

# Adaptive Edge Detection with Directional Wavelet Transform

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*Abstract:* - This paper presents adaptive edge detector using a novel directional wavelet transform. The proposed algorithm has two stages: a) directional wavelet transform and b) edge detection on space-scale-directional plane with maximum entropy measure. Preliminary results with synthetic images show that directional wavelet transforms gives excellent results. The proposed method was tested on synthetic images at different signal-to-noise ratios (SNRs) and visually assessed on medical image. We assessed its reliability, accuracy and robustness using the mean absolute distance (MAD) metrics.

*Key-Words:* - Wavelet transform, Directional filter, Edge detection

## 1 Introduction

The process of edge detection is based on the hypothesis that the edge is a point where an image has sharp intensity transitions [1, 2, 3]. Important regions of interest (ROI) are separated by different level of pixel intensity value. Upon this assumption, many edge detectors have been proposed. Most of them depend on the local pixel intensity gradient, done by differencing [2] as a calculation of convolution of weighted matrix called local gradient mask. This group consists of well-known edge detectors, such as Sobel, Roberts, Prewitt, Robinson, Kirsch, Frei-Chen and Marr-Hildreth [3]. Their major drawbacks are high sensitivity to noise and disability of discrimination edges versus textures. Because of these limitations more advance edge detectors have been proposed which do not only detect edges but also try to connect neighbouring edge points into a contour. In this way, many authors have developed different edge detectors based on the scale space [6], active contours [5], and morphological operations [10]. Among all, the fundamental one is Canny edge detector [4], which is fast, reliable, robust and generic, but the accuracy is not satisfactory, because of the parameter  $\sigma$  which is the weakest point in the procedure [7].

The purpose of this paper is to present an edge detector based on directional wavelet transform. Analyzing a signal at different scales and directions increases the accuracy and reliability of edge detection. Focusing on localized signal structures, e.g. edges, with a zooming procedure enables simultaneous analysis from a rough to a fine shape [8]. Progressing between scales also simplifies the discrimination of edges versus textures [9]. Because

of this ability the wavelets have also been used for edge detection in different applications.

Wavelets are well adapted to singularities that are commonly found in real-life signals. In multidimensional cases, most often tensor product wavelets are employed. Therefore, wavelets are well adapted in higher dimensions for pointlike phenomena. But this is the only type of singularities that wavelets can efficiently represent. This problem was raised recently by Candes [12] who argued that in higher dimensions, there are many other kinds of intermittency such as singularities along lines and curves which wavelets do not deal with efficiently. In order to overcome this weakness, we have develop a new system of representations named directional wavelet transform which can effectively deal with linelike phenomena in 2-D.

The paper is organized as follows: in Section 2 the directional wavelet transform is presented. Section 3 deals with the edge detector. Section 4 demonstrates the experimental results, while Section 5 concludes the paper.

## 2 Directional Wavelet Transform

Natural images are not simply stacks of 1-D piecewise smooth scan-lines; discontinuity points (i.e. edges) are typically located along smooth curves (i.e. contours) owing to smooth boundaries of physical objects. Thus, natural images contain *intrinsic geometrical structures* that are key features in visual information [11].

The wavelet transform [8, 9] has a long and successful history as an efficient image processing

tool. However, as a result of a separable extension from 1-D bases, wavelets in higher dimensions can only capture very limited directional information. For instance, 2-D wavelets only provide three directional components, namely horizontal, vertical, and diagonal. Furthermore, the 45° and 135° directions are mixed in diagonal subbands.

A number of approaches in providing finer directional decomposition have been proposed. Some notable examples include 2-D Gabor wavelets [13], the steerable pyramid [14], the directional filter bank [6], 2-D directional wavelets [15], complex wavelets [16], curvelets [12], ridgelets [12], and contourlets [11]. However, there is a question “Can we extend the 1-D wavelet transform with finer directionality, while still retaining its structure and desirable features?” We give an affirmative answer in this paper by proposing a simple directional extension.

The main idea here is to find some directional extension to 1-D wavelets. In particular, we want to have a system which has characteristics of wavelets plus directions.

