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The Distribution Principle of Benefits Before Trade in Power Market

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Abstract: - Market equilibrium and market failures are two opposite concepts. Generally, power market is not able to work on the intersection of supply-demand curve because of the particularity of power network. It is the result of market failures. That is to say, power market cannot work in equilibrium state. Nowadays, there are two ways to deal with market failures, such as coordination from government and coordination between participants. In this paper, the two methods are compared and one kind of coordination between participants is put forward to make power market work in equilibrium state.

Key-Words: - market equilibrium; market failures; network losses allocation; Pigou principle; Coase Principle; Spot market;

1 Introduction

The equilibrium in power market is a key aspect regarding price making and load dispatching [1]~[4]. Power market, however, doesn't work in equilibrium state because of some network characteristics such as losses and congestion.

Market failure, often emerging in ordinary markets, is generally caused by external factors. Usually, there are many external factors blocking the market's reaching equilibrium. Considering power market, for generators and consumers, network characteristics can be treated as external factors independent from energy market, leading to market failures, and thus making transaction unperformed at the intersection of supply-demand curve.

When market failures occur, government usually resorts to administrative coordination so as to pilot competition towards the increasing of social benefits, which is the famous Pigou Principle, a key measure to deal with market failures. It regulates the market by government's tax-allowance precept. However, it met a challenge later when Coase wrote a paper to express his view on this subject. He believed that the failure could be corrected through the coordination between participants instead of through that from government, making them realized the influence of external factors, letting them adjust themselves and thus maximizing the social benefits.

In today's power market, the methods based on the Pigou principle are prevalently used to eliminate the influence of network characteristics on power transactions. Discussing about equilibrium and failure of the market in detail, this paper found the Coase Principle's advantage in the transaction. The principle will help power market work in equilibrium state.

2 Power Market Equilibrium

The equilibrium based on competition is the main point of economics' market theory. It represents the balance between supply and demand where the social benefit reaches its maximum and the market is full of efficiency. At that time, the distribution of social benefit is considered fair and reasonable. Therefore, the equilibrium is the ideal state of market operation.

The competitive equilibrium can realize the Pareto efficiency, which can be explained as follow math model. Suppose there are two generators in the market and there exists competition between them. For generator 1, the producing function is:

$$Y_1 = F_1(L_1) \tag{1}$$

For generator 2, it is:

$$Y_2 = F_2(L_2) \tag{2}$$

where: L_1 and L_2 separately represent the production volume of generator 1 and 2. Y_1 and Y_2 show the producing functions of each company and $\partial F_1 / \partial L_1 > 0$, $\partial F_2 / \partial L_2 > 0$. According to the economic laws, the Pareto efficiency or the efficient distribution of production will be given as the solution of optimization with constraints. If the objective is:

$$\min \quad P_1F_1(L_1) + P_2F_2(L_2) \tag{3}$$

then the problem will be to find L_1 and L_2 under the constraints of

$$L_1 + L_2 = L \tag{4}$$

where, L is the market demand. Obviously, the optimal condition will be

$$P_1F_1^{'} = P_2F_2^{'} \tag{5}$$

and Equation (4), Where, P_1,P_2 are fixed non-negative constants. For different P_1 and P_2, there will be different Pareto efficient point. As for each generator, if a certain product's price is ω, the decision will become as follow:

$$\max \quad \omega L_1 - P_1F_1(L_1) \tag{6}$$

and

$$\max \quad \omega L_2 - P_2F_2(L_2) \tag{7}$$

then,

$$P_1F_1^{'} = P_2F_2^{'} = \omega \tag{8}$$

It is coincident with the Pareto efficiency, which means the market has gotten the optimal point via competition. Meanwhile, the optimization as a whole is based on optimizing of each generator.

In power market, for example, the equilibrium state can be expressed as Figure 1, when there are multi-participants in the market with a flexible demand. Line 1 is the biding curve of generator, and line 2 that of consumer. ω shows market price. Furthermore, the shadowed area signals the benefit of the market, where the area above the margin price represents customer's benefit, while the counter part means generator's benefit.

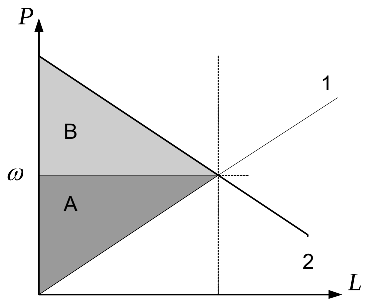

Fig. 1 Supply and demand equilibrium figure in the power market

When a market reaches equilibrium, there is a balance not only between supply and demand, but also between buyer's output and seller's input (receipts and expenditure of both sides). So market equilibrium features in following aspects. First, the prices of participants are the uniform margin price of the market. Secondly, supply balances demand. Thirdly, the output and input are in equilibrium among the participants.

3 Power Market Failure

External factors, if they exist, will influence production of each generator. For generator 1 and 2, their producing function are given as follows:

$$Y_1 = F_1(L_1, G_1(L_1)) \tag{9}$$

$$Y_2 = F_2(L_2, G_2(L_2)) \tag{10}$$

where, $G(L_1)$ and $G(L_2)$ illustrate the impact of external factors as negative elements on the producing benefit of a generator and $\partial G_1 / \partial L_1 > 0$, $\partial G_2 / \partial L_2 > 0$, $\partial F_1 / \partial G_1 < 0$, $\partial F_2 / \partial G_2 < 0$. Then the Pareto efficiency will be expressed as:

$$\min \quad P_1F_1(L_1, G_1) + P_2F_2(L_2, G_2) \tag{11}$$

The optimization is:

$$P_1(F_1^{'} + \frac{\partial F_1}{\partial G_1}\frac{\partial G_1}{\partial L_1}) = P_2(F_2^{'} + \frac{\partial F_2}{\partial G_2}\frac{\partial G_2}{\partial L_2}) \tag{12}$$

Regarding each generator, if producing decision follows Equation. (8), the deviation from the whole optimal will emerge because of the un-control of external factors. At that time, the power market will work in failure state. It can be expressed as Equation (13)

$$Y_L = \int (P_1 \frac{\partial F_1}{\partial G_1}\frac{\partial G_1}{\partial L_1} + P_2 \frac{\partial F_2}{\partial G_2}\frac{\partial G_2}{\partial L_2}) \, dL \tag{13}$$

The failure of power market can be illustrated as Fig. 2. Area C is the part of market failure. Compared with equilibrium market, there will be some losses in the profit of either generator or customer. Furthermore, the losses are indispensable, for the external factors can't be totally controlled by participants.

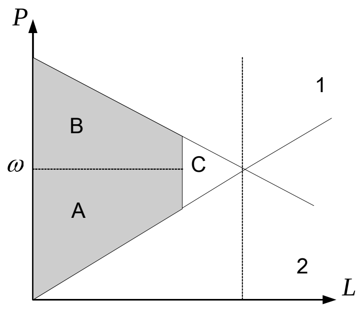

Fig. 2 Power market failure analysis

4 Pigou Principle

To deal with market failures, the measure "tax-allowance" can be used, which is called Pigou principle. It can be deduced as follows. If the government imposes taxes of t_1% and t_2% on generator 1 and 2 separately, the profit after tax will be:

$$\max \quad \omega L_1 - (1-t_1)P_1F_1(L_1, G_1) \tag{14}$$

and:

$$\max \quad \omega L_2 - (1-t_2)P_2F_2(L_2, G_2) \tag{15}$$

The optimization will be:

$$(1-t_1)P_1F_1' = (1-t_2)P_2F_2' = \omega \tag{16}$$

To satisfy the Pareto efficiency, the government is requested to regulate tax rates of t_1 and t_2. That is:

$$\begin{aligned} & -t_1P_1F_1' + t_2P_2F_2' \\ & = P_1\frac{\partial F_1}{\partial G_1}\frac{\partial G_1}{\partial L_1} - P_2\frac{\partial F_2}{\partial G_2}\frac{\partial G_2}{\partial L_2} \end{aligned} \tag{17}$$

Afterwards, the market will work in another Pareto efficient point.

Regarding power market, it can be illustrated as Fig. 3, where t is the tax rate of generator and s the rate of customer. The latter, a kind of allowance, is negative. Area D presents the whole tax of government.

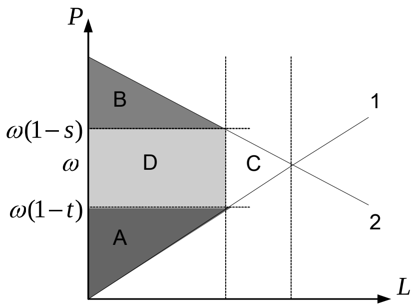

Fig. 3 Figure of the network losses allocation based on Pigou principle

In power market where the administration department of grids replaces the government in some functions, tax is in the form of electricity price. Certainly, it is not only for grid companies. Then, it is a matter of fairness on how to decide the coefficients of t and s. In other words, it is how to allot the profits of transaction. Unfortunately, it seems hard to be absolutely impartial no matter how to do so. Nothing rather than policies and codes are requested to apportion the profits.

