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Evaluation of Various Approaches to Frequency Response Calculation of Switched Circuits in Spice

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Abstract: – This paper is a presentation of frequency response calculation of switched circuits in Spice program. Evaluated methods are based on transient analysis of circuits followed by Fourier transformation of the computed time response. Three types of circuit excitation are taken into account – pulse, set of harmonic signals and the same set but with Sample and Hold circuit. Fourier transformation is computed either by DFT or by numerical calculation of Fourier integral.

Key-Words: – Analysis of switched circuits, Switched capacitors, Fourier Transformation, Spice

1 Introduction

There are several methods for computing frequency response of switched circuit. The basic method is charge equations method for switched capacitor (SC) circuits [2, 3] or nodal equations method modified for switched currents (SI) circuits [6, 3]. This method is applicable only for idealized circuits. For linear switched circuits with parasitical effects we can solve general linear differential equation of a circuit by a method based on the Fourier transformation [4] or the Laplace transformation [5], see section 5. The third one is general solution of the circuit by transient analysis using program like SPICE. The frequency response is obtained consecutively by Discrete Fourier Transformation (DFT) of output signal samples [7, 9]. The comparison of this method was shortly described in [1]. The main motivation of this contribution is to find conditions for reliable analysis of frequency response of switched circuit which is excited by a pulse in comparison with excitation by sinusoidal signals.

This paper is focused on the third method using Spice. Compared to [1] there are used more types of exciting signals and alternative method of DFT computing based on [8], besides of `spec` command of Winspice.

The frequency response of switched circuit is obtained from computed output signal spectrum. However the excitation signal type affects the output spectrum, specially for real dissipation circuit and for circuit, where the part of input signal comes to output by direct (“non-swithed”) way.

2 Exciting Signals

There are used three different signals for exciting the circuit. The first signal is sinusoidal or set of sinusoidal signals as was used in [1]. The set includes all frequencies in which frequency response will be calculated. The conditions mentioned in [1] must be kept.

The circuit can be also excited through a sample and hold circuit with same results of magnitude response in case of idealized switched circuit (without parasitical capacitors and resistors) and where any part of input signal does not pass directly to output. The highest frequency of exciting signal must be less than half of clock frequency $f_c/2$. However the frequency response (output spectrum) can by calculated up the frequency $f_c/2$. The spectrum up to this frequency is valid only for ideal circuit.

The following figure shows these types of signals.

The first one (sin) is sinusoidal where the clock frequency is $f_c = 10f$. The second one (sinSH) is sampled sinusoidal signal. The last used type of exciting signal is a pulse about width equal just $1/(2f_c)$. See figure 1.

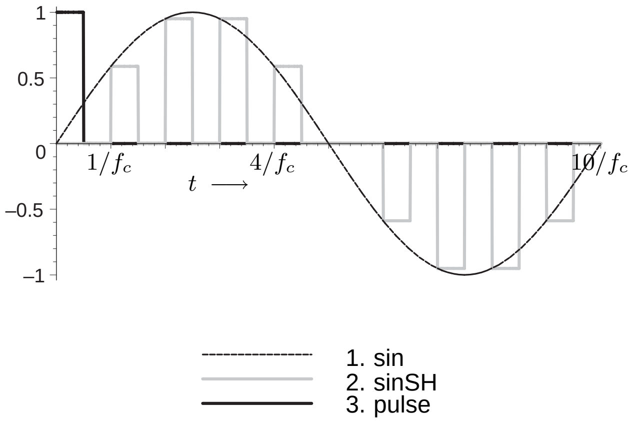

Figure 1: Exciting signals.

3 DFT

Setting parameters of the transient analysis is pivotal for validity of whole analysis. For set of sinusoidal signals it is necessary to subside of transient response. The next two periods may be saved then as input data for Fourier analysis [1]. This data must exactly cover two period of the smallest input signal frequency, i.e. the difference between stop-time and start-time of the transient analysis must be an entire (two) multiple of the longest input signal period. It is necessary for correct computation frequency response by DFT.

The data (output signal vector) as a result of the transient analysis for DFT must be saved from beginning (form zero time) in case of exciting of circuit by pulse signal. `Linearize` command must be always applied to the data for equidistant time step by contrast of sinusoidal sources.

Then the `spec` command of WinSpice calculates the DFT of input data according to known formula (1). FFT cannot be used because number of samples in vector mostly does not correspond to number 2^n.

$$X[k] = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-ikn(2\pi/N)} \tag{1}$$

The second way how to calculated the output spectrum is using direct calculation of Fourier integral. It is described the following sections.

4 Fourier Integral Calculation

The method of spectrum calculation using the Fourier Integral calculation is based on [8].

For a periodic waveform x, the cosine Fourier coefficients are computed with the following formula

$$a_k = \frac{2}{T} \int_T x(\tau) \cos(k\omega_0 \tau)\, d\tau \tag{2}$$

where $T = \frac{1}{f_0}$ is signal period and $f_0 = \frac{\omega_0}{2\pi}$ is signal frequency.