Firstly, we extended 1-D wavelet  $\psi_\alpha(n); n=0, \dots, N-2$  to a 2-D wavelet kernel as:

$$\kappa_\alpha(m, n) = \psi_\alpha(n) \text{ for any } m \in [0, M] \quad (1)$$

where  $\alpha$  is the scale parameter. The result is 2-D wavelet kernel which can be used in wavelet transform as follows:

$$K_s(\alpha, \tau_1, \tau_2) = \sum_{m=0}^M \sum_{n=0}^N \frac{1}{\sqrt{\alpha}} s(m, n) \kappa_\alpha(\tau_1 - i, \tau_2 - j) \quad (2)$$

where  $M$  and  $N$  stand for the image size,  $s(m, n)$  is input image and  $\kappa_\alpha(m, n)$  is 2-D mother wavelet kernel. This kernel is a prototype from which all other kernels are generated. The supplement of (2) with directional characteristics is given by:

$$K_s(\alpha, \tau, \delta) = \sum_{i=0}^M \sum_{j=0}^N \frac{1}{\sqrt{\alpha}} s(i, j) Q(\delta) \cdot \kappa_\alpha(\tau_1 - i, \tau_2 - j) \quad (3)$$

where  $Q(\delta)$  is rotator that rotates kernel  $\kappa_\alpha$  in image counterclockwise through an angle  $\delta$ . The result of directional wavelet transform is a image-scale-directional space.

The main benefit of directional wavelet transform is singularities detection along lines.

### 3 Edge detector

We transform the problem of image edge detection into a search of sudden amplitude changes in image signals taken along neighbouring rows or columns.

#### 3.1 Adaptive edge detection using directional wavelet transform

Directional wavelet transform maps an image into a four-dimensional space of  $\alpha, \tau_1, \tau_2$  and  $\delta$ . Parameter  $\alpha$  scales the transform by compressing or stretching it. Parameters  $\tau_1, \tau_2$  corresponds to the translation of the wavelet function along the image rows and columns. Parameter  $\delta$  rotates the wavelet kernel  $\kappa_\alpha$ . An important property of wavelet transform is its ability to focus on localized signal structures with a zooming procedure that progressively reduces the scale parameter  $\alpha$ . In this way, rough and fine signal structures are simultaneously analysed at different scales. It will be shown that this property is also important for real edges detection.

We decided for wavelet kernel which is made from Haar wavelets (3), because it is orthogonal, compact and without spatial shifting in the transform domain. Its main property is ability to present the magnitude variation between adjacent intervals in signal as a modulus maximum on the time-scale plane [8]. This property is also preserved in the image-scale-directional space.

Natural images are corrupted by the noise which is presented as magnitude variation in image. Such noise appears in image-scale-directional space the same way as additional edges. But there are two principal differences between edges and noise: a) modulus maxima of edges are larger than noise modulus maxima if the signal-to-noise ratio (SNR) is low, and b) the influence of noise decreases with progressing toward higher scales if noise is additive with zero mean, because Haar wavelet kernel perform averaging. Hence, the influence of noise is gradually filtered out going toward higher scales and its maxima become negligibly small.

Considering the findings above, we made two major conclusions about the edge detector, which are: a) the corresponding maxima between scales must be linked if we wanted to avoid multiple detection of the same edge at different scales, and b) a decision whether a particular maximum, regarding to other

maxima, is significantly different or not must be adaptive.

Modulus maxima carry a significant degree of information about the position of edges. Therefore, our goal is to identify the local maxima of the directional wavelet transform in all scales. But the edges are, in general, presented as significant peaks in more than one scale and therefore we need to connect them. Maxima line is an answer to this demand. It is a connected curve along which all points are maxima, but it could break up and stop before reaching the finest scales [8]. It links the same edge points from finer to coarser scales.

When dealing with low SNRs, the number of maxima lines may be very large. In order to select the maxima lines of image edges, we introduced the following assumption: if the maxima line has at least one modulus maximum which is above a certain threshold  $T$ , then this maxima line is edge maxima line and the position of maximum at the finest scale reflects the edge position. In the proposed edge detector a maximum entropy thresholding technique is used [17].

