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Flexible Takagi-Sugeno Neuro-Fuzzy Structures

KRZYSZTOF CPAŁKA[1, 2], LESZEK RUTKOWSKI[1, 2]
(1) Department of Computer Engineering
Technical University of Częstochowa
and
(2) Department of Artificial Intelligence
WSHE University in Łódź
POLAND

Abstract:-In the paper we study a new class of Takagi-Sugeno fuzzy systems. Due to incorporation of various parameters and weights into construction of such systems their performance is significantly improved. The results are illustrated on a typical benchmark problem.

Key-Words:-Takagi-Sugeno systems, Weighted triangular norms, Parameterized triangular norms, Soft fuzzy norms.

1. Introduction

The construction of most neuro-fuzzy structures [2, 3, 4, 6-8] is based on the Mamdani-type reasoning, describing by a t-norm, e.g. product or min. An alternative approach [1, 10-13, 17] is based on the logical method. Another method is based on the Takagi-Sugeno scheme [15] which involves a functional dependence between IF and THEN parts of the rules in the form

$$R^{(r)}: \text{IF } \bar{\mathbf{x}} \text{ } is \text{ } \mathbf{A}^r \text{ THEN } y^r = f^{(r)}(\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n) \quad (1)$$

where $\bar{\mathbf{x}} = [\bar{x}_1, \ldots, \bar{x}_n] \in \mathbf{X}$, $y \in \mathbf{Y}$, $A^r = A_1^r \times A_2^r \times \ldots \times A_n^r$, $A_1^r, A_2^r, \ldots, A_n^r$ are fuzzy sets characterized by membership functions $\mu_{A_i^r}(\bar{x}_i)$, $i = 1, \ldots, n$, $r = 1, \ldots, N$. The aggregation in the Takagi-Sugeno model is described by formula

$$\bar{y} = f(\bar{\mathbf{x}}) = \frac{\sum_{r=1}^{N} f^{(r)}(\bar{\mathbf{x}}) \cdot \mu_{A^r}(\bar{\mathbf{x}})}{\sum_{r=1}^{N} \mu_{A^r}(\bar{\mathbf{x}})} \quad (2)$$

where $\mu_{A^r}(\bar{\mathbf{x}}) = \mathop{T}_{i=1}^{n} \left\{ \mu_{A_i^r}(\bar{x}_i) \right\}$ and T is a t-norm.

Model (2) has been extensively studied in many papers (see e.g. [3]). In this paper we propose an extension of formula (2). Due to incorporation of various parameters and weights into construction of Takagi-Sugeno system the performance of our structure is significantly improved comparing with previous results [5, 9, 10].

2. Flexibility in Fuzzy Systems

2.1. Weighted triangular norms

We will explain the idea of weighted triangular norms. The weighted t-norm in the two-dimensional case is defined as follows

$$T^*\{a_1, a_2; w_1, w_2\} = T\{S\{1 - w_1, a_1\}, S\{1 - w_2, a_2\}\} \quad (3)$$

Parameters a_1 and a_2 can be interpreted as antecedents of the rule. The weights w_1 and w_2 are corresponding certainties (credibilities) of both antecedents in (3). Observe that

- If $w_1 = w_2 = 1$ then the weighted t-norm is reduced to the standard t-norm. In the context of linguistic values we assign the truth to both antecedents a_1 and a_2 of the rule.
- If $w_1 = 0$ then

$$\begin{aligned} T^*\{a_1, a_2; 0, w_2\} &= T\{S\{1, a_1\}, S\{1 - w_2, a_2\}\} \\ &= T\{1, S\{1 - w_2, a_2\}\} \\ &= S\{1 - w_2, a_2\} \end{aligned} \quad (4)$$

Therefore, antecedent a_1 is discarded since its certainty is equal to 0. Similarly if $w_2 = 0$ then antecedent a_2 vanishes, i.e.

$$\begin{aligned} T^*\{a_1, a_2; w_1, 0\} &= T\{S\{1 - w_1, a_1\}, S\{1, a_2\}\} \\ &= T\{S\{1 - w_1, a_1\}, 1\} \\ &= S\{1 - w_1, a_1\} \end{aligned} \quad (5)$$

If $0 < w_1 < 1$ and $0 < w_2 < 1$ then we assume a partial certainty of antecedents a_1 and a_2. It is easily seen that formula (3) can be applied to the evaluation of an importance of input linguistic values. The values of weights will be depicted in the form of diagrams.

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In Fig. 1 we show an example of a diagram for a linear Takagi-Sugeno system having three rules ($N=3$) and two inputs ($n=2$) which is given by

$$
\begin{aligned}
R^{(1)}:&\left\{\begin{array}{l}\text{IF } \bar{x}_1 \text{ is } A_1^1\left(w_{1,1}^{\tau}\right) \text{AND } \bar{x}_2 \text{ is } A_2^1\left(w_{2,1}^{\tau}\right)\\ \text{THEN } f^{(1)}(\bar{\mathbf{x}})=c_{0,1}^f+\sum_{i=1}^{2} c_{i,1}^f \bar{x}_i\end{array}\right\}\left(w_1^{\text{def}}\right)\\
R^{(2)}:&\left\{\begin{array}{l}\text{IF } \bar{x}_1 \text{ is } A_1^2\left(w_{1,2}^{\tau}\right) \text{AND } \bar{x}_2 \text{ is } A_2^2\left(w_{2,2}^{\tau}\right)\\ \text{THEN } f^{(2)}(\bar{\mathbf{x}})=c_{0,2}^f+\sum_{i=1}^{2} c_{i,2}^f \bar{x}_i\end{array}\right\}\left(w_2^{\text{def}}\right) \qquad (6)\\
R^{(3)}:&\left\{\begin{array}{l}\text{IF } \bar{x}_1 \text{ is } A_1^3\left(w_{1,3}^{\tau}\right) \text{AND } \bar{x}_2 \text{ is } A_2^3\left(w_{2,3}^{\tau}\right)\\ \text{THEN } f^{(3)}(\bar{\mathbf{x}})=c_{0,3}^f+\sum_{i=1}^{2} c_{i,3}^f \bar{x}_i\end{array}\right\}\left(w_3^{\text{def}}\right)
\end{aligned}
$$

where $w_{i,r}^{\tau}$, $i=1,2,\ldots,n$, $r=1,2,\ldots,N$, w_r^{def}, $r=1,2,\ldots,N$, and $c_{i,r}^{\tau}$, $i=0,2,\ldots,n$, $r=1,2,\ldots,N$, are certainty weights of antecedents, certainty weights of rules and parameters of consequences, respectively.

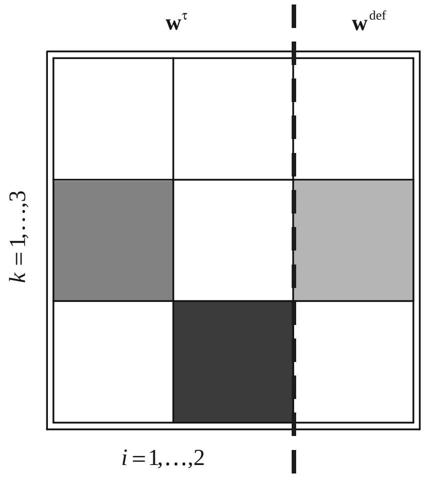

Fig. 1. Exemplary weights representation in a fuzzy system with three rules and two inputs (dark areas correspond to low values of weights and vice versa)

Observe that the second rule is "weaker" than the others and the linguistic value A_2^1 corresponds to a low value of $w_{2,1}^{\tau}$.

Example 1. The algebraic t-norm with weighted arguments is given as follows

$$T_P^*\{a_1,a_2;w_1,w_2\}=(1-w_1(1-a_1))(1-w_2(1-a_2)) \qquad (7)$$

The 3D plots of function (7) are depicted in Fig. 2.

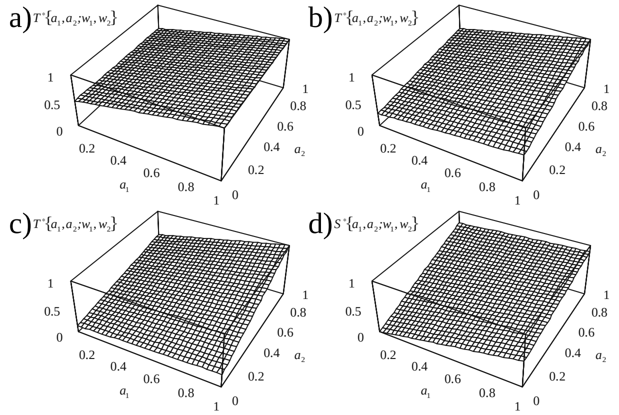

Fig. 2. 3D plots of function (7) for $w_1=0.50$ and a) $w_2=0.00$, b) $w_2=0.50$, c) $w_2=0.75$, d) $w_2=1.00$

2.2. Soft triangular norms

In this paper we apply the concept of soft fuzzy norms proposed by Yager and Filev [17] to the construction of Takagi-Sugeno systems. Let $a_1,\ldots,a_n$ be numbers in the unit interval that are to be aggregated. The soft version of a t-norm is given by

$$\tilde{T}\{\mathbf{a};\alpha\}=(1-\alpha)\frac{1}{n}\sum_{i=1}^{n} a_i+\alpha \mathop{T}_{i=1}^{n}\{a_i\} \qquad (8)$$

where $\alpha\in[0,1]$. Formula (8) allows to balance between the arithmetic average aggregator and the triangular norm aggregator depending on parameter α.

Example 2. The soft algebraic t-norm is described as follows

$$\tilde{T}\{a_1,a_2;\alpha\}=(1-\alpha)\frac{a_1+a_2}{2}+\alpha a_1 a_2 \qquad (9)$$

The 3D plots of function (9) are depicted in Fig. 3.

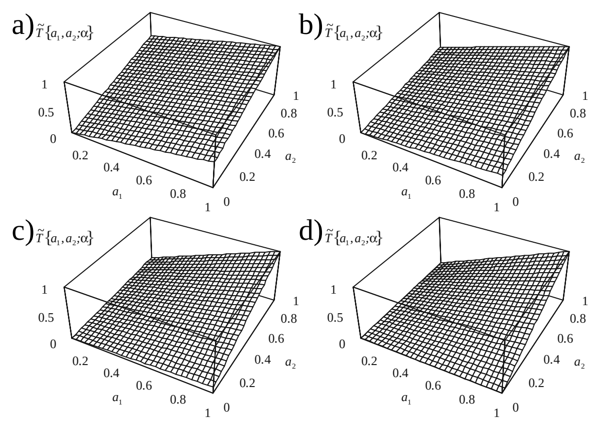

Fig. 3. 3D plots of function (9) for a) $\alpha=0.00$, b) $\alpha=0.50$, c) $\alpha=0.75$, d) $\alpha=1.00$

2.3. Parameterized triangular norms

Most fuzzy systems are based on non-parameterized triangular norms, e.g. algebraic or Łukasiewicz. In this paper we suggest an application of parameterized t-norms [4], denoted by $\vec{T}\{a_1, a_2, \ldots, a_n; p\}$, for the construction of Takagi-Sugeno systems. The hyperplanes corresponding to them can be adjusted in the process of learning of parameter p.

Example 3. The parameterized t-norm of Yager's is given as follows ($p > 0$)

$$\vec{T}\{a_1, a_2; p\} = 1 - \min\left\{1, \sqrt[p]{(1-a_1)^p + (1-a_2)^p}\right\} \quad (10)$$

The 3D plots of function (10) are depicted in Fig. 4.

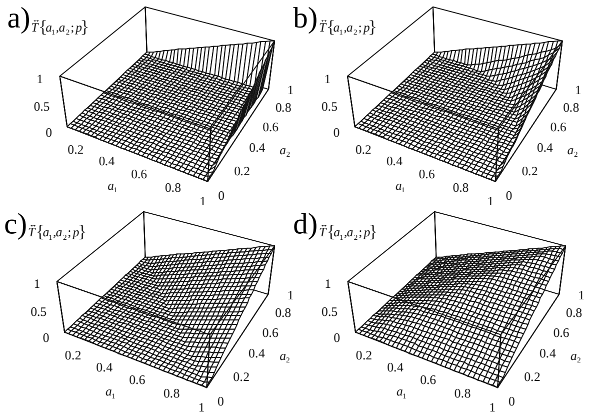

Fig. 4. 3D plots of function (10)
for a) $p = 0.1$, b) $p = 0.5$, c) $p = 1.0$, d) $p = 10.0$

3. Flexible Fuzzy Systems

We will now incorporate flexibility parameters and weights described in section 2 to the construction of flexible Takagi-Sugeno neuro-fuzzy systems. We consider multi-input, single-output fuzzy system mapping $\mathbf{X} \rightarrow \mathbf{Y}$, where $\mathbf{X} \subset \mathbf{R}^n$ and $\mathbf{Y} \subset \mathbf{R}$. Assume the following the rule base

$$R^{(r)}: \left\{ \begin{array}{l} \text{IF } \bar{x}_1 \text{ is } A_1^r\left(w_{1,r}^\tau\right) \text{AND } \bar{x}_2 \text{ is } A_2^r\left(w_{2,r}^\tau\right) \\ \text{THEN } f^{(r)}(\bar{\mathbf{x}}) = c_{0,r}^f + \sum_{i=1}^{2} c_{i,r}^f \bar{x}_i \end{array} \right\} \left(w_r^{\text{def}}\right) \quad (11)$$

where $r = 1,2,\ldots,N$. The construction of a new Takagi-Sugeno systems will be based on the following parameters and weights:

- fuzzy sets $A_1^k, A_2^k, \ldots, A_n^k$ characterized by membership functions $\mu_{A_i^k}(\bar{x}_i)$ with unknown parameters to be learnt;
- parameters $c_{0,r}^f$, $c_{i,r}^f$, $i = 1,2,\ldots,n$, $r = 1,2,\ldots,N$, in linear models describing consequences;
- certainty weights $w_{i,r}^\tau$, $i = 1,2,\ldots,n$, $r = 1,2,\ldots,N$, describing importance of antecedents in the rules;
- parameters p_r^τ, $r = 1,2,\ldots,N$, of parameterized families of t-norms;
- softness parameters α_r^τ, $r = 1,2,\ldots,N$, in connectives of antecedents;
- certainty weights w_r^{def}, $r = 1,2,\ldots,N$, describing importance of the rules.

Consequently, the general architecture of flexible Takagi-Sugeno systems studied in the paper can be presented in the form

$$\bar{y} = \frac{\sum_{r=1}^{N} w_r^{\text{def}} \left(\left(\begin{array}{c} \left(1-\alpha_r^\tau\right)\text{avg}\left(\mu_{A_1^r}(\bar{x}_1),\ldots,\mu_{A_n^r}(\bar{x}_n)\right) + \\ + \alpha_r^\tau \vec{T}^* \left\{ \begin{array}{c} \mu_{A_1^r}(\bar{x}_1),\ldots,\mu_{A_n^r}(\bar{x}_n)\ ; \\ w_{1,r}^\tau,\ldots,w_{n,r}^\tau, p_r^\tau \end{array} \right\} \end{array} \right) \cdot \left(c_{0,r}^f + \sum_{i=1}^{n} c_{i,r}^f \cdot \bar{x}_i \right) \right)}{\sum_{r=1}^{N} w_r^{\text{def}} \left(\begin{array}{c} \left(1-\alpha_r^\tau\right)\text{avg}\left(\mu_{A_1^r}(\bar{x}_1),\ldots,\mu_{A_n^r}(\bar{x}_n)\right) + \\ + \alpha_r^\tau \vec{T}^* \left\{ \begin{array}{c} \mu_{A_1^r}(\bar{x}_1),\ldots,\mu_{A_n^r}(\bar{x}_n)\ ; \\ w_{1,r}^\tau,\ldots,w_{n,r}^\tau, p_r^\tau \end{array} \right\} \end{array} \right)} \quad (12)$$

