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Fractional Feedback Control for a Hinged Flexible Beam Attached to a Rigid Arm via Diffusive Representation

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Abstract: This paper presents the stabilization of a hinged flexible beam attached to a rigid arm. The applied control at the articulation point consists of the absorbing waves with the goal of minimizing the reflection of the energy at this point. The stabilization of the beam involves fractional operators that can be achieved in the non-hereditary way using a diffusive model. The numerical simulation results clearly show the powerful and the effectiveness of the control using diffusive representation.

Key-Words: Flexible beam, Wave absorbing control, Traveling wave, Fractional operator, Diffusive representation

1 Introduction

Control of a flexible structure is one of the main themes in control engineering. The traveling wave approach used in [ ][ ]is based on the property that the response of the flexible structure can be viewed as a superposition of the waves traveling in a flexible structure. The boundary conditions at an actuator might be written as the relation of reflective waves with the incident waves.

Von Flotow and Shafer [ ] developed a wave absorber for flexible cantilevered beam by using a control force which leads to the impedance matching at the free end of the beam.

This paper proposed a control of a hinged-free flexible beam attached to a rigid arm by means force control and torque applied at the articulation point. The idea works on the basis of absorbing the traveling waves introduced in [ ]. Such wave-absorbing controller involves fractional integrals and derivators. The major inconvenient associated to the fractional operators is the hereditary behavior. Therefore, the employment of mathematical analyses tools, such as stability analysis and numerical approximation is very difficult. We used a new approach called "diffusive representation" to alleviates these difficulties.

The diffusive representation introduced in [ ] allows to achieved the fractional operators in a non-hereditary way and simplify the study as much as the stability and the numerical approximation. The application of such representation in the flexible beam control leads to obtain a global system that can be expressed in the standard form dX/dt = AX.

This paper is organized as follows. In section 2, we present the model of the hinged flexible beam. In section 3, we describe the dynamics of the beam in term of traveling waves, and the fractional feedback controller. Section 4 introduces the diffusive representation of the fractional operators in simplified way. Its application, for the stabilization of the beam, leads to a global system. In section 5, we present the numerical simulation of the fractional operator based on the diffusive model

2 Model of flexible beam

A dynamic model of the beam is based on the Euler-Bernoulli theory. In this paper, the angle θ is assumed to be small (Fig.1). Therefore, the stabilization problem can be regarded as the stabilization of a hinged-free beam by a force and torque controls applied to hinged point (fig.2). The partial differential equation governing the dynamics of the beam (fig.1) is written in the non-dimensional form:

$$\partial_t^2 y(x,t) + \partial_x^4 y(x,t) = 0 \quad x \in [0,1] \qquad (1)$$

Together with the boundary conditions:

$$\begin{aligned} &\partial_x^2 y(0,t) + M = 0 \\ &\partial_x^3 y(0,t) + F = 0 \\ &\partial_x^2 y(1,t) = 0 \\ &\partial_x^3 y(1,t) = 0 \end{aligned} \qquad (2)$$

and the initials conditions:

$$\begin{aligned} &y(x,0) = y_0(x) \\ &\partial_t y(x,0) = y_1(x) \end{aligned} \qquad (3)$$

where $y(x,t)$ is the vertical displacement and x is space variable.

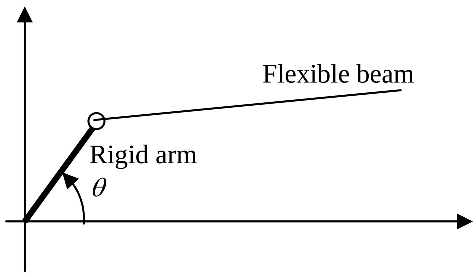

Fig.1: Flexible beam attached to a rigid arm

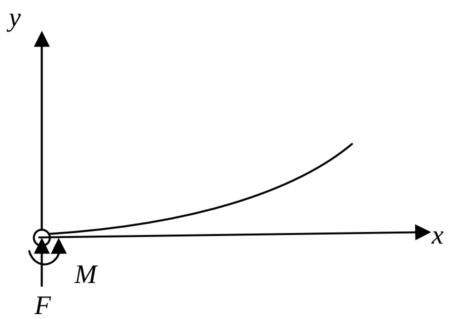

Fig.2: Flexible hinged-free flexible beam

3 Controller Design

3.1 Wave representation

The analysis proceeds with the introduction of the cross-sectional state vector $z = \left(y, \partial_x y, \partial_x^2 y, \partial_x^3 y\right)^T$ Where $\partial_x y$ is the slope, $\partial_x^2 y$ is the internal bending moment, $\partial_x^3 y$ is the internal shear force.

