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Both Chaotic and Harmonic Signals Generation: Closing the Loop around the Duffing System

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Abstract — A new autonomous oscillator topology is proposed, based on a feedback loop around the Duffing non-autonomous system and capable of generating both chaotic and almost harmonic signals. The main idea is to filter the wide-band chaotic output signal by means of a high selectivity band pass filter (BPF), implemented with quartz crystals and connected in a feedback loop. Simulation results for several important particular cases confirm the predicted behavior.

Key-Words: - nonlinear systems, chaotic signal generation, Duffing system

1 Introduction

Generating both harmonic and wide band signals inside the same system is desirable in many applications, including communication systems test equipment, computer clock generation, chaotic encryption and chaos communication.

Non-linear non-autonomous systems have received increasing attention recently as regards their possibility to behave chaotically, under certain particular conditions, and in this way generating useful unpredictable signals. Their non-autonomous character is generally treated for the particular case of harmonic input signal. Depending on the particular implementation of the non-linear circuit, series/parallel RLC circuits with one of its elements being nonlinear were studied e.g. in [4], [5].

Special attention was paid to the Duffing differential equation, regarded as a particular case of the aforementioned class of chaotic systems. In-depth studies on the dynamics of circuits described by a Duffing like differential equation were made. Simulation results were reported in [1], highlighting the phase portraits and attractor-basin for the forced Duffing circuit.

The most general equation for the forced Duffing circuit is:

$$\frac{d^2y}{d\tau^2} + 2k\frac{dy}{d\tau} + y + \lambda y^3 = B\sin(\nu\tau + \gamma) \quad (1)$$

Its simplified form, as it can be found in [1], [3], is:

$$\frac{d^2y}{dt^2} + k\frac{dy}{dt} + y^3 = B\sin(\varpi t) \qquad (2)$$

The factor k, in the second term, shows that the circuit is dissipative and it is important to note that its magnitude is a determining parameter of how fast/slow the oscillations damp/latch-up rather than as a chaos control parameter.

For state space characterization, the equation (2) is re-written as a system of two first order differential equations:

$$\begin{cases} \dfrac{dx_1}{dt} = -k\dfrac{dx_2}{dt} - x_1^{\,3} - B\sin\varpi t \\ \dfrac{dx_2}{dt} = x_1 \end{cases} \qquad (3)$$

which, in turn, can be re-written in vector-matrix form:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}' = \begin{bmatrix} -k & 0 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} -x_1^{\,3} - B\sin\varpi t \\ 0 \end{bmatrix} \quad (4)$$

The new variables denoted as:

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$$\begin{cases} x_1 = \frac{dy}{dt} \\ x_2 = y \end{cases} \quad (5)$$

are the state variables of the Duffing circuit and the link between the equations (3) and (2).

In almost all situations the amplitude of the excitation signal, $e(t)$, denoted as B, is the main control parameter:

$$e(t) = B \cdot \sin\omega_0 t \quad (6)$$

Other control parameters are the dissipative term coefficient, k in equations (1-5) and the input signal -frequency $f_0 = \omega_0 / 2\pi$ in equations (6). Moreover, in [3], a control strategy for such a non-autonomous circuit is developed, in order to produce appropriate chaotic and non-chaotic reference trajectories or switching between the two. But, regarded as a block diagram, the entire circuit proposed in [4] is still non-autonomous, needing a harmonic signal as in equation (6) as input.

2 The Autonomous Duffing like Oscillator

The basic block diagram of the proposed system is shown in Fig.1.

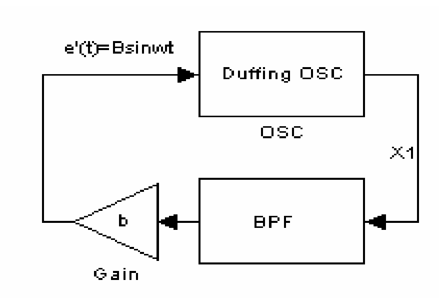

Fig.1 Block diagram of the proposed autonomous system

The main idea of the proposed circuit is that of transforming the classic, non-autonomous one into an autonomous oscillator by band-pass filtering the output signal of the Duffing circuit, amplifying it, and driving with the filter output signal, $e_f(t)$, the non-autonomous Duffing circuit. The newly obtained signal, $e_f(t)$, plays the part of $e(t)$ in the classic forced Duffing circuit.

The transfer function of the BPF of Fig.1 is given by:

$$H(s)_{BPF} = \frac{b_1 s}{s^2 + 2\alpha s + \omega_0^{\ 2}} \quad (7)$$

where $G_0 = b_1/2\alpha$ is the filter's gain at the central frequency ω_0, and $Q = \omega_0 / 2\alpha$ is its quality factor.

