

# Asymmetrical Switched Reluctance Drive with an Unequal Number of Turns per Motor Phases - Torque Ripple Evaluation

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*Abstract:* - In this paper the torque ripple in the 3-phase asymmetrical switched reluctance motor (SRM) drives is considered. Analyzed asymmetrical SRMs have the same geometry as the symmetrical one, but they have an unequal number of turns per phases. The optimal control parameters that provide maximal output power are determined for a referent symmetrical drive and for a number of asymmetrical drives. Using these parameters, the torque ripple for various arrangements of phase windings is determined as a function of rotor speed. Typical current and torque waveforms of the referent symmetrical and the asymmetrical SRM drives are also presented. The results are obtained using suitable SRM model whose validity has been experimentally verified.

*Key-Words:* - computer aided analysis, reluctance motor, asymmetrical drive, torque ripple evaluation.

## 1 Introduction

The main area of application of the SRM is electric traction and home appliances, where it is particularly important that the motor develops rated power in wide speed range. The width of this range significantly depends on the motor geometry, topology of power converter and skill of the applied control [1]-[3]. Investigations described in [4]-[6] give relation between motor geometry and its constant power region width. The results show that the wider constant power range is developed by SRM drives using motor with lower number of phases and poles with reduced pole width. An interesting idea to resolve the problem of low torque at high speed is "pole-changing" method described in [7]. The 6/2 asymmetrical topology of SRM for high-speed range is proposed in [8], where the "low-speed" poles winding are switched off as soon as the motor starts. Significant contribution to improve exploitation characteristics of the SRM in home appliances is also made in [9]. On the other hand, many new topologies of the power converter [8], [10], [11] and control techniques [12]-[15] are proposed with intention to improve the process of electromechanical conversion in the SRM, and, in such a way, to extend the constant power range.

The paper considers the asymmetrical configuration of the three-phase SRM drive that can provide wide constant power region. In this configuration, the SRM has unequal number of turns per phase. The model suitable for performance prediction of the asymmetrical SRM drive is used to obtain the valid results. In analyzed examples, the computer simulation is employed to determine the optimal control parameters (referent

current, and turn on and turn off angles) that provide maximal output power. Using these parameters, the torque ripple is determined as a function of speed for drives with various arrangements of phase windings. Typical current and torque waveforms for the referent symmetrical and asymmetrical SRM drives are also presented.

## 2 Basic Consideration

The constant power regime of SRM is ensured by moving the turn on ( $\theta_{on}$ ) and the turn off ( $\theta_{off}$ ) angles in advance when rotor speed ( $\omega$ ) increases. At certain speed ( $\omega_c$ ), the turn on angle can not be further moved in advance, and, with further increasing of speed, the time interval between  $\theta_{on}$  and  $\theta_{off}$ , is decreased. Thus, the flux ( $\phi$ ) decreases proportionally to  $1/\omega$ , and, consequently, the output torque ( $T$ ) and power ( $P$ ) decrease proportionally to  $1/\omega$  and  $1/\omega^2$ , respectively, as  $T$  is proportional to  $\phi^2$  and  $P = T\omega$ , [1].

One way to increase  $\omega_c$  (and thus the constant power range) is to ensure slower changes of  $\theta_{on}$  providing faster rise of  $\phi$ . It requires increase of voltage supply or decrease the number of turns per phase ( $N$ ), and, thus, increase volt-ampere (VA) and speed requirements of switches in the power converter. The same VA requirements can be ensured only if an unequal number of turns per phases are applied.

Asymmetrical 6/4 SRM, which has two "low-speed" (LS) phases and one "high-speed" (HS) phase, is considered in this paper. LS phases have larger number of turns than HS phase. As a consequence, they will ensure lower VA requirements for same magneto-motive force (MMF), but the HS phase will have higher speed

$\omega_c$  and, thus, will provide dominant torque at high speed. Optimal combination of phases' number of turns should ensure wide constant range of maximal output power.

In order to obtain valid results, the output characteristics of asymmetrical drives and referent symmetrical ones will be compared. Therefore, although the speed of switches in converters will be different, all drives will have approximately the same total VA requirements of power converters and the same motor volumes. Considerations are limited on drives with classic power converters [8]. Also, it is assumed the same slot-fill factor for any phase number of turns i.e. any wire diameter.

In the case of symmetrical SRM drive all phases have the same current waveforms. If the relative increase of the input filter capacitor voltage is neglected, the total VA requirements of power switches for three-phase SRM, fed by classic converter, are [8]:  $6 \cdot V \cdot I_{pm}$ , where  $V$  is input dc voltage, and  $I_{pm}$  is maximum of the phase current peak value ( $I_p$ ). The same requirements for the asymmetrical three-phase SRM are:  $2 \cdot V_1 \cdot I_{pm1} + 4 \cdot V_2 \cdot I_{pm2}$ , where subscript 1 is referred to one phase (less number of turns) and subscript 2 is referred to other two phases (larger number of turns). Thus, if we want to compare output characteristics of asymmetrical SRM drive with referent symmetrical one, the equation (1) is to be satisfied:

$$6 V I_{pm} = 2 V_1 I_{pm1} + 4 V_2 I_{pm2}. \quad (1)$$

If we assume the same values of phase input dc voltage i.e.  $V = V_1 = V_2$ , than condition (1) becomes:

$$6 I_{pm} = 2 I_{pm1} + 4 I_{pm2}. \quad (2)$$

The same current density in the phase conductors and the same magnetic field structure may be achieved by same values of the MMF in all phases, i.e.:

$$N I = N_1 I_1 = N_2 I_2, \quad (3)$$

where  $N$ ,  $N_1$  and  $N_2$  are number of turns of corresponding phases, and  $I$ ,  $I_1$  and  $I_2$  are rms values of phase currents.

In that case, the following expression is valid:

$$I_{pm} / I = I_{pm1} / I_1 = I_{pm2} / I_2 = k_i, \quad (4)$$

where  $k_i$  is constant. Than, equation (2) can be written as:

$$3 I = I_1 + 2 I_2. \quad (5)$$

Combining (3) and (5), and defining coefficient  $k = N / N_1$ , we obtain the system:

$$N_1 = N / k,$$

$$N_2 = 2 N / (3 - k), \quad (6)$$

$$I_1 = k I,$$

$$I_2 = (3 - k) I / 2,$$

If  $N_1 < N < N_2$  and  $1 < k < 3$ , the system (6) ensures the same VA requirements to power converters for symmetrical and asymmetrical SRM drive. Thus, changing the value of coefficient  $k$ , with given phase current ( $I$ ) and number of turns ( $N$ ) of the referent symmetrical SRM drive, we obtain currents and numbers of turns for different asymmetrical SRM drives. Note, the value of coefficient  $k = 1$  refers to symmetric SRM drive ( $I_1 = I_2 = I$ ,  $N_1 = N_2 = N$ ), and the value  $k = 3$  refers to the case:  $N_1 = 3N$ ,  $N_2 \rightarrow \infty$ ,  $I_1 = 3I$ ,  $I_2 \rightarrow 0$ .

If the phase resistance  $R$  for symmetrical SRM is known, than the resistance of asymmetrical SRM phases can be obtained as:

$$R_j = (N_j / N)^2 R, \quad (j=1,2) \quad (7)$$

Similarly, if  $L_u$  value of unaligned inductance of the symmetrical SRM phases, