On Nonlinear Adaptive Control of a Handling Crane

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Abstract: - This paper presents some nonlinear adaptive control techniques for the control of a handling crane. The nonlinear model of the handling crane is widely analyzed and an exactly linearizing feedback control law is designed. This nonlinear control law, aggregated with our nonlinear system achieves input-output linearization. When some parameters of the system are imprecisely known or unknown, an adaptive control strategy is designed. Several computer simulations are included to demonstrate some theoretical aspects and the performances of the controlled system.

Key-Words: - Nonlinear systems, Adaptive control, Linearizing control, Handling crane

1 Introduction

In the last years, significant advances have been made in the development of ideas such as feedback linearizing techniques. The problem of exact linearization via feedback and diffeomorphism consists in transforming a nonlinear system into a linear one using a state feedback and a coordinate transformation of the systems state [5], [7]. Several application of the exactly linearization technique are reported for the control of the electric motors, chemical and biochemical reactors, robotic manipulators and so on [2], [3], [4], [5].

In this paper, by using the feedback linearizing technique, a nonlinear control law for a handling crane is obtained. The control goal is the regulation of the position of the load of a handling crane. This nonlinear control method provides an alternative solution to existing classical linear methods for the control of handling cranes. For the implementation of the nonlinear control law we suppose that all states are measurable (otherwise a state observer can be used in order to estimate the state variables). In many practical situations, some handling crane parameters (such as load, height) are unknown; therefore an adaptive control strategy is required in order to maintain the performances of the controlled system (for a general point of view regarding the adaptive control theory see [1], [8]).

In this paper, an adaptive control law based on reference model for the exactly linearized model is designed.

The paper is organized as follows: in Section 2, the mathematical theoretical fundaments of the exact linearization technique are briefly presented, while the Section 3 deals with the nonlinear model of a handling crane. Section 4 presents the input – output linearization technique for the handling crane, used in order to obtain the nonlinear control law; also the adaptive control law is designed. Some computer simulation results for different parameters of the controlled system are presented. Finally, in Section 5 some concluding remarks are collected.

2 Theoretical fundaments of the exact linearization technique

The nonlinear system that we consider is described in state space by equations of the following kind:

$$\dot{x} = f(x) + \sum_{i=1}^{m} g_i(x)u_i$$
$$y_j = h_j(x) \qquad j = 1...m \tag{1}$$

in which $f(x), g_1(x), g_2(x),...., g_m(x)$ are smooth vector fields.

The exact linearization via feedback and diffeomorphism consists in transforming the nonlinear system (1) into a linear one using a state feedback and a coordinate transformation of the systems state. We do not develop the details of input-output linearization techniques (for details see [7]) but directly show the application on the handling crane. This can be done introducing the Lie derivative of a function $h(x): R^n \to R$ along a vector field $f(x) = [f_1(x),...f_n(x)]$

$$L_f h(x) = \sum_{i=1}^{n} \frac{\partial h(x)}{\partial x_i} f_i(x) \tag{2}$$

Definition 2.1. [7]. A multivariable nonlinear system of the form (1) has a relative degree $\{r_1,...,r_m\}$ at a point x^0 if:

1) $L_{g_j} L_f^k h_i(x) = 0$ (3)

for all $1 \le j \le m$, for all $1 \le i \le m$ for all $k \le r_i - 1$, and for x in a neighborhood of x^0,

2) the $m \times m$ matrix

$$A(x) = \begin{bmatrix} L_{g_1} L_f^{r_1-1} h_2(x) & . & . & L_{g_m} L_f^{r_1-1} h_1(x) \\ L_{g_1} L_f^{r_2-1} h_2(x) & . & . & L_{g_m} L_f^{r_2-1} h_2(x) \\ . & . & . & . \\ L_{g_1} L_f^{r_m-1} h_m(x) & . & . & L_{g_m} L_f^{r_m-1} h_m(x) \end{bmatrix}$$ (4)

is nonsingular at $x = x^0$.

Theorem 2.2. [7]. Let be the nonlinear system of the form (1). Suppose the matrix $g(x^0)$ has rank m. Then, the State Space Exact Linearization Problem is solvable if and only if:
1) for each $0 \le i \le n-1$, the distribution G_i has constant dimension near x^0;
2) the distribution G_{n-1} has dimension n;
3) for each $0 \le i \le n-2$, the distribution G_i is involutive.