The application of the Fourier transformation to Eq.1(in order to avoid a new symbols, the transformed variables hereafter have the same notation as their times-dependent equivalents), give the ordinary differential equation :

$$\omega^2 \partial_t^2 y(x,t) + \partial_x^4 y(x,t) = 0 \qquad (4)$$

In terms of state vector, the beam dynamics are written in the form:

$$\frac{dz}{dx} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \omega^2 & 0 & 0 & 0 \end{bmatrix} z \qquad (5)$$

This equation is diagonalized by the transformation:

$$z = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -i\sqrt{\omega} & -\sqrt{\omega} & i\sqrt{\omega} & \sqrt{\omega} \\ -\omega & \omega & -\omega & \omega \\ i\omega^{3/2} & -\omega^{3/2} & -i\omega^{3/2} & \omega^{3/2} \end{bmatrix} \cdot W \qquad (6)$$

This diagonalization can be interpreted in term of the traveling waves, where each term of the new - sectional state vector W is the amplitude of traveling wave mode. The amplitudes of these waves mode vary according to:

$$\frac{dW}{dx} = \begin{bmatrix} -i\sqrt{\omega} & 0 & 0 & 0 \\ 0 & -\sqrt{\omega} & 0 & 0 \\ 0 & 0 & i\sqrt{\omega} & 0 \\ 0 & 0 & 0 & \sqrt{\omega} \end{bmatrix} \cdot W \qquad (6)$$

The cross-sectional state vector *W* has been ordered as $W = (a_1, a_2, b_1, b_2)^T$, where a_1 and a_2 are the amplitudes of the waves mode incoming into the hinged end of the beam, b_1 and b_2 are the amplitude of waves mode departing the hinged end of the beam.

3.2 Fractional feedback control

The boundary conditions can be rewritten in terms of the state vector *z* and the externally applied force control *F* and torque *M* as follows:

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} z(0,\omega) = \begin{bmatrix} M \\ F \end{bmatrix} \qquad (7)$$

This Equation can be written in wave-mode coordinates by:

$$\begin{bmatrix} i\omega & -i\omega & i\omega & i\omega \\ i\omega^{/2} & -i\omega^{/2} & -i\omega^{/2} & i\omega^{/2} \end{bmatrix} W(0,\omega) = \begin{bmatrix} M \\ F \end{bmatrix} \qquad (8)$$

The Eq.8 may be rewrite in the general form:

$$b = S_{OL}\, a + BF_{ext} \qquad (9)$$

where S_{OL} is the open loop scattering matrix, a is the outgoing wave modes, b is the incoming wave modes and F_{ext} are the external forces.

In this paper, we limited to linear wave absorbers that given by:

$$\begin{bmatrix} M \\ F \end{bmatrix} = C(\omega) \cdot \begin{bmatrix} y \\ \partial_x y \end{bmatrix}_{(x=0)} \qquad (10)$$

For evaluate and design the absorber controller it's convenient to derive an expression for the closed loop. Substituting eq.10 to eq.9 leads to:

$$b = S_{CL}\, a \qquad (11)$$

where S_{OL} is the closed loop scattering matrix that is a function C.
For guarantied any reflection at hinged point it's necessary to put S_{CL}=0. Therefore, the controller matrix C is given by:

$$\begin{bmatrix} M \\ F \end{bmatrix} = \begin{bmatrix} -i\omega & -(1+i)\, i\omega / \sqrt{\omega} \\ (-1+i)\,\omega^{3/2} & i\omega \end{bmatrix} \cdot \begin{bmatrix} y \\ \partial_x y \end{bmatrix}_{(x=0)} \qquad (12)$$

The temporal domain expression is given by:

$$\begin{aligned} M(t) &= \partial_t y(0,t) - \sqrt{2}\, I^{1/2} \partial_t \partial_x y(0,t) \\ F(t) &= \sqrt{2}\, \partial_t^{1/2} \partial_t y(0,t) + \partial_t \partial_x y(0,t) \end{aligned} \qquad (13)$$

where $I^{1/2}$ and $\partial^{1/2}$ are a fractional operators defined by the classical Riemann –Liouville formulas:

$$I^{1/2} f(t) = \frac{1}{\sqrt{\pi}} \int_0^t \frac{f(s)}{\sqrt{t-s}}\, ds \qquad (14)$$

$$\partial_t^{\,1/2} f(t) = \partial_t \left(I^{1/2} f(t) \right) \qquad (15)$$

The difficulties generated by these operators, due to the hereditary behavior, are multiple in particular the stability study and numerical approximation.
From Eq. (13), we note that the traveling waves are suppressed by a fractional controller. The system can be presented by the following functional diagram:

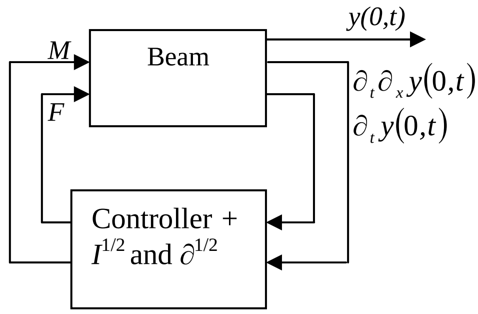

Fig.3: Fractional feedback control of the beam

4 Diffusive controller

4.1 Diffusive representation

The diffusive realization noted $\mu(\xi)$ of the pseudo differential operator H: $u \rightarrow g = H\left(d/dt\right)u$ is defined by the dynamic input-output system [ ][ ]:

$$\begin{cases} \partial_t \varphi(\xi,t) = -\zeta\varphi(\xi,t) + u(t) \\ g(t) = \int_0^\infty \mu(\xi)\varphi(\xi,t)\, d\xi \\ \varphi(\xi,0) = 0, \;\; \xi \succ 0 \end{cases} \qquad (16)$$

The impulse response $h{:}=L^{-1}H$ is clearly expressed from $\mu(\xi)$by [ ]:

$$h(t) = \int_0^{+\infty} e^{-\xi t} \mu(\xi)\, d\xi \qquad (17)$$

so the diffusive symbol is also given by

$$\mu{:}L^{-1}h \qquad (18)$$

In the particular case of fractional integrators

$$H\left(\frac{d}{dt}\right) = \left(\frac{d}{dt}\right)^{-\alpha}, \;\; 0 < \alpha < 1 \qquad (19)$$

The diffusive symbol is expressed as:

$$\mu(\xi) = \frac{\sin(\pi\alpha)}{\pi} \frac{1}{\xi^{\alpha}}, \;\; \xi > 0 \qquad (20)$$

where α is the order of integration

The diffusive symbol of the fractional derivators may be derives directly from (16).

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4.2 Global system

From Eqs.(1), (2) and (16) we can construct a global system as follows:

$$\begin{cases} \partial_t^2 y + \partial_x^4 y = 0 \\ M = -\sqrt{2}\int_{-\infty}^{+\infty} \mu(\xi)\varphi d\xi + \partial_t y(0,t) \\ F = \sqrt{2}\int_{-\infty}^{+\infty} (-\mu(\xi)\partial_t \psi) d\xi + \partial_t \partial_x y(0,t) \\ \partial_t \varphi + \xi\varphi = \partial_t \partial_x y(0,t) \\ \partial_t \psi + \xi\psi = \partial_t y(0,t) \end{cases} \tag{21}$$

The system (21) can be expressed under standard abstract form $\frac{dX}{dt} = AX$, where $X = (y, \partial_t y, \varphi, \psi)^T$

We can easily proof that the global system (21) is dissipative in the sense $\frac{dE_X}{dt} \le 0$.

5 Numerical simulation

5.1 Numerical approximation of diffusive representation

The numerical approximation of *H(d/dt)* can be constructed by discretizing the variable ξ and involving standard quadrature methods on (16). This leads to input-output approximation $u \to \tilde{g} \cong g = H\left(d/dt\right)u$ of the form:

$$\begin{cases} \partial_t \varphi_k(t) = -\xi_k \varphi_k(t) + u(t) \\ \tilde{g}(t) = \sum_{k=1}^{K} c_k \varphi_k(t) \\ \varphi_k(0) = 0,\ \mathrm{k} = 1,...,\mathrm{K} \end{cases} \tag{22}$$

with $c_k = \int_{-\infty}^{+\infty} \mu(\xi) \Lambda_k d\xi$

where Λ_k are convenient piecewise affine functions with bounded support.

5.2 Results and comments

Figures 4 and 5 present the bode diagram of the half integrator and derivator. It's clearly show that their numerical approximations are correctly carried out by diffusive representation on the pulsation domain [10^{-5}, 10^5].(the amplitude and the phase of half integrator or derivator are equal to -10dB or 10 dB and-45° or 45° respectively).

Figure 6 shows the evolution of the beam in the autonomous case, and reveals complex vibrating behaviors of the beam due to the traveling waves reflection.

Figures 7 and 8 show the effect of the wave absorbing control to the flexible beam. It does not have no reflection at the articulation *(x=0)*. The absorption is slow at the beginning but always tends towards to a minimal value.

Figure 9 shows the tip deflection of the beam. we see that the tip exhibits visible vibration and the magnitude is quite to constant value.

Figure 10 shows the evolution of the mechanical energy of the beam, it is obvious that the energy is decreasing.

The responses of the flexible beam and a rigid arm are shown in fig.11.