The circuit simulator (Spice) discretizes time at $N+1$ time-point, t_0, t_1, ... t_N. Equation (2) is rewritten as a sum of integrals over each time step,

$$a_k = \frac{2}{T} \sum_{n=1}^{N} \int_{t_{n-1}}^{t_n} x(\tau) \cos(k\omega_0 \tau)\, d\tau \tag{3}$$

where $t_N - t_0 = T$.

Between time-points, waveform x can be approximated with a low-order polynomial.

$$x(\tau) \approx \sum_{m=0}^{M} c_{mn} \tau^m , \tag{4}$$

where M is the order of the approximating polynomial. Substituting into (3) gives

$$a_k \approx \frac{2}{T} \sum_{n=1}^{N} \sum_{m=0}^{M} c_{mn} \int_{t_{n-1}}^{t_n} \tau^m \cos(k\omega_0 \tau)\, d\tau \tag{5}$$

$$\alpha_{kmn} = \int_{t_{n-1}}^{t_n} \tau^m \cos(k\omega_0 \tau)\, d\tau \tag{6}$$

For first order of polynomial approximation ($M = 1$) we get:

$$x(\tau) \approx c_{k0n} + c_{k1n}\tau \tag{7}$$

$$\alpha_{k0n} = \frac{\sin(k\omega_0 t_n) - \sin(k\omega_0 t_{n-1})}{k\omega_0} \tag{8}$$

$$\alpha_{k1n} = \frac{\cos(k\omega_0 t_n) + t_n k\omega_0 \sin(k\omega_0 t_n)}{k^2\omega_0^2} - \\ - \frac{\cos(k\omega_0 t_{n-1}) - t_{n-1} k\omega_0 \sin(k\omega_0 t_{n-1})}{k^2\omega_0^2} \tag{9}$$

$$c_{n0} = \frac{x_n - x_{n-1}}{t_n - t_{n-1}}\,, \tag{10}$$

$$c_{n1} = x_n - c_{n0} t_n\,. \tag{11}$$

where $n = \langle 1, N \rangle$ and N is number of time samples of transient analysis, t_n is time vector element of transient analysis, and x_n is sample value in time t_n.

Computation of DC component a_0 and b_k components can be done by the same way.

The frequency response then can be calculated from the a_k and b_k components by

$$\mathrm{mod}_k = \sqrt{a_k^2 + b_k^2} \tag{12}$$

$$\mathrm{phase}_k = \arctan\left(\frac{-b_k}{a_k}\right) \tag{13}$$

5 Generalized Transfer Function Method

Generalized transfer function (GFT) are based on modeling of periodically switched networks by mixed s-z description [5]. The method is based on assumption that linear systems with the time-varying parameters can be modelled by nonstationary transfer functions $K(s,t)$. If the parameters vary periodically (e.g. in SC and SI circuits with externally controlled switches), then the system response contains both continuous and discrete time parts and it can be described by a generalized transfer function GTF $K(s,z)$. The frequency response of the system is obtained by double substitution $s = i\omega$ and $z = e^{i\omega T}$, where T is switching period. The dynamic properties of the system is determined by pole locations of the GTF in s and z domains.

6 Switched Capacitors Simple Example

Very simple example of SC circuit was chosen to evaluate just influence of different types of excitation signals and computing methods for frequency response calculation. The circuit is shown on figure 2. Resistors R_1 and R_2 represent on-resistance of switches. The values of both resistors were $10\,\Omega$ for ideal circuit and $10\,\mathrm{k}\Omega$ for real circuit. The capacitance were $C_1 = C_2 = 1\,\mu$F.

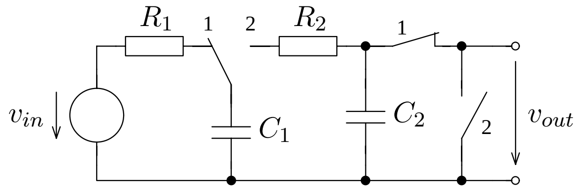

Figure 2: Analyzed SC circuit

The transfer function for ideal circuit were derived using charge equations method, which supposes ideal switches.

$$H(z) = \frac{0.5}{z - 0.5} \tag{14}$$

The other transfer functions was derived using GTF. The first one (15) is for real circuit excited by sampled and held sinusoidal signal. The second one (16) is for the same circuit excited just by sinusoidal signal.

$$H(z) = \frac{0.4558650694 z}{z^2 - 0.544688015 z + 0.55308438 \cdot 10^{-3}} \tag{15}$$

$$H(z,s) = \\ = \frac{0.4966310265\,(1\sqrt{z} - 0.08208499862)\,z}{(z^2 - 0.544688015 z + 0.55308438 \cdot 10^{-3})} \cdot \\ \cdot \frac{1}{1 + 0.01 s} \tag{16}$$

where $z = e^{(i2\pi f/f_c)}$ and $s = 2\pi f$.

