# **Evaluation of Rock Acoustic Properties by a Parametric Spectral Analysis**

A. Lachouri<sup>1</sup> N. Doghmane<sup>2\*</sup>, T. Bouden<sup>3</sup>, B. Harhoud<sup>2</sup> and H. Boumaaraf<sup>1</sup> 1 Laboratoire d'Automatique Appliquée LAASK, Université de Skikda.
2\* Dpt. d'Electronique, Faculté des Sciences de l'ingénieur, Université de Annaba. 3 Dpt d'électronique Université de Jijel

ALGERIA

*Abstract:* In this paper, a new approach on non-destructive evaluation by ultrasonic signal is proposed. This work is based on a parametric spectral analysis. Indeed, the ultrasonic velocity measurement in rocks is often obtained by transmitting ultrasonic pulses through the specimen. The longitudinal waves (P) are chosen in the experiment analysis (the case of transversal waves (S) will be tested in other studies). This measurement is simple and can allow an easy identification of the acoustics parameters of the rock. The rock acoustic characteristics give several indications on its state of alteration, the grains size and micro crakes.

Keywords: Ultrasonic, Attenuation, Porosity, Micro cracks, parametric estimation, PSD, Prony, Resolution.

# **1. Introduction**

Ultrasonic rocks characterization techniques can be used for the recognition of the internal constitution of specimen (in this case: The rock). This is possible by using existing pulse method and appropriate digital signal processing. Different parameters extracted from the processed ultrasound have been considered in the literature, with degrees variable of success. The present work deals with rock characterizations using ultrasonic nondestructive evaluation method. The ultrasonic velocity measurement in rocks is often obtained by transmitting ultrasonic pulses through the materials (rock sample) [1, 2]. This measurement is easy, fast and offers the possibility for the identification and the estimation of rock acoustic parameters. These parameters can be used to give significant indications on the state of alteration, grain size and micro cracks of the sample (rock) [3] etc...

# 2. Problem position

Digital signal processing involves techniques that improve our understanding of information contained in received ultrasonic data. Normally, this signal is measured and viewed in the time domain. When the frequency content of the signal is of interest, it makes sense to study the signal in the frequency domain (spectral analysis).

The spectral analysis of the ultrasonic signals can be used as a processing technique to permit, then, a nondestructive diagnosis of the rocks. The goal is to evaluate several acoustic characteristics of the rocks.

Generally, the diagnosis is done by the calculation of the attenuation of the ultrasonic signal transmitted through the rock (attenuation and/or quality factor) or by the measurement of the delay of this same signal (for wave velocity measurement inside the solid). This method, which is known as quantitative analysis, does not provide enough information and some times its results are not reliable.

The purpose of this study is to improve the characterization using a spectral estimation with linear prediction for a sharp and accurate analysis. In others words, the defaults, lacunae and pores, which may exist in the samples, are identified and isolated. The method is based on the emission reception of an acoustic longitudinal wave passing through rocks.

This paper contains a description of quantitative analysis method of rock acoustic properties and a description of qualitative analysis method based on autoregressive (AR) spectral estimation.

# **3.** Description of the experiment

A bloc diagram of the experiment set-up is given in fig.1. In these tests, the pulse transmission method is used. The experiment set-up allows the study of attenuation coefficient and longitudinal wave velocity obtained from the pulse transmitted through cylindrical samples of rocks.

The pulses generated from an ultrasonic generator and transducers (transmitter and receiver) of 150KHz, 300KHz and 500KHz central frequencies. Acoustic contact, between transducers and samples, is made with couplant (grease) and a weight of about 0.2Mpa is also applied to improve wave propagation. The waveform of the signal is visualized by a digital scope and transferred to the computer by the liaison RS232C as digital data file for further analysis. The experiment was carried out on two different types of rocks and two samples of Aluminum (Al1 and Al2) as a reference.



Fig 1: Experiment set-up

The samples tested (three marbles and two dolomites) are cylinders respectively of 57mm in diameters and 91mm in length and 63mm in diameters and 72mm in length. The table 1 below gives different parameters of tested rocks with 150KHz transducers, and the figures are obtained with 60KHz transducers.