3 The nonlinear model of the handling crane

The structure of the handling crane is presented in Fig. 1. The model of the handling crane can be obtained using the fundamental law of the dynamics for the directions Ox and Ox_1. The dynamical model consists of two nonlinear differential equations, both of order two [4]:

$$\ddot{\theta} = \frac{h\left[-bm_2\dot{x}\cos\theta + c\dot{\theta}\left(m_1 + m_2 - m_2\cos^2\theta\right)\right]}{h^2 m_2^2 \cos^2\theta - (m_1 + m_2)(J + m_2 h^2)} +$$

$$+ \frac{h\left[hm_2^2\dot{\theta}^2\sin\theta\cos\theta + hm_2(F\cos\theta + g\sin\theta(m_1 + m_2))\right]}{h^2 m_2^2 \cos^2\theta - (m_1 + m_2)(J + m_2 h^2)}$$

(5)

$$\ddot{x} = \frac{-h^2 m_2 \cos\theta(c\dot{\theta} + gm_2\sin\theta)}{h^2 m_2^2 \cos^2\theta - (m_1 + m_2)(J + m_2 h^2)} -$$

$$- \frac{(J + m_2 h^2)(F - b\dot{x} - c\dot{\theta}\cos\theta + hm_2\dot{\theta}^2\sin\theta)}{h^2 m_2^2 \cos^2\theta - (m_1 + m_2)(J + m_2 h^2)}$$
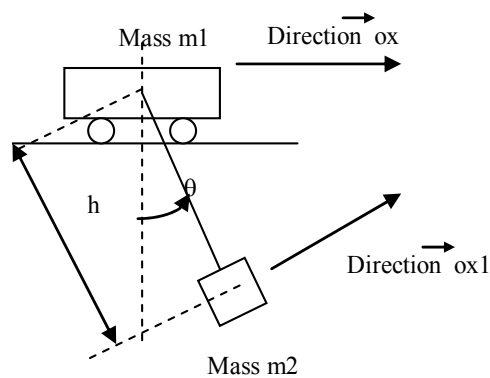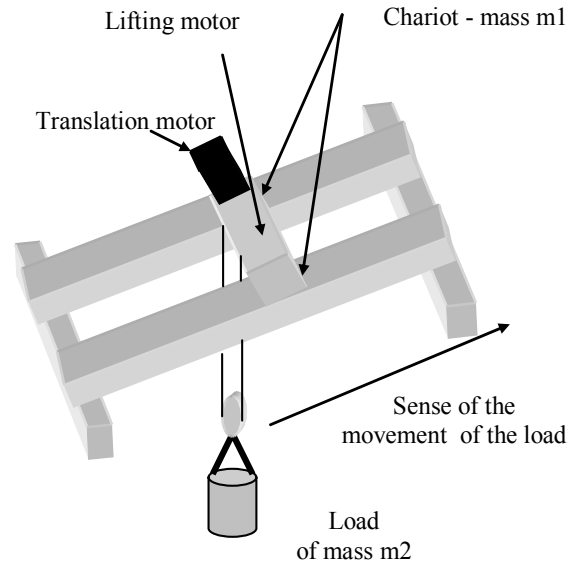

Fig. 1. The schema of the handling crane

The control purpose is the regulation of the output:

$$y = x + h\sin\theta$$ (6)

In equations (5), (6) we have: F – the force developed by the translation motor, m_1 – the mass of chariot, m_2 – the mass of the load, b – the viscosity friction coefficient for the chariot, c – the friction coefficient opposing to the oscillation of the load, h – the height, g – the acceleration due to the gravity, J – the inertia moment, θ - the angular position, x – the position of the chariot (direction Ox), y – the position of the load (direction Ox_1).