and depicted in Fig. 5.

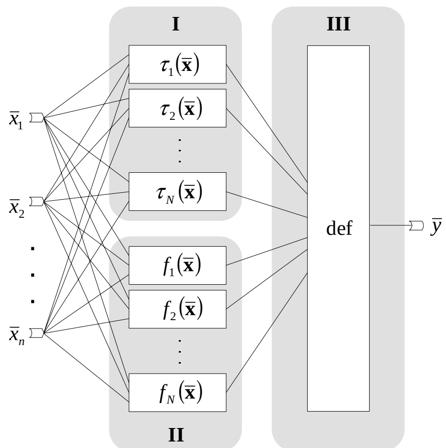

Fig. 5. The scheme of system (12)

4. Simulation Results

The flexible Takagi-Sugeno system, described by formula (12), is simulated on the Chemical Plant problem [14].

We deal with a model of an operator's control of a chemical plant. The plant produces polymers by polymerisating some monomers. Since the start-up of the plant is very complicated, men have to perform the manual operations at the plant. Three continuous inputs are chosen for controlling the system: monomer concentration, change of monomer concentration and monomer flow rate. The output is the set point for the monomer flow rate.

The experimental results for the Chemical Plant problem are depicted in Tables 1 and 2 for the not-parameterised (algebraic) and parameterised (Yager) t-norms, respectively. The final values (after learning) of weights $w_{i,r}^{\tau} \in [0,1]$ and $w_r^{\text{def}} \in [0,1]$, $i = 1,\ldots,3$, $r = 1,\ldots,6$, are shown in Fig. 6.

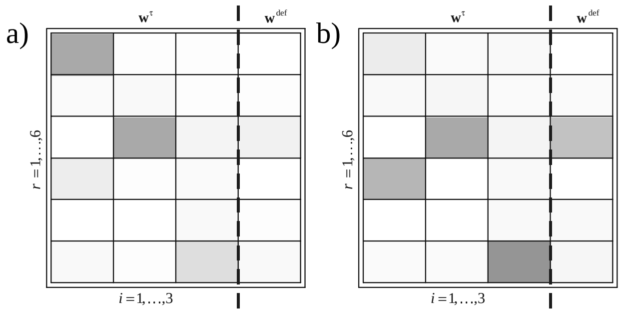

Fig. 6. Weights representation in the Chemical Plant problem for system (12) and a)soft weighted product t-norm, b)soft weighted Yager t-norm

Table 1. Values of flexible parameters

rule	System with algebraic t-norms	System with Yager's t-norms	
	α_r^τ	α_r^τ	p_r^τ
1	1.00	0.99	4.9952
2	0.98	0.98	4.9617
3	0.99	1.00	4.9882
4	0.98	0.97	5.0290
5	0.97	1.00	5.0064
6	1.00	0.98	4.9973

Table 2. Experimental results

Type of the system	RMSE (learning sequence)
System with algebraic t-norms	0.0042
System with Yager's t-norms	0.0035

Table 3. Comparison table

Lin and Cunningham [5]	0.0079
Pal and Chakraborty [9]	0.0092
Rutkowski [10]	0.0042
our result	0.0035

5. Final remarks

In the paper we studied flexible Takagi-Sugeno neuro-fuzzy systems. The results were illustrated on the Chemical Plant problem. Table 3 shows that, we achieved the best performance comparing with previous approaches.

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