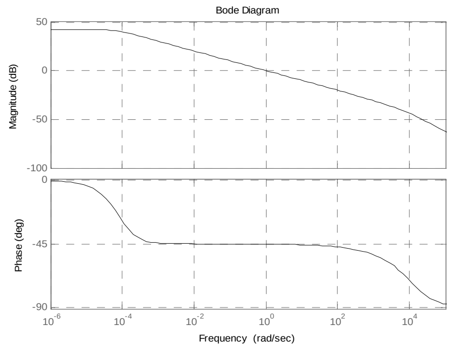

Fig.4: Numerical approximation of $I^{1/2}$

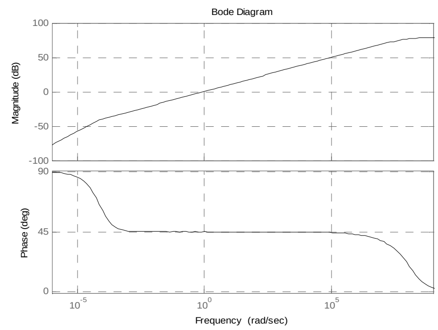

Fig.5: Numerical approximation of $\partial^{1/2}$

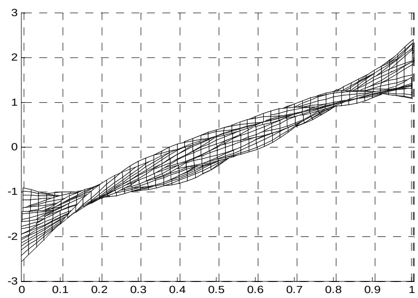

Fig.6: Autonomous beam deflection

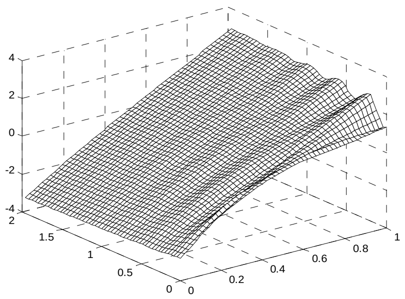

Fig.7: Beam deflection with control

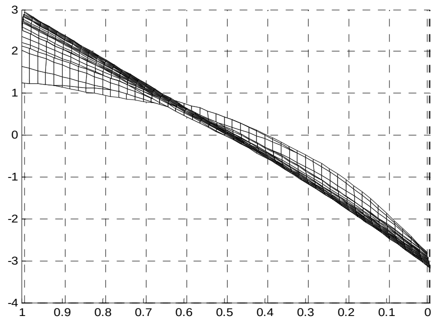

Fig.8: Beam deflection with control in 2D Representation

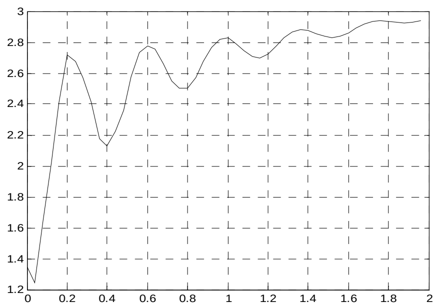

Fig.9: Tip deflection of the beam

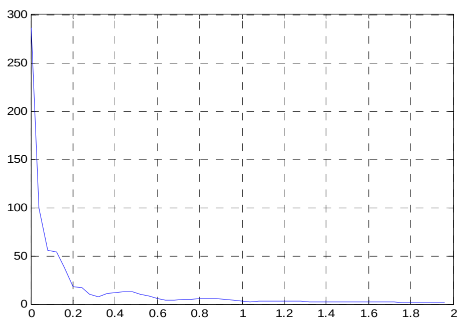

Fig.10: Mechanical energy of the beam

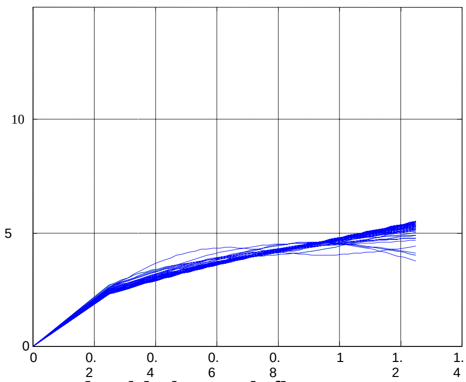

Fig.11: Flexible beam deflection associated to rigid arm

6 Conclusion

Stabilization of a hinged-free flexible beam attached to a rigid arm has been investigated using wave-absorbing control. Such control involved fractional integrators and derivators. The non-hereditary realization of the fractional feedback has been achieved by the diffusive representation. . The numerical simulation results clearly has been showed the powerful and the effectiveness of the wave absorbing control using diffusive representation.

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