The chariot is displaced using an induction motor and a reduction gear. The force F will be considered the input variable for the nonlinear model (5). Choosing, as state variables,

$$x^T = [x_1, x_2, x_3, x_4] = [\theta(t), \dot{\theta}(t), x(t), \dot{x}(t)]$$ (7)

the mathematical model (5), (6) is described in the state space by the following equations

$$\dot{x}(t) = f(x) + g(x)u \tag{8}$$

$$y = x + h\sin\theta \tag{9}$$

where

$$f(x) = \begin{bmatrix} x_2 \\ \dfrac{\Omega_1}{h^2 m_2^2 \cos^2\theta - (m_1 + m_2)(J + m_2 h^2)} \\ x_4 \\ \dfrac{\Omega_2}{h^2 m_2^2 \cos^2\theta - (m_1 + m_2)(J + m_2 h^2)} \end{bmatrix}$$

$$\tag{10}$$

$$g(x) = \begin{bmatrix} 0 \\ \dfrac{hm_2 \cos x_1}{h^2 m_2^2 \cos^2\theta - (m_1 + m_2)(J + m_2 h^2)} \\ 0 \\ \dfrac{-2J}{h^2 m_2^2 \cos^2\theta - (m_1 + m_2)(J + m_2 h^2)} \end{bmatrix}$$

$$u = F$$

with:

$$\Omega_1 = a_1 x_4 \cos x_1 + a_2 x_2 - a_3 x_2 \cos^2 x_1 + \\ + a_4 x_2^2 \sin x_1 \cos x_1 + a_5 \sin x_1$$

$$\Omega_2 = -a_6 x_2 \cos x_1 - a_7 \sin x_1 \cos x_1 - a_8 x_2^2 \sin x_1 + \\ + a_9 x_4 + a_{10} x_2 \cos x_1$$

$$a_1 = hbm_2; \, a_2 = hc(m_1 + m_2); \, a_3 = hcm_2;$$

$$a_4 = m_2^2 h^2; \, a_5 = hm_2 g(m_1 + m_2); \, a_6 = m_2 h^2 c;$$

$$a_7 = gh^2 m_2^2; \, a_8 = 2Jhm_2; \, a_9 = 2Jb; \, a_{10} = 2Jc \tag{11}$$

4 Nonlinear adaptive control laws and simulation results

4.1 Design of linearizing and nonlinear adaptive control laws

The problem of exact linearization via feedback and diffeomorphism consists in transforming a nonlinear system (8), (9) in a linear one using a state feedback and a coordinate transformation of the systems state. We do not develop the details of input-output linearization techniques (see [5], [7]) but directly show the application on the handling crane. The quantities which will be controlled are differentiated with respect to time until the input u appears and the derivatives of the state variables are eliminated using the state-space representation (8), (9).

We consider like output the variable y from (9):

$$y = h(x) = x + h\sin\theta$$

Using the Lie derivatives we have

$$L_g L_f^1 h(x) = -\frac{2J(1 + \sin^2 x_1)}{h^2 m_2^2 \cos^2\theta - (m_1 + m_2)(J + m_2 h^2)} \neq 0 \tag{12}$$

$$L_f^2 h(x) = \frac{a_9 x_4 - a_6 x_2 \cos x_1 - a_7 \sin x_1 \cos x_1 - a_8 x_2^2 \sin x_1}{h^2 m_2^2 \cos^2\theta - (m_1 + m_2)(J + m_2 h^2)} + \\ \frac{a_{10} x_2 \cos x_1 - a_1 h x_4 \cos^2 x_1 + a_2 h x_2 \cos x_1 - a_3 h x_2 \cos^3 x_1}{h^2 m_2^2 \cos^2\theta - (m_1 + m_2)(J + m_2 h^2)} + \\ \frac{a_4 h x_2^2 \sin x_1 \cos^2 x_1 + a_5 h \sin x_1 \cos x_1}{h^2 m_2^2 \cos^2\theta - (m_1 + m_2)(J + m_2 h^2)} - h x_2^2 \sin x_1 \tag{13}$$

Thus, we see that the system has relative degree $r = 2$. In this situation, the state feedback:

$$u = \frac{1}{L_g L_f^1 h(x)} (-L_f^2 h(x) + v) \tag{14}$$

transforms the system (8), (9) into a system whose input-output behavior is identical to that of a double integrator

$$H(s) = \frac{1}{s^2} \tag{15}$$

On the linear system thus obtained one impose a feedback control of the form:

$$v = c_0(y_{ref} - y) - c_1 \dot{y} \tag{16}$$

then, the obtained system has a linear input-output behavior, described by the following transfer function

$$H(s) = \frac{c_0}{s^2 + c_1 s + c_0}$$

The implementation of the obtained nonlinear control law (14), (16) is hampered if some of handling crane parameters are unknown or variable in time (slowly). In order to overcome this disadvantage, an adaptive control law, based on reference model approach, can be designed.