In order to force a chaotic behavior in the proposed circuit the BPF should have the highest selectivity possible, which means, for a second order filter, a high quality factor. This is the reason why the BPF is thought of as being made in quartz technology, ensuring a quality factor of, at least, 10^4.

The main control parameter remains the amplitude of $e_f(t)$ as in the case of the classic forced Duffing oscillator.

It must be mentioned that the BPF's gain G_0 and the gain b are interdependent, the sense that the amplitude B of $e_f(t)$ can be set at a certain value by modifying the value of any of the control parameters.

In order to study the circuit dynamics and compare it to that of the classic forced Duffing one, waveforms of the state variables x_1, x_2, and the forcing signal are presented. In Fig.2.a) and Fig.2.b) are presented examples of simulated variation in time of the state variable x_1 for the proposed circuit and classic Duffing circuit, respectively.

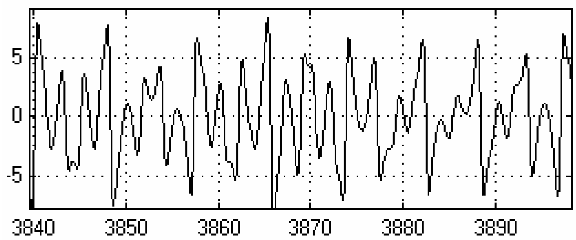

Fig.2.a) Time variation of the state variable x_1 in the proposed circuit

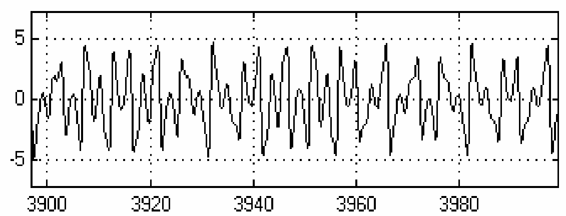

Fig.2.b) Time variation of the state variable x_1 in the forced Duffing circuit

From their power spectra, depicted in Fig.3.a) and Fig.3.b) we can further argue their chaotic character.

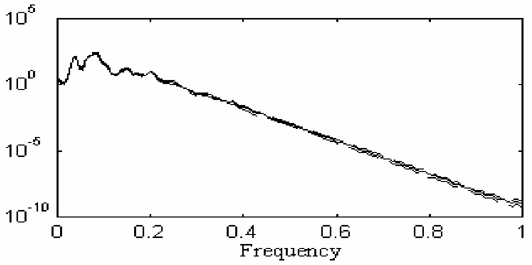

Fig.3.a) The power spectrum of the chaotic signal shown in Fig.3.a

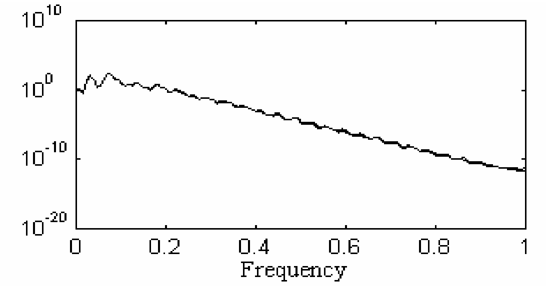

Fig.3.b) The power spectrum of the chaotic signal shown in Fig.3.b

The output of BPF, $e_f(t)$, which plays the part of the classic forcing harmonic signal, depicted in Fig.4.b), is shown in Fig.4.a). It can be seen that $e_f(t)$, the filtered chaotic signal, is a reasonable approximation of a harmonic one.

It must be noted that BPF's selectivity is a useful control parameter, in that it modifies the degree of approximation of the ideal input signal. For small values of the filter quality factor, chaotic behavior could not be achieved, as can be seen in Fig.5, for Q=100 and the rest of the parameters kept unmodified. This is the cause for the necessity of a quartz implementation. A value of $Q=10^4$ or greater is enough to determine, by setting appropriate G_0 or b, a chaotic behavior as it can be seen in Fig.3.a).

The following numerical values were used in the simulation examples presented: k=0.0015, ω_0=1 rad/sec, 2α = 0.0001, a_1 = 0.8, b = 1, and the initial conditions $(x_{10}, x_{20}) = (0.1, -1)$.

For the presented numerical values the phase portrait (x_2 vs. x_1) shown in Fig.6 was obtained, suggesting the existence of a chaotic attractor.