Designation	Density	Volumique-	Porosity	Velocity	Attenuation
	g/cm <sup>3</sup>	mass	%	m/s	Coef a
	-	g/cm <sup>3</sup>			
MARBLE1	2.650	2.580	2.630	$2.42 \times 10^3$	0.131
(72mm length)					
MARBLE2	2.640	2.560	3.010	$2.62 \times 10^3$	0.096
(91mm length)					
MARBLE3	2.680	2.620	2.200	$2.41 \times 10^{3}$	0.138
(105mm length)					
ALUMINIUM1	/	/	/	$2.53 \times 10^3$	0.000
72mmlength)					
ALUMINIUM2	/	/	/	$2.50 \times 10^3$	0.000
(72mmength)	,	,	,		

Tab	le 1	:	different	parameters	of	tested	roc	ks
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# 4. Quantitative Analysis

It is understood by quantitative analysis of rock acoustic properties the methods used to determine attenuation factor ( $\alpha$ ) and/or quality factor (Q) and propagation velocity ( $V_p=L/t_p$ , L is the length of the sample and  $t_p$  is the propagation time ) of ultrasonic signal transmitted through rock samples. Two methods exist for determining Q or  $\alpha$ :

### 4.1 Rise time method [4]

This method is based on characterization the transmitted signal by the rise time of the first arrivals, propagation time and magnitude of the first arrival. Example of The time representation of ultrasonic signal can be obtained from experiment is shown in fig.2.

#### 4.2 Spectral ratio method

The spectral ratio (SR) method consists on evaluating the attenuation of a material by comparing the obtained signals (longitudinal waves), in the same experimental conditions, to the signal of the material (Al1) of reference (see fig 3, 4 and 5). The Al1 is used because of its very low attenuation [5]. The attenuation of different spectra of ultrasonic signal transmitted through different rocks is noticeable compared to the attenuation of A1. this gives information on the state inside samples. The attenuation is mainly due to porosity, micro cracks; size of grains, etc...



Figure 2: Example of ultrasonic signal

### **5.** AR parametric spectral analysis

With this method, an analysis of the signal with an AR (Autoregressive) estimator is proposed [6]. Indeed, the ultrasonic signal transmitted through rock is considered as the exit of AR filter:

$$\widetilde{y}(k) = -\sum_{i=1}^{M} a_i y(k-i) + b(k) \tag{1}$$

where y(k): is the output sequence;  $a_i$  the AR filter coefficients; b(k) the noise of measurement.

This enables to write

$$b(k) = Y^{T} A_{M}$$
(2)  

$$A_{M}^{T} = [a_{1}, a_{2}, ..., a_{M}]$$
  

$$Y^{T} = [y(n), y(n-1), ..., y(n-m)]$$
  
The multiply of equation (2) by Y<sup>\*</sup> gives  

$$Y^{*}b(k) = Y^{*}Y^{T}A_{M}$$
(3)

(4)

 $P_M = \Phi_M A_M$ Where

$$\Phi_M = E[Y^*Y^T] = \begin{bmatrix} \Phi_0 & \Phi_1 & \dots & \Phi_M \\ \Phi_1^* & \Phi_0 & \dots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \Phi_M^* & \Phi_{M-1}^* \dots & \Phi_0 \end{bmatrix}$$

 $P_M$ : is the power spectral density (PSD) of a white noise.

The system of equation represented by (4) is often called: equations of Yule-walker. The solution of the vector  $A_M$  of AR model is given by a recursive algorithm (example recursive last square).

### 6. Prony method

To consider the characteristics of the anomalies affecting the rock, a method of high resolution is chosen. Indeed, for example the acoustic characteristics of a pore or a slit, by using a spectral analysis, can present much closed frequencies [6]. The Prony method is often used in this type of spectral analysis.

The process to analyze is assumed to consist of M sinusoidal components:

$$x(n \Delta T) = \sum_{i=1}^{M} A_i \cos(2\pi f_i \Delta T + \phi)$$
  
$$= \sum_{i=1}^{M} \left[ \frac{A_i}{2} \exp(j2\pi f_i \Delta T + j\phi_i) + \frac{A_i}{2} \exp(j2\pi f_i + j\phi_i) \right]$$
  
$$= \sum_{i=1}^{M} \left[ C_i \phi_i^n + C_i^* \phi_i^*^n \right]$$
(5)

where  $C_i = \frac{A_i}{2} \exp(j\Phi i), \Phi_i = \exp(j2\pi n f_i \Delta T)$ 

And \* denotes complex conjugate. The equation (5) may be defined by this expression:

$$\prod_{i=1}^{M} (\psi - \phi_i) (\psi - \phi_i^*) = \sum_{i=1}^{2M} a_i \psi^{2m-1} = 0$$
(6)

with  $a_0=1$  where the  $\{\phi_i\}$  are the roots of unity.