We consider that the nonlinear process is described by the state equations

$$\dot{x}(t) = f(x, c, t, \theta)$$

where $x(t)$ is the state vector, c is the vector of tuning parameters and θ is the vector of unknown parameters of the process (and eventually external disturbances).

The adaptation criterion consists in the minimisation of a functional Q_t, with the derivative of the form

$$\frac{dQ_t}{dt} = \Psi(x(t), c(t), t)$$

where Ψ is a derivable function with respect to the components of vector c.

For the synthesis of the adaptive algorithm, the method of the gradient is used, choosing the following criterion [1], [8]:

$$Q_t = \frac{1}{2} e^T H e \qquad (17)$$

where $e(t) = x(t) - x_m(t)$ and matrix $H > 0$ is the solution of the Lyapunov equation

$$H A_m + A_m^T H = -G \qquad (18)$$

where G is a symmetric positive definite matrix and A_m is reference model matrix. The adaptive algorithm will be:

$$\frac{dc}{dt} = -D g_c \Psi(x, c, t) \qquad (19)$$

where D is a positive definite matrix and $g_c^T \Psi = \left[\ldots \frac{\partial \Psi}{\partial c_i} \ldots \right]$ is the gradient of Ψ in rapport with c_i parameters.

The adaptation law for the controller parameters is of the form

$$\frac{dc_0}{dt} = -\gamma_0 [g_0(y - y_m) + g_1(\dot{y} - \dot{y}_m)] y$$
$$\frac{dc_1}{dt} = -\gamma_1 [g_0(y - y_m) + g_1(\dot{y} - \dot{y}_m)] \dot{y} \qquad (20)$$

where $\gamma_0, \gamma_1, g_0, g_1$ are design parameters [x], [x].

For the closed loop model (14), (16) of the controlled handling crane, we choose as a reference model a transfer function of order two associated with the Integral of Time – Multiplied Absolute Value of Error (ITAE) criterion (see [6]). The structure of the adaptive closed loop system is presented in Fig. 2.
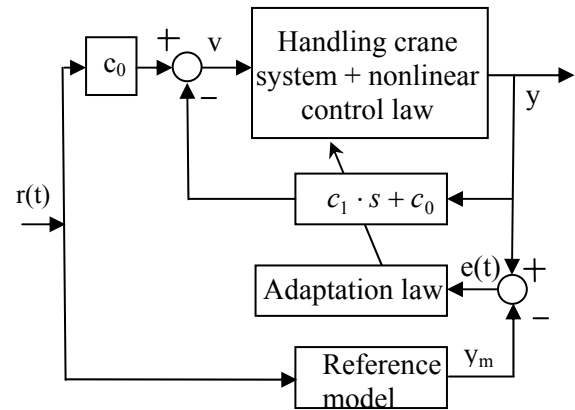

Fig. 2. Structure of the adaptive controlled system

4.2 Simulation results
In order to test the performances of the obtained nonlinear adaptive controller, extensive computer simulations were performed in Matlab/Simulink [9], using the following handling crane parameters:

$$m_1 = 200 kg, \ m_2 \in [100, 10000] kg, \ g = 9.81 m/s^2$$
$$b = 1000 Ns/m, \ c = 10 Ns/rad, \ h \in [1, 10] m$$

Three simulation cases were considered:

(i) In first simulation case, the control goal was the regulation of the position of the load to a prescribed set point, using the exactly linearizing controller (14), (16). In fact, the profile of the reference comprises some step changes (first 4 m and for the last half of the simulation time 8 m).