All transfer function should be, in addition, multiplied by spectrum of sampling function, i.e. by

$$\frac{sin\left(\frac{\pi f}{2 f_c}\right)}{\frac{\pi f}{2 f_c}} \tag{17}$$

where $f_c = 20\,\mathrm{Hz}$.

Figure 3 shows magnitude responses of ideal and real circuit using different types of excitation signals. These responses were obtained by relations 14, 15 and 16. These results were used also for error evaluations in tables 1, 2 and 3.

The errors in given tables were calculated for following Winspice setting: `method = gear`, `trtoll = 4`, `chgtol = 1e-16`, `specwindow = none`. The other parameters had default values. The tables compare computing of magnitude response for direct computation of Fourier integral method

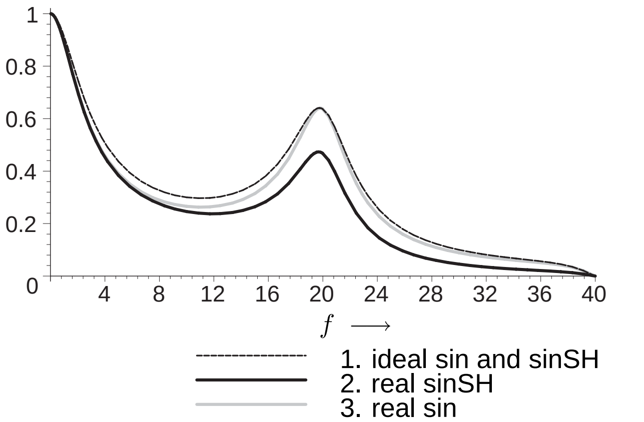

Figure 3: Magnitude responses for ideal circuit (ideal sin and sinSH) and for real circuit exciting once by sinusoidal signal (sin), once by pulse or sampled and held sinusoidal signal (sinSH). The clock frequency was set to $f_c = 20\,\mathrm{Hz}$.

ideal	sin	sinSH	pulse
fourier	0.18	0.17	0.047
DFT	0.29	0.29	0.21

Table 1: Maximum errors of magnitude responses in % calculated for ideal circuit, different types of excitation signals and computing methods. There were calculated 250 time samples per half of the clock signal period.

real	sin	sinSH	pulse
fourier	0.072	0.32	0.0047
DFT	0.069	0.45	0.0047

Table 2: The same as table 1 but real circuit.

real	sin	sinSH	pulse
fourier	0.016	0.037	0.0019
DFT	0.024	0.066	0.0019

Table 3: The same as table 2 but for 1000 time samples per half of the clock signal period.

(fourier), introduced in section 4 and DFT method (DFT), introduced in section 3. The errors were referenced to transfer functions 14, 15 and 16.

Listing of Winspice commands for direct computing of Fourier integral according section 4 is shown in table 4.

7 Conclusion

The paper shows computation of frequency response of switched capacitors circuit by DFT and direct computing of Fourier integral. The frequency response was computed for different types of exciting signals.

Circuit excitation by pulse gives some advantage in comparison with excitation by a set of sinusoidal signals. The frequency response could be easily computed for more frequencies. The computation is more quick because it is not necessary to wait for periodical steady state in transient analysis. The errors of computation are comparable for both types of excitation. However it is necessary to be careful in case of real circuit (with parasitical effects) without sample and hold input circuit. There is need to use just a set of sinusoidal signals for excitation, because the analysis with pulse excitation gives different (bad) results. The reason is that there are different transfer functions for these types of excitation signals.

Acknowledgement

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*vectors definition
let l=length(time)
let ak=vector(kmax+1)
let bk=vector(kmax+1)

*first order polynomial approximation
let c1=(out[1,l-1]-out[0,l-2])/(time[1,l-1]-time[0,l-2])
let c0=out[0,l-2]-c1*time[0,l-2]

*computation of DC coefficient
let a0=time[1,l-1]-time[0,l-2]
let a1=(time[1,l-1]^2-time[0,l-2]^2)/2
let ak[0]=2/T*sum(c0*a0+c1*a1)

*computation of Fourier coefficients
let k=1
while k <= kmax
let kom=k*om
let sinkom=sin(kom*time)/kom
let coskom=cos(kom*time)/kom
let sintim=sinkom*time
let costim=coskom*time
let a0=sinkom[1,l-1]-sinkom[0,l-2]
let b0=-coskom[1,l-1]+coskom[0,l-2]
let a1=-b0/kom+(sintim[1,l-1]-sintim[0,l-2])
let b1=a0/kom-(costim[1,l-1]-costim[0,l-2])
let ak[k]=2/T*sum(c0*a0+c1*a1)
let bk[k]=2/T*sum(c0*b0+c1*b1)
let k=k+1
end

*computation of magnitude and phase
let as=sqrt(ak^2+bk^2)
let ps=ph(ak-i*bk)
```

Table 4: Listing of Winspice commands for direct computing of Fourier integral according section 4.

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