5 Coase Principle

Regarding the solution of market failure, Coase found a different way. He believed that each company resorts to coordination itself to get the market's Pareto efficiency. When it makes decision on production, the company takes external factors into account, and then coordinates with other companies that effected by those factors. The procedure can be expressed as following. To company 1 and 2, after the coordination between the effected companies, the optimization of production becomes as Equation (18) and Equation (19).

$$P_1\left(F_1' + \frac{\partial F_1}{\partial G_1}\frac{\partial G_1}{\partial L_1}\right) = \omega \tag{18}$$

$$P_2\left(F_2' + \frac{\partial F_2}{\partial G_2}\frac{\partial G_2}{\partial L_2}\right) = \omega \tag{19}$$

Then, the optimization of each company's production is accordant to the Pareto efficiency of the market, so as to get the whole efficiency. Regarding completely competitive market, it equals to the modification to producing curve of each company. Shown in Fig. 4, the extent of modification is demonstrated in the part of $\frac{\partial F_1}{\partial G_1}\frac{\partial G_1}{\partial L_1}$ and $\frac{\partial F_2}{\partial G_2}\frac{\partial G_2}{\partial L_2}$. So, the key point is how to coordinate with the effected companies to decide the extent of external factors' influence. In other words, the change of G needs to be calculated when the production varies.

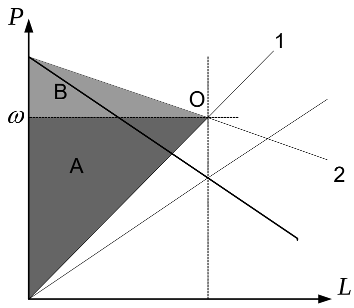

Fig. 4 Figure of the network losses allocation based on Coase principle

Both the measures above have advantages and disadvantages. Regarding micro-market, Coase principle, the relatively new one, injects energy into the market, because it avoids from the complexities brought by policies and codes. Meanwhile, it achieves better performance if combined with the Pigou principle. In this case, during the coordination among companies, government's functions are also performed, especially in power market, because of its feature in strong supervision. This paper illustrates the application of Coase principle to power market with an example of network losses allocation.

6 Application of Coase Principle — Network Losses Allocation

Network loss is one of the main characteristics of power network. Although generator's bidding curve can signal the maximum of its producing benefits, it doesn't consider the influence of network losses. Therefore, when determining the final market benefit at the intersection of the conventional supply and demand curves, the power market is inevitable to work in failure state if taking real network losses into account. Meanwhile, we have to face a matter of fairness if we resort to the coordination from government.

At present, there are many network losses allocation methods [5]~[8], some of which are based on the node injected power (or current) or based on the branch power (or current) to allocate the losses. These methods are typical applications of the Pigou principle to power market's losses allocation that is made after power market trade.

However, the principle of network losses allocation must be established before power market trade when the Coase principle is applied. For instance, if a simplest principle of network losses allocation that is according to the direct proportion of nodal injected power is adopted, the losses can be given in the form of the sum of all node injected power as:

$$P_L = \sum_{i=1}^{N} p_i \tag{20}$$

where, $i = 1,2,3\cdots N$ is the number of generators and customers and p_i is the injected power of each participator. To the injected power p_i of each given node, a modification coefficient is defined of each node's power.

$$\eta_i = (P_L / \sum_{i=1}^{N} |p_i|) \times (p_i / |p_i|) \tag{21}$$

As shown in the above equation, there are two parts in it. The latter part gives the sign of power modification coefficient. The coefficient is negative for a load node, and positive for a generator node. The equivalent network loss is $\eta_i p_i$ for node i. The total real power is $(1+\eta_i)p_i$. It can be considered the modification to the bidding curve of participants, with the coefficient of $1+\eta_i$. The final bidding curve of generator is shown as line 1 in fig.4. Line 2 is the bidding curve of customers. Point O signs a new equilibrium of the market. At that time, the equilibrium can be expressed as following model.

Objective function:

$$\min \sum_{i=1}^{N} \omega(1+\eta_i)p_i \tag{22}$$

Constraints:

$$\sum_{i=1}^{N} (1+\eta_i)p_i = 0 \tag{23}$$

It is the manner of Coase principle for network losses allocation according to above equilibrium model.