The design parameters are computed using a pole-placement design technique. Fig. 3 presents the load position evolution for the nonlinear controlled system.
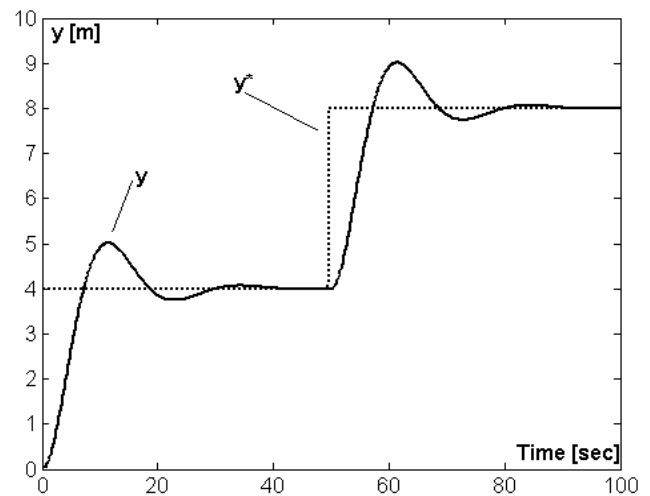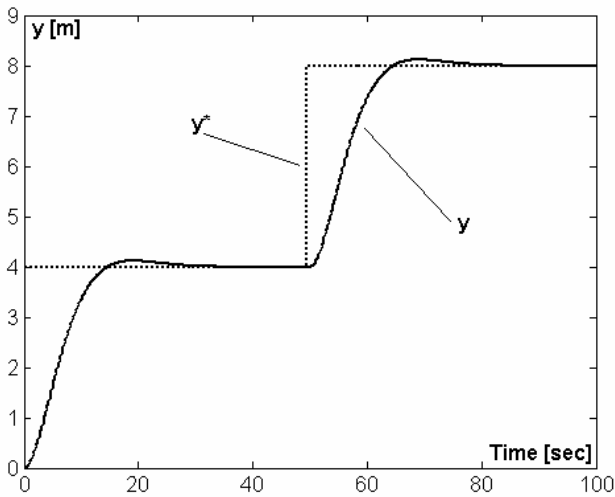

Fig. 3. Load position versus reference (i)

Fig. 4. Time evolution of the load position (i)

The values of the parameters are $m_2 = 500\,kg$, $h = 6m$ and the control design parameters are set to $c_0 = 0.09, c_1 = 0.24$. It can be observed that we have an important overshoot and the settling time is over 40 sec. An improvement of the performances can be obtained using the control design parameters $c_0 = 0.0625, c_1 = 0.375$ - see Fig. 4. The overshoot is reduced and the settling time is less than 35 sec. For all simulations, the control action F takes values in admissible limits (maximum 3000 N).

(ii) In second simulation case, the control goal and the handling crane parameters are the same as in the first simulation case, but the adaptive control law (14), (16), (20) was implemented. The tuning parameters for the adaptation law (20) were set to $\gamma_0 = \gamma_1 = 0.2$ and g_0, g_1 are obtained using the Lyapunov equation (18).
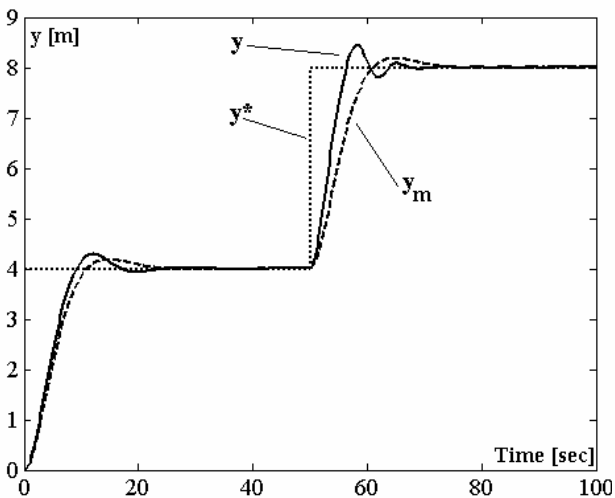

Fig. 5. Time profiles of the load position, model output and reference – adaptive case (ii)

Fig. 5 depicts the time evolution of the load position versus the output model and versus the desired reference. It can be observed that the evolution of the load position is quite good, comparable with the results obtained using the exactly linearizing controller (the settling time is very good and the overshoot is acceptable). In Fig. 6 the time profiles of the parameters c_i obtained using the adaptation law (20) are presented. Another simulation is presented in Fig. 7, where another profile of the reference is utilised (the handling crane parameters are the same).