The algorithm of the proposed wavelet-based edge detector is executed in six major steps which are:

1. Directional wavelet transform with Haar wavelet kernel at dyadic scales from 1 to  $\lfloor \log_2(\min(I, J)) \rfloor$  is calculated, at directions between 0 and  $\pi$  with increments  $\pi/12$ . Dyadic scaling sequence minimizes the time complexity of the algorithm, while the quality of edge detection is not reduced. The result is image-scale-directional space.
2. Dimension reduction thus that maximal absolute value in dimension vector is kept. The result of dimension cutting down is image-scale space  $S(\alpha, \tau_1, \tau_2)$ .
3. Modulus maxima detection is done along  $\tau_1$  and  $\tau_2$  directions in image-scale space  $S(\alpha, \tau_1, \tau_2)$ . The result is a plane,  $MM$ , of modulus maxima.
4. Connecting the corresponding maxima from  $MM$ , according to the procedure introduced in [8], the maxima lines are obtained. The result is set  $S$  of modulus maxima lines.
5. Adaptive maxima selection: a threshold value  $T$  is calculated upon the absolute value in space  $S(\alpha, \tau_1, \tau_2)$ . for each scale and detection direction (row or column). The edge is considered where maxima surpass the threshold  $T$ .

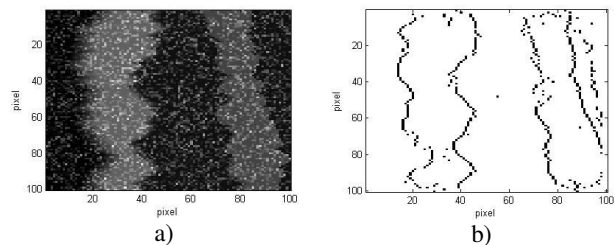
6. Edge position determination: the position of modulus maximum at the lowest detected scale determinates the edge position.

The proposed edge detector relies on the directional wavelet transform and the adaptive edge modulus maxima detection. It does not require explicit specification of any additional input regulation parameters and it is able to detect real edges which actually appear as edge region with different widths. Detection of edges solely depends on the information in a row or column signal in space  $S(\alpha, \tau_1, \tau_2)$  and consider information of the adjacent rows or columns. Inclusion of a wider area brings additional information about the edge outline.

## 4 Experimental results

The proposed edge detector was tested on a set of real images, as well as on synthetic images. Firstly, several tests were performed on synthetic images. The performance of the proposed edge detector was evaluated by mean absolute distance (MAD) [12]. Beside statistical evaluation we made also a visual inspection.

Synthetic images used in our experiment were corrupted by additive zero-mean Gaussian noise with SNRs interval from 3 dB to 40 dB. Figure 1a is an example of them (SNR was 4 dB). Figure 1b is the edge image obtained by the proposed edge detector. The results are good. The contour lines are almost entirely detected. The MAD measures 0.75 pixels.

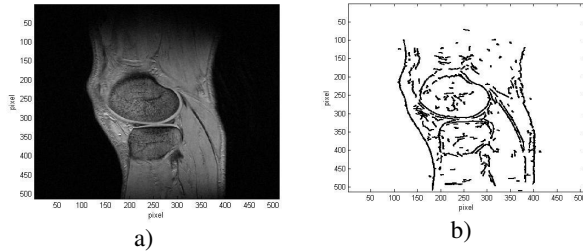


**Figure 1: Example of synthetic images (a) and their corresponding edge detection (b).**

Differences between the actual and detected edges are less than 1 pixel, which means that the obtained edge positions are really accurately detected. The algorithm was tested on 30 synthetic images. The accuracy of our detector was around 0.53 pixel in almost noiseless images. Excellent results were also obtained in very noisy images (around 6 dB) where the errors were around 1 pixel. The average MAD distance was 0.68 pixels, the standard deviation was 0.12 pixels, the minimum distance was 0.50 pixels,

and the maximum distance was 1.04 pixels. This indicates that the edge points have been very accurately and consistently detected.

Furthermore, we experimented edge detection in MR images (Fig. 2a). Figure 2b depicts the result of edge detection. With the visual assessment we can confirm that significant structures are outlined.



**Figure 2: MR image edge detection with our proposed method (a) MR-image and the corresponding edge detection (b).**

## 5 Conclusion

We have introduced adaptive edge detection with directional wavelet transform, which is robust to the outliers and very accurate. With directional wavelet transform modulus maxima are detected, which are connected into maximum lines. Then with maximum entropy measure, the edge maxima lines are selected and the position of modulus maximum at the lowest detected scale determinates the edge position. The result is an edge image. We showed that the introduced edge detector is robust, accurate and effective even in images where the presence of considerable noise and outliers is unavoidable.

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