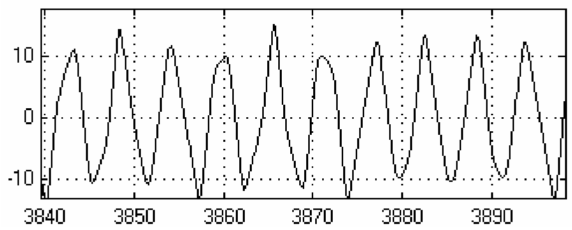

Fig.4.a) The almost harmonic signal e_f (t) in the proposed circuit

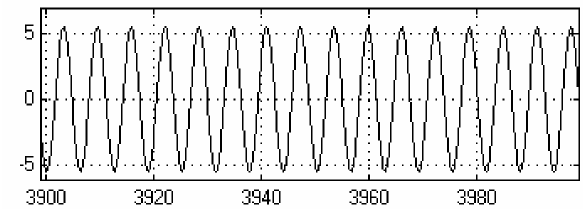

Fig.4.b) The forcing harmonic signal e(t) in the classic Duffing oscillator

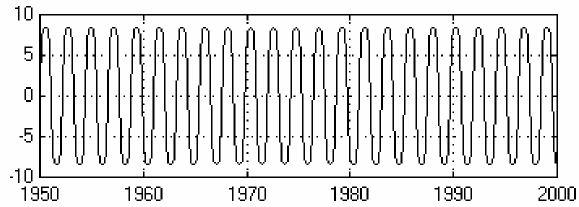

Fig.5 Time variation of the state variable x_1 of the proposed circuit for Q=100

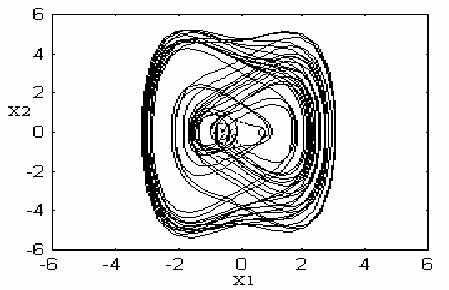

Fig.6 The phase portrait suggesting the existence of an chaotic attractor

Since it is known that in the case of the classic forced Duffing oscillator the frequency of the forcing sinusoidal signal is also a control parameter, the central frequency of BPF from Fig.1 was modified around nominal value. At central frequencies ω_0 greater than 3rad/sec, numerical instability was observed. At frequencies less than 1rad/sec the output signals x_1 and x_2 tend to become periodical, as it can be seen in Fig.7.a) and b).

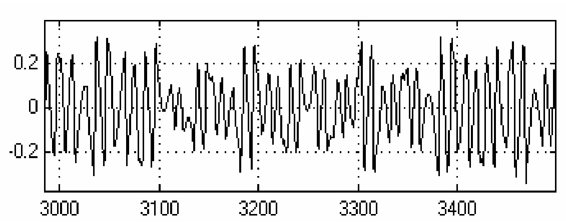

Fig.7.a) Time variation of the state variable $x_1$of the proposed circuit for ω_0=0.1rad/sec

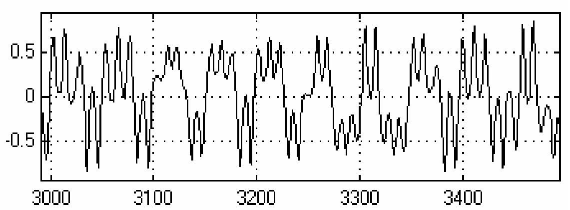

Fig.7.b) Time variation of the state variable $x_2$of the proposed circuit for ω_0=0.1rad/sec

The greater Q is (>10^5-10^6), the closer to a harmonic signal the output of BPF will be but with its amplitude smaller. This means that a greater gain b will be needed in order to determine the circuit to behave chaotically. A tradeoff must be made between a quality factor as large as possible and a gain b not to large in order for a chaotic response to be get and numerical stability not to be obtained.

The bifurcation diagram presented in Fig. 8, was developed as it was considered in [2], [3], in order to see the b-ranges, where the circuit behaves chaotically.

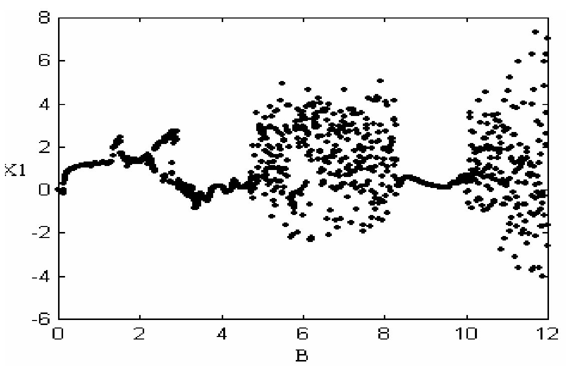

Fig.8 The bifurcation diagram x_1 vs. the gain b in the loop of the autonomous Duffing oscillator

3 Conclusions

We proposed a non-autonomous oscillator based on Duffing circuit, capable of generating chaotic output signal and an almost harmonic one. The chaotic behavior can be controlled by means of the classic control parameter, the amplitude of the harmonic signal, and using the filter gain, center frequency and quality factor as new ones. Some simulation results are reported as arguments for the desired operation of the proposed circuit.

Further research is needed to analyze system parameters, such as Lyapunov exponents, and atractor dimensions in order to demonstrate rigorously its chaotic dynamics.

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