7 Case Study

The case study is made at IEEE-30 nodes system, the nodal data are listed in table 1 and other data are omitted. It is explained for table 1 as follows: 1) Suppose that the bidding or offering curve of the participator are a straight line as $c_i = a_i p_i + b_i$; 2) Node type 1 means generation node when type 2 expresses load node and type 3 expresses slack node.

When the initial marginal price is 280Y/MW, the $P_N - p_N$ reached a smaller value (10^{-3}) by 13 iterations as showed in table 2.

Table 1 Node Data for IEEE-30 Nodes System

Node	Volt. Mag.	Volt. Angle	Coefficient a for Generator	Coefficient b for Generator	Coefficient a for Consumer	Coefficient b for Consumer	Node Yype	Cap.
1	1.0500	0.0000	128.00	160.00	0.0000	0.0000	1	0
2	1.0338	2.7374	195.00	171.00	0.0000	0.0000	1	0
3	1.0309	4.6722	0.0000	0.0000	-752.00	382.00	2	0
4	1.0258	5.5963	0.0000	0.0000	-761.00	388.00	2	0
5	1.0058	9.0005	0.0000	0.0000	-760.00	388.00	2	0
6	1.0214	6.4821	0.0000	0.0000	0.0000	0.0000	2	0
7	1.0073	8.0435	0.0000	0.0000	-753.00	380.00	2	0
8	1.0230	6.4864	206.80	189.00	0.0000	0.0000	3	0
9	1.0583	8.1508	0.0000	0.0000	0.0000	0.0000	2	0
10	1.0527	10.0086	0.0000	0.0000	-734.00	385.00	2	0.19
11	1.0913	6.3003	195.00	186.00	0.0000	0.0000	1	0
12	1.0564	9.2015	0.0000	0.0000	-508.00	388.00	2	0
13	1.0883	8.0216	185.00	178.00	0.0000	0.0000	1	0
14	1.0428	10.0986	0.0000	0.0000	-740.00	384.10	2	0
15	1.0393	10.2212	0.0000	0.0000	-888.00	385.11	2	0

16	1.0476	9.8207	0.0000	0.0000	-762.00	384.00	2	0
17	1.0459	10.1598	0.0000	0.0000	-750.00	386.00	2	0
18	1.0319	10.8362	0.0000	0.0000	-762.00	383.00	2	0
19	1.0307	11.0109	0.0000	0.0000	-761.00	386.00	2	0
20	1.0354	10.8178	0.0000	0.0000	-750.00	383.00	2	0
21	1.0404	10.4668	0.0000	0.0000	-763.00	387.00	2	0
22	1.0409	10.4598	0.0000	0.0000	0.0000	0.0000	2	0
23	1.0314	10.6662	0.0000	0.0000	-791.32	388.22	2	0
24	1.0292	10.9159	0.0000	0.0000	-893.00	380.00	2	0.04
25	1.0298	10.8036	0.0000	0.0000	0.0000	0.0000	2	0
26	1.0124	11.2117	0.0000	0.0000	-750.00	383.00	2	0
27	1.0388	10.4761	0.0000	0.0000	0.0000	0.0000	2	0
28	1.0177	6.8955	0.0000	0.0000	0.0000	0.0000	2	0
29	1.0192	11.6689	0.0000	0.0000	-762.21	383.23	2	0
30	1.0080	12.5242	0.0000	0.0000	-752.32	386.34	2	0

Table 2 The Results of Power Flow Calculation

Iteration	P_N	p_N	C_{0M}
1	0.512519	0.361538	266.6
2	0.500374	0.362073	266.7
3	0.488232	0.362609	266.8
4	0.476095	0.363144	266.9
5	0.463963	0.363679	267.0
6	0.451837	0.364215	267.1
7	0.439715	0.36475	267.2
8	0.427599	0.365285	267.3
9	0.415487	0.365821	267.4
10	0.403381	0.366356	267.5
11	0.391279	0.366891	267.6
12	0.379183	0.367427	267.7
13	0.367092	0.367962	267.8

8 Conclusions

Generally, power market doesn't work in equilibrium state due to the influences of some aspects such as network losses, which leads to market failure. The reasons causing failure of power market and the distribution manner of market bnafits are analysized, and based on Coase principle, this paper provides a new method to allocate network losses in this paper. It enables power market return to the equilibrium state. Tested by simulation of the network losses allocation, it is effective and enable to achieve impartiality under market circumstance.

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