(iii) In order to test the tracking performances of the proposed nonlinear adaptive control law, in Fig. 8 are depicted the evolution of load position, model output and reference for a composite profile of the reference, and when the load is $m_2 = 10000\,kg$ and $h = 8m$, with satisfactory results.
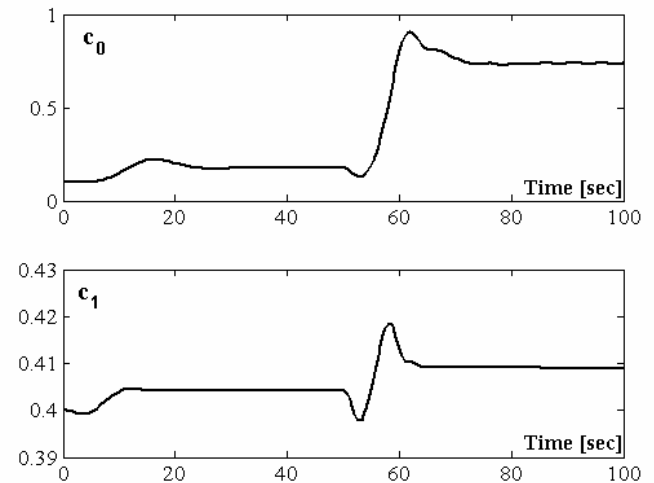

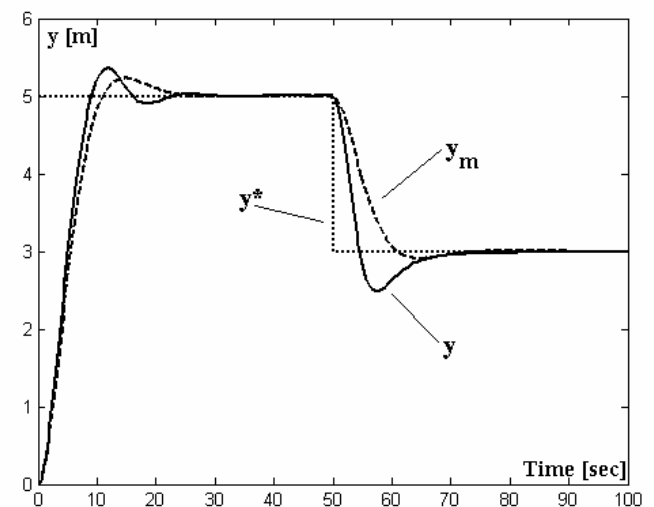Fig. 6. Evolution of parameters c_0 and c_1 – case (ii)

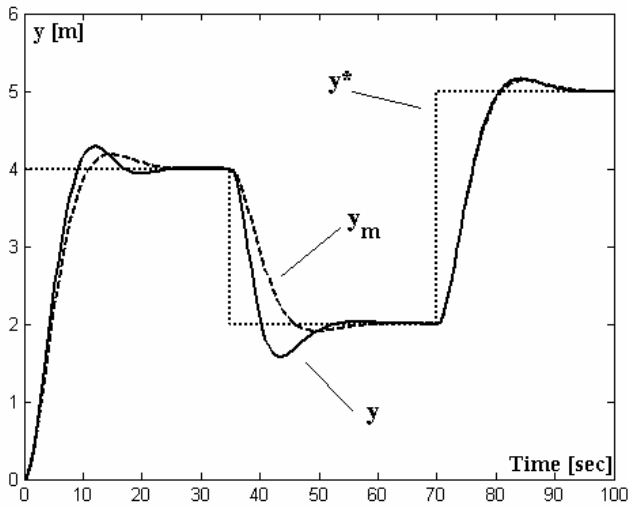Fig. 7. Evolution of outputs for a "negative" step (ii)

Fig. 8. Time profiles of the load position, model
output and reference – adaptive case (iii)

5   Conclusions

In this work, some nonlinear strategies were developed in order to control the load position of a handling crane. Starting from the nonlinear model of the handling crane, an exactly linearizing feedback control law is designed. The exact linearization via feedback and diffeomorphism consists in transforming the nonlinear system into a linear one using a state feedback and a coordinate transformation of the system state.

An adaptive control law, based on reference model approach is designed in order to overcome the disadvantage of parametric uncertainties. In fact, the nonlinear adaptive controller consists of the exactly linearizing control law combined with an adaptation law. For the synthesis of the adaptive algorithm, the method of the gradient is used.

Computer simulations are performed in order to test and validate the proposed adaptive controller. From the simulation point of view, the results show a good behavior of the controlled system.

6   Acknowledgements

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