As the sum of the M sinusoidal components can be modeled with the error:  $\varepsilon_i$ , so  $\varepsilon_i = \sum_{i=0}^{2M} a_k x_{2m+1-k}$ 

for 
$$I=1,2,\ldots,N-2M$$
 (7)  
satisfy the polynomial equation:

The roots of (5) also satisfy the polynomial equation:  $\sum_{k=0}^{2M} a_{k}\psi^{i} = 0$ ,  $a_{2M} = 1$ (8)

The coefficients  $a_j$  can be extracted:  $a_j = a_{2M-j}$ , with  $a_0 = a_{2M}$ , and the number of coefficients to be determined is reduced by one half.

$$\varepsilon_{i} = \sum_{i=0}^{2M} (a_{k} x_{2m+1-k} + x_{2M-k+i})$$
(9)

The coefficients  $a_1, \ldots, a_M$  are determined, using the least square method, by minimizing the total square error  $E = \sum^{N-2M} \varepsilon_i^2$ (10)

$$\sum_{j=0}^{M} \left[ \sum_{i=0}^{2M} (x_{2m-k} + x_{k+i}) (x_{2M-j+i} + x_{j+1}) \right] = 0$$

for k=1,...,M from the estimated  $\{a_i\}$  values, the  $\{\phi_i\}$  are determined using equation (6). This gives the frequencies estimates. To obtain the  $\{C_i\}$  a second set off normal equation is solved.

$$\sum_{j=0}^{N} \left[ C_{i} \sum_{j=0}^{N} \phi_{k}^{j} \phi_{i}^{j} + C_{i}^{*} \sum_{j=0}^{N} \phi_{k}^{*j} \phi_{i}^{*j} \right] = \sum_{j=0}^{N} 2 \left( real \phi_{i}^{j} \right)_{x_{i}}$$
(12)

For  $k=0,1,\ldots,M$  the  $\{C_i\}$  provide both amplitude A (or power) and phase  $\{\phi_i\}$ .

### 7. Results and discussion

Several rock samples are tested in this work (ultrasonic signals of Al1, Al2 and of marble 1, 2 and 3 are shown on figures 3, 4, 5, 6 and 7). Their prediction errors are shown on the same figures. The PSD obtained, using the FFT and AR model, are shown on figures (8, 9, 10, 11 and 12).

The PSD is calculated starting from the coefficients of the AR filter. These coefficients, which the number indicates the filter order, are calculated by an adaptive algorithm. First, the fig.13 shows the attenuation factor between the estimated spectra of aluminum, as reference, and the analyzed samples. The results obtained are satisfactory for the PSD estimation by the parametric methods (Prony, or the predictor filters). In fact, the diagnosis can be done by the prediction error when the spectra (or PSD) present quick changes which indicate the existence of micro cracks and/or pores inside the specimen. From these spectra, a localization of different faults and pores inside the specimen can be detected. The table2 gives the filter coefficients of the AR model calculated by the Prony algorithm. The normalized estimated frequencies obtained by different methods are shown on the table 3. From these frequencies, a considerable amount of information which can be used in identification of different faults existing inside the rocks.

Coef	$a_0$	a1	<b>a</b> <sub>2</sub>	a <sub>3</sub>	a4
Al1	1.00	0.1946	0.7355	-0.217	-0.0129
Al2	1.00	-0.1236	0.8345	-0.362	-0.2093
MA1	1.00	0.4699	0.1164	-0.612	-0.0161
MA2	1.00	0.5146	-0.013	-0.801	-0.1889
MA3	1.00	0.6693	0.1436	-0.688	-0.2479

Table 2: AR estimated by Prony

Freq	F estimated by burg		F estimated by prony		
Al1	0.2881	0.5000	0.2881	0.5000	
Al2	0.1537	0.2891	0.1542	0.2890	
MA1	0.3525	0.5000	0.3526	0.5000	
MA2	0.3521	0.5000	0.3520	0.5000	
MA3	0.3487	0.5000	0.3487	0.5000	

Table 3: frequencies estimated by Prony and Burg

### 8 Conclusion

(11)

The nondestructive analysis by ultrasonic signals ensure the acoustic characterization of rocks and objects without their destruction. In fact, a parametric spectral estimation (AR, ARMA, etc...) of the system representing the material to be studied, can solve the problem of quantitative analysis of rocks. This can help to identify and to estimate the system power spectra density in a recursive manner. From these spectra, a localization of different faults and pores inside the specimen can be detected.



Fig 5: Signal and spectra of marble 1



Fig 8: Signal and PSD (by FFT, Prony and Burg) of aluminum 1



Fig 9: Signal and PSD (by FFT, Prony and Burg) of aluminum 2





Fig 11: Signal and PSD (by FFT, Prony and Burg) of marble2





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