

# Parallel Particle Swarm Optimization on Power Transmission Network Planning Take Account of Congestion Management and Residual Capacity

YI-XIONG JIN<sup>1</sup>, HAO-ZHONG CHENG<sup>1</sup>, CHENG-MIN WANG<sup>1</sup>, JIAN-YONG YAN<sup>2</sup>, LI ZHANG<sup>2</sup>

1. Dept. Electrical Engineering, Shanghai Jiao Tong University  
1954 Hua Shan Road, Shanghai. Post code: 200030

2. Shanghai Urban Power Supply Company  
Shanghai 200080  
P.R.China

*Abstract:* - A novel model for transmission network planning which takes account of network configuration, generation planning, power flow control device installation, and residual capacity evaluation based on congestion management technology is presented in this paper. The character of this model is analyzed, and a parallel Particle Swarm Optimization algorithm is put forward according to the type of optimization variables. This method provides a flexible expansion technique for multifactor & multivariable transmission network planning. Simulation results demonstrate the model can improve the economical efficiency of planning scheme, and the parallel Particle Swarm Optimization can improve the velocity and convergence performance of planning.

*Key-Words:* - congestion management; power transmission network planning; parallel algorithm; Particle Swarm Optimization

## 1 Introduction

Any generator should supply power to any load in ideal power market so as to ensure the maximum freedom of the market. Therefore the function of power transmission network should guarantee the accomplishment of power transaction, and satisfy the demand of consumer. The main obstacle for fulfilling of this function is the congestion problem due to the limited transmission capacity of transmission line. Therefore, how to deal with the congestion becomes a widely concerned problem. Setting down the dispatch planning can be a short-term method, while the ultimate method is to build a reasonable and optimal transmission network. Congestion management model for dispatch planning has being discussed widely, such as community benefit maximization, electric power purchase maximization and cost of electric power purchase minimization [1],[2]. However, little Power Transmission Network Planning (PTNP) has cast an eye on congestion management problem in power market environment. The task of traditional PTNP is to minimize the investment and running expense of the network at given generation and load distributing under the promise of security. It doesn't consider the congestion control method.

Furthermore, traditional PTNP method hasn't considered the utilization ratio of the network. Therefore, many lines only have a little power flow

appeared in the planning network, this is especially distinct on those lines in existence. For example, the power flow on the line 3-7 is only 1/10 of its capacity in the planning network of paper [3]. This is waste of resources apparently.

A novel model for PTNP which takes account of network configuration, generation planning, power flow control device installation, and residual capacity evaluation based on congestion management technology is presented in this paper. However, the number and type of decision variables is expanded due to the consideration of many new factors. It baffles the optimization process.

Particle Swarm Optimization (PSO) method is an evolutionary computation technique developed by Kennedy and Eberhart in 1995<sup>[4], [5]</sup>. This method achieves efficient search by remembrance and feedback mechanisms. Because it imitates the behaviors of biome, it very fits for parallel calculation, and has perfect performance on large-scale optimization problems. By comparing PSO with some other bionic optimization methods such as Genetic Algorithm (GA), paper [6] and [7] have pointed out that PSO is much simpler and quicker. However, PSO is easy to converge to local optimum when the optimization scale is large. A simpler method to deal with this problem is parallel PSO (PPSO). PPSO can use several computers to deal with a problem, so it can accelerate the calculation

speed and get more careful search of feasible space. PPSO [8], [9] in existence only considered its intrinsic parallel character; less PPSO has been used on PTNP and hasn't considered the characters of PTNP either. A new PPSO which takes the character of PTNP into account is put forward.

## 2 Congestion Management Model

Congestion management is to adjust generation, load and some control device to make the power flow under the capacity constraint of transmission lines. Common model based on optimal power flow (OPF) is as follows.

$$\left. \begin{array}{l} \min(\max) f(u, x) \\ \text{s.t. } h(u, x) = 0 \\ g(u, x) \leq 0 \end{array} \right\} \quad (1)$$

The optimal object of model (1) often has following form:

a cost of electric power purchase minimization:

$$\min \sum_i C(P_i)$$

b community benefit maximization:

$$\max(\sum_j B(P_j) - \sum_i C(P_i))$$

in witch:  $h(u,x)=0$  is equation constraint, i.e. power flow constraint;  $g(u,x)\leq 0$  is inequation constraint, including generation constraint, capacity constraint and security constraint;  $C(P_i)$  and  $B(P_j)$  are quote curves of  $i^{\text{th}}$  generator and  $j^{\text{th}}$  load;  $P_i$  and  $P_j$  are power injection of  $i^{\text{th}}$  generator and  $j^{\text{th}}$  load.

The difference between "a" & "b" is that "b" considered the quote curve of load. Since the object of PTNP is to ensure enough power supply to every customer, the load value should adopt the fixed value forecasted by load forecast in PTNP. Therefore, model "a" should be used in PTNP model.

Apart from the OPF method, Flexible Alternating Current Transmission Systems (FACTS) also provide control measure for congestion management, such as Static Var Compensator (SVC), Thyristor Controlled Series Compensator (TCSC), and Unified Power Flow Controller (UPFC). Their basic function is to adjust the power flow to make the network under the constraint of line's capacity and increase the transmission capacity of system. This paper took TCSC for example and added it to PTNP model.

TCSC model is as follows

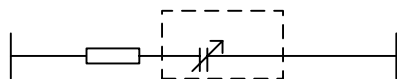


Fig.1 Model of TCSC

TCSC can be modeled as a capacitive reactance  $x'_{ij}$  in steady state. The result of capacitive reactance connecting with inductive reactance in series is that the reactance of the line is decreased. So TCSC can adjust the power flow. Usually the compensation ratio of TCSC is less than 60% for fear of series resonance. Inductive reactance of the line is follow

$$x_{ij}^{new} = x_{ij} + x'_{ij} = x_{ij}(1 - \alpha) \quad (2)$$

In which:  $\alpha$  is compensation ratio,  $0 \leq \alpha \leq 0.6$ .

TCSC investment is as follow

$$T = \sum_{i \in (m+m^0)} H_i q \quad (3)$$

In which:  $H_i$  is TCSC cost on  $i^{\text{th}}$  line;  $q$  is capital recovery factor;  $m$  is branch that can add line;  $m^0$  is branch that cannot add line.

## 3 Residual Capacity Model

For the purpose of make full use of network capacity, residual capacity can be added to objective function of PTNP as a index of utilization degree of network, economy evaluation of residual capacity is as follow

$$Q = \sum_{i \in (m+m^0)} A_i q \max((0.8 p_i^{\max} - p_i), 0)^2 / p_i^{\max} \quad (4)$$

In which:  $p_i$  is active power flow on  $i^{\text{th}}$  line;  $p_i^{\max}$  is capacity limit of  $i^{\text{th}}$  line;  $A_i$  is construction cost of  $i^{\text{th}}$  line;  $q$  is capital recovery factor.

## 4 TNEP Model

TNEP model Taking Account of Congestion Management and Residual Capacity is as follows

$$\min(\sum_{i \in N_G} C(P_i) + \sum_{i \in m+m^0} A_i (p_i^{\max} - p_i) / p_i^{\max} + \sum_{i \in m+m^0} H_i + \sum_{i \in m} A_i x_i q + V(\sum_{i \in m^0} I_i^2 r_i t + \sum_{i \in m} I_i^2 r_i t)) \quad (5)$$

$$\begin{cases} P_i = V_i \sum_{j \neq i} V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) \\ Q_i = V_i \sum_{j \neq i} V_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}) \end{cases} \quad i \in N \quad (6)$$

$$\begin{cases} P_i^{\min} \leq P_i \leq P_i^{\max} \\ Q_i^{\min} \leq Q_i \leq Q_i^{\max} \end{cases} \quad i \in N_G \quad (7)$$

$$\begin{cases} -p_i^{\min} \leq p_i \leq p_i^{\max} \\ -q_i^{\min} \leq q_i \leq q_i^{\max} \end{cases} \quad i \in m + m^0 \quad (8)$$

$$0 < x_i < x_i^{\max}, x \in Z, i \in m \quad (9)$$

In which:  $x_i$  is circuits added on  $i^{\text{th}}$  right-of-way;  $A_i$  is investment of new added circuit;  $t$  is equivalent runtime of system;  $V$  is cost of loss per kilowatt-hour;

$I_i$  is current on circuit  $I$ ;  $r_i$  is resistance of circuit  $i$ ;  $m$  is right-of-way number allowed to add line;  $m^0$  is right-of-way number in existence;  $N_G$  is generation

node;  $q$  is capital recovery factor,  $q = \frac{s(1+s)^y}{(1+s)^y - 1}$ ,  $s$

is interest rate,  $y$  is life-span of equipment;  $P_i$  and  $Q_i$  is active and reactive power injection of  $i^{\text{th}}$  node;  $P_i^{\min}$ ,  $Q_i^{\min}$ ,  $P_i^{\max}$  and  $Q_i^{\max}$  is maximal & minimal active & reactive power generate of  $i^{\text{th}}$  generation respectively.  $N$  is all node of the network;  $p_i$  and  $q_i$  is active and reactive power flow on  $i^{\text{th}}$  line;  $p_i^{\max}$  and  $q_i^{\max}$  is maximal active and reactive power can flow on  $i^{\text{th}}$  line; equation (6) is power flow constraint, which is represented by power flow calculation; inequation (7) is node injection constraint; inequation (8) is overload constraint, which can be represented by penalty factor added to object function for overload of circuit; inequation (9) is integer constraint and upper & lower bound constraint of circuit number can be added on each branch.

### 5 New Parallel PSO

PTNP is a multi-dimension, multimodal optimization problem. Traditional optimization methods, such as linear programming and heuristic algorithm, are difficult to solve this problem. PSO shows great advantage in solving this problem. However, “two steps forward & one step back” phenomenon occurs [10]. A simple example to illustrate this concept follows. Consider a three-dimensional vector  $X(x_1, x_2, x_3)$ , and the fitness function is:

$$\min f(x) = x_1^2 + x_2^2 + x_3^2 \quad (10)$$

Its optimal point is (0, 0, 0). If the current best point searched is (10, 0, 10), its fitness value is 200, if the next iteration find point (5, 5, 5), its fitness value is 75. PSO thinks the latter point is better than the former. It is caused by the fact that the fitness function is computed only after all the components in the vector have been updated to their new values. This means an improvement in two components (two steps forward) will overrule a potentially good value for a single component (one step back). In order to overcome this flaw, direct method is to optimize the three variables separately, that is to say, takes one variable as optimal variable, other variables as environment variable. However, if the three variables are associated, how to harmonize their relations is a critical problem. This phenomenon is more serious in high dimension optimization problem such as PTNP. To overcome the “two steps forward & one step back” flaw, parallel optimization model shown in fig.2 is put forward for PSO.

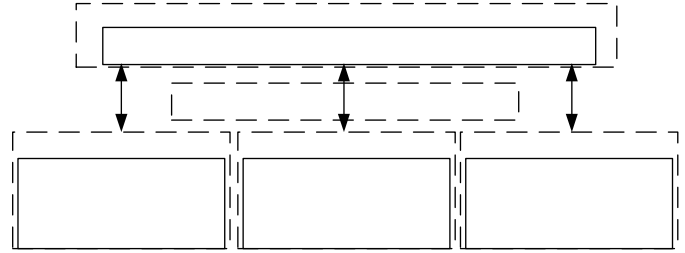


Fig.2 server-client model for parallel PSO

The Optimization process is as follows:

#### A. Particle initialization:

The initialization of PSO is usually random. However, we find out that many (about 1/2 in an 18-bus system [3]) networks represented by the initial particles is unconnected; this makes many initial particles wasted. So, for the purpose of making the initial network connected, random topology-tree search is adopted: start from a random bus, randomly select a branch, add random number of circuit on this branch, perform the former random search process from the end bus of the branch, and so on, until all the buses having been searched. All initial particles formed by this random topology-tree search are connected.

#### B. The optimization process of server

1) Initialize  $f_s, f_1, f_2, f_3$  and  $k$  to 0,  $f_s$  is new optimum found symbol of main PSO,  $f_1, f_2, f_3$  are new sub-optima found symbol of every sub-PSO respectively;  $k$  is iteration times; initialize particle position and velocity. network configuration:  $X_i = (x_{i1}, \dots, x_{im})$ ,  $V_i^X = (v_{i1}^X, \dots, v_{im}^X)$ , TCSC installation:

$$T_i = (\alpha_{i1}, \dots, \alpha_{i(m+m^0)})$$

$$V_i^T = (v_{i1}^T, \dots, v_{i(m+m^0)}^T)$$

generation planning:  $P_i = (P_{i1}, \dots, P_{iN_G})$ ,

$$V_i^P = (v_{i1}^P, \dots, v_{iN_G}^P)$$

$i = 1, 2, \dots, n$ ,  $n$  is particle's population number;  $m$  is number of branch that can add line; send initial particle position and velocity of different type to corresponding sub-PSO client;

- 2) Power flow calculation ;
- 3) Calculate the total investment, net loss, cost of power purchase, economic index of residual capacity and penalty value for overload according to power flow results, and then fitness value.
- 4) Update gbest and pbest of every particle. If new gbest is found, make  $k=0, f_s=1$ ; else make  $k=k+1$ ;
- 5) If  $k$  is bigger than given iteration times, go to step 7); else continue;
- 6) If any new sub-optimum found symbol  $f_i$  is 1 ( $i=1,2,3$ ), then read corresponding new sub-optimum to the same position of 5~8 particle who have the worst fitness value and set  $f_i$  to 0; update the other particles by normal PSO updating strategy; go to step 2);

7) Send end message to every client, and output optimization result.

**C. The communication process of server**

8) If  $f_s$  is 1, send connection-request message to every client; if confirm message is received from client, then transmit gbest particle to every sub-PSO as environment variables, and set  $f_s$  to 0;

9) If connection-request message of client is received, send confirm message to client, receive sub-optimum from corresponding client, and set corresponding  $f_i$  to 1;

10) Go to step 8);

**D. The optimization process of  $i^{th}$  client**

11) Set  $f_i$  to 0; read environment variables.

12) Power flow calculation;

13) Calculate the network investment and penalty value for overload according to power flow results in sub-PSO of network optimization, calculate the cost for electric power purchase in sub-PSO of generation planning, and calculate the TCSC cost in sub-PSO of TCSC optimization as fitness value respectively;

14) Update gbest and pbest of every particle. If new sub-optimum is found, make corresponding  $f_i = 1$ ;

15) If end message is received, stop calculation; else continue;

16) Update particles' position and velocity by normal PSO updating strategy; go to step 12);

**E. The communication process of client**

17) If  $f_i$  ( $i=1, 2, 3$ ) is 1, send connection-request message to server; if confirm message is received from server, then transmit sub-optimum planning scheme to server, and set corresponding  $f_i$  to 0;

18) If connection-request message of server is received, send confirm message to server, and receive planning scheme of server as environment variables;

19) Go to step 17)

As can be seen from the above optimization process, PPSO based PTNP take advantage of the relative independence of different variable type, and also give attention to the interrelation of different variables by the server's optimization and assignment. "read corresponding new sub-optimum to the same position of 5~8 particle who have the worst fitness value" in step 6) instead of to the particle who have the best fitness value can not only provide optimal search direction for main PSO and prevent the "two step forward & one step back" phenomenon, but also prevent the mode loss of optimization. Since the sub-PSO provide optimal search direction, population size needed of main PSO is decreased, so quicker calculation speed and less computer memory consumption is achieved

**6 Numerical Simulation**

Adopting four P4 1.7G processor (a server and three client) in 100M ether LAN, 256M DDR, VC6.0 programming, Socket communication.

**6.1. IEEE Garver-6 system**

Generation data of IEEE Garver-6 is shown in tab.1. Other data is detailed in paper [11]. Right-of-ways numbers allowed adding line is 15. Upper bound of line on each right-of-way is 4. Line cost is \$0.017 million /km. DPSO population size is set to 30, electric price is \$0.02/Kwh, cost of TCSC is \$25,000/Mvar, interesting rate is 0.02, and life-span of equipment is 15 year.

**Tab.1 generation data of IEEE Garver-6**

bus	Upper limit of generator(MW)	Lower limit of generator (MW)	quote(\$/Mwh)
3	200	0	8
6	600	0	10

Compared with the result that doesn't consider congestion management and residual capacity, this result decreased the construction cost, residual capacity of network, and cost for electric power purchase at the same time. The whole economic performance of the network is increased. The two results are shown is tab.2.

**Tab.2 compression of optimization result**

	Without congestion management and residual capacity consideration	With congestion management and residual capacity consideration
Adding lines	2-6(4), 3-5, 4-6(2)	2--6(3), 3-5, 4-6(2)
Cost of line's construction per year /\$thousand	2334.77	1984.55
Compensation degree of TCSC	No	2-6(0.142),4-6(0.24)
Cost of TCSC	0	59.535
Network loss per year /\$thousand	446.79	492.42
index of residual capacity /\$thousand	452.03	375.98
Cost for electric purchase /\$thousand	25800	25200
whole economic performance /\$thousand	29033.59	28112.485

PPSO and Basic PSO convergence curves are shown is fig.3 and fig.4. As can be seen from fig.3, basic PSO has good convergence performance at the beginning of the iteration, however, its convergence velocity is decreased with the calculation time increasing, and doesn't meet the stop rule (gbest doesn't change is 50 times iteration). Fig.4 is the convergence curve after 670 times iteration of basic PSO. It calculation didn't stop until 720 times iteration. While the convergence performance of this PPSO is better and quicker because the sub-PSO can prevent the "two step forward & on step back" phenomenon, and provide pertinent search direction for main PSO when main PSO run into convergent

dilemma. PPSO get optimal result only in 219 times iteration.

Simulation result considering A: generation planning only, B: A & TCSC configuration and C: B & Residual Capacity respectively are shown in tab.3. As can be seen from the difference of scheme B and C in tab.3, residual capacity index can affect the TCSC configuration to decrease the residual capacity of the network, increasing the rationality and flexibility of the power flow. The whole economic performance of scheme C is the best one.

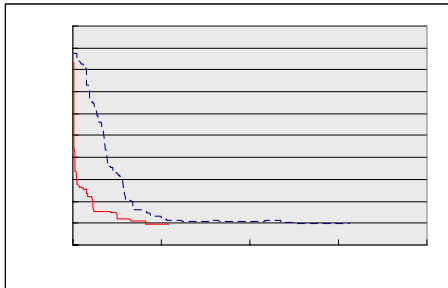


Fig.3 convergence curves of parallel & basic PSO

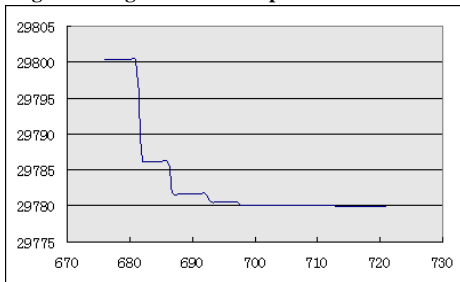


Fig.4 tail of convergence curve of basic PSO

Tab.3 optimal results of three different control schemes

Scheme	A	B	C
Adding lines	2-6(3), 3-5(2), 4-6(2)	2--6(3), 3-5, 4-6(2)	2--6(3), 3-5, 4-6(2)
Cost of line's construction per year /\$thousand	2218.03	1984.55	1984.55
Compensation degree of TCSC	无	3-5(0.6)	2-6(0.142), 4-6(0.24)
Cost of TCSC	无	63.1	59.535
Network loss per year /\$thousand	362.39	494.37	492.42
index of residual capacity /\$thousand	445.31	437.17	375.98
Cost for electric purchase /\$thousand	25200	25200	25200
whole economic performance /\$thousand	28225.73	28179.19	28112.485

\$30,000/Mvar, interesting rate is 0.02, and life-span of equipment is 15 year. Optimal result is shown is tab.5. Convergence curves are shown is fig.5. Optimal results of three schemes are shown is tab.6. As can be seen from fig.5, convergence speed of both the basic PSO and PPSO are quick at the beginning of iteration, while the convergence velocity of basic PSO decreased with the calculation time increasing as that of the Garver-6 system. PPSO get optimal result only in 483 times iteration. It is obvious that the PPSO is an effective extension of basic PSO, and exhibit good convergence performance especially in high-dimension optimization problem.

Tab.4 generation data of Southern part of the Brazilian interconnected 46-bus

bus	Upper limit of generator(MW)	Lower limit of generator (MW)	quote(\$/Mwh)
14	1000	0	10
17	1500	0	11
19	900	0	10
27	100	0	12
28	850	0	11
31	400	0	12
32	600	0	10
34	300	0	11
37	250	0	12
39	150	0	10
46	700	0	11

Tab.5 compression of optimization result

	Without congestion management and residual capacity consideration	With congestion management and residual capacity consideration
Adding lines	20-21, 42-43(2), 5-11(2), 19-25, 31-32, 28-30, 26-29(2), 46-11(2), 24-25(2), 29-30(2), 40-41, 2-3, 5-6, 9-10, 14-15	20-21,42-43,5-11 46-6,19-25,31-32 28-30,26-29(3),41-43(2) 24-25(2), 29-30(2), 40-41 2--3,5--11,9--10
Cost of line's construction per year /\$thousand	15178.16	14246.86
Compensation degree of TCSC	No	19-25(0.6), 42-43(0.3), 5-6(0.57)
Cost of TCSC	No	199.401
Network loss per year /\$thousand	775.91	825.8
index of residual capacity /\$thousand	33943.25	29442.9
Cost for electric purchase /\$thousand	29000.8	27932.14
whole economic performance /\$thousand	78898.12	72647.1

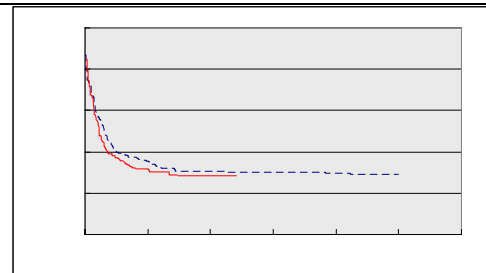


Fig.5 convergence curves of parallel & basic PSO

## 6.2. Southern part of the Brazilian interconnected 46-bus network

Generation data of southern part of the Brazilian interconnected 46-bus is shown in tab.4. Other data is detailed in paper [11]. Right-of-ways numbers allowed adding line is 79. DPSO Population size is set to 60. Electric price is \$0.03/Kwh, cost of TCSC is

\$thousand  
75000  
70000  
65000  
60000  
55000  
50000  
45000  
40000

**Tab.6 optimal results of three different control schemes**

Scheme	A	B	C
Adding lines	20-21, 42-43(2), 5-6, 46-6, 19-25, 31-32, 28-30(3), 26-29(2), 24-25(2), 29-30(2), 40-41, 2-3, 46-10, 5-11, 15-16	20-21,42-43(2),5-6(2) 46-6,19-25,31-32 28-30,26-29(3),24-25(2) 29-30(2),40-41,2-3 5-11,9-10,14-15	20-21,42-43,5-6 46-6,19-25,31-32 28-30,26-29(3),41-43(2) 24-25(2), 29-30(2), 40-41 2--3,5--11,9--10
Cost of line's construction per year /\$thousand	14899.16	14631.58	14246.86
Compensation degree of TCSC	无	42-43(0.6),29-30(0.21),5-6(0.21)	42-43(0.3), 5-6(0.57), 19-25(0.6), 29-30(0.44)
Cost of TCSC	无	168.47	199.401
Network loss per year /\$thousand	800.91	879.13	825.8
index of residual capacity /\$thousand	32011.87	31125.08	29442.9
Cost for electric purchase /\$thousand	27818.4	27987.14	27932.14
whole economic performance /\$thousand	75530.34	74791.4	72647.1

## 7 Conclusion

A novel model for PTNP which takes account of network configuration, generation planning, power flow control device installation, and residual capacity evaluation based on congestion management technology is presented in this paper. The character of this model and “two steps forward & one step back” phenomenon is also analyzed. A parallel PSP algorithm is put forward according to the type of optimization variables based on the character of this new model. This method provides a flexible expansion technique for multifactor & multivariable transmission network planning. Simulation results demonstrate the model can improve the economical efficiency of planning scheme, and the new PPSO can improve the velocity and convergence performance of planning.

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