_t^{(2)} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}}_{\mathbf{H}} \cdot x_t + \underbrace{\begin{bmatrix} w_t^{(1)} \\ w_t^{(2)} \end{bmatrix}}_{\mathbf{w_t}} \quad (19)$$

Program 2. Velocity $x_{2,t}$ is only measured by two different sensors. In this case

$$y_t = \begin{bmatrix} y_t^{(1)} \\ y_t^{(2)} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}}_{\mathbf{H}} \cdot x_t + \underbrace{\begin{bmatrix} w_t^{(1)} \\ w_t^{(2)} \end{bmatrix}}_{\mathbf{w_t}} \quad (20)$$

Program 3. Position and velocity are measured. Then

$$y_t = \begin{bmatrix} y_t^{(1)} \\ y_t^{(2)} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{\mathbf{H}} \cdot x_t + \underbrace{\begin{bmatrix} w_t^{(1)} \\ w_t^{(2)} \end{bmatrix}}_{\mathbf{w_t}} \quad (21)$$

The point of interest is the mean square error (MSE) in the estimate of state components,

$$P_t^{opt} = E\left[e_t^{opt} \left(e_t^{opt}\right)^T\right] = \begin{bmatrix} P_{11,t}^{opt} & P_{12,t}^{opt} \\ P_{12,t}^{opt} & P_{22,t}^{opt} \end{bmatrix},$$

$$P_t^{sub} = E\left[e_t^{sub} \left(e_t^{sub}\right)^T\right] = \begin{bmatrix} P_{11,t}^{sub} & P_{12,t}^{sub} \\ P_{12,t}^{sub} & P_{22,t}^{sub} \end{bmatrix},$$

where $e_t^{opt} = x_t - \hat{x}_t^{opt}$ is the estimation error of the state components under consideration at time **t** with optimal KF, and similarly for ROSF. These are the quantities shown in Fig.2 and 3. Fig. 2 shows the estimation result comparison of position measurement program 1 (19) for KF and ROSF with close estimation results.

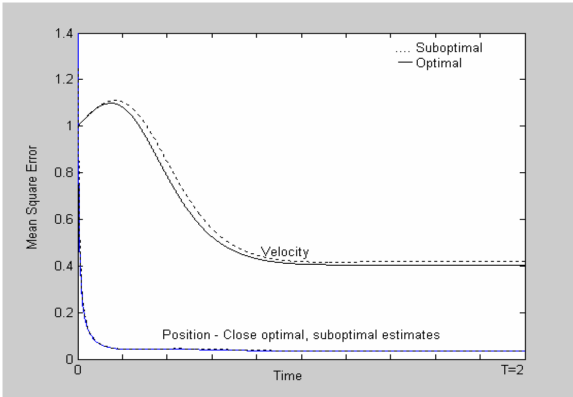

Fig. 1: MSE analysis of measurement Program 1

The system (17), (18) becomes unobservable for velocity measurement Program 2 (20) for both filters KF and ROSF. In this case the rank of the observable matrix Θ is equal 1, i.e.,

$$\mathrm{rank}(\Theta) = 1, \quad \Theta = \left[\mathrm{H}^{\mathrm{T}} \quad \mathrm{F}^{\mathrm{T}}\mathrm{H}^{\mathrm{T}}\right].$$

Fig.3 shows that optimal and suboptimal estimates are very close for Program 3 (21).

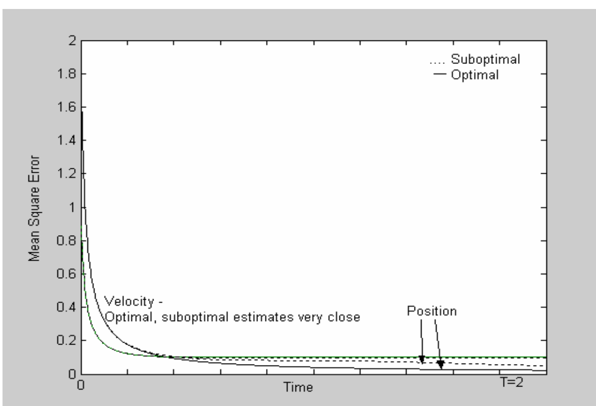

Figure 2: MSE analysis for measurement Program 3

5. Conclusion

In this paper, a new reduced order suboptimal filter supporting parallel processing of individual observations in multisensor environment is proposed. Results produced are very close to optimal filter. Proposed filter is well suited for real-time results in multi-sensor environment because of its parallel structure.

The proposed filter can be used in fields like surveillance, guidance, military and industry [1], [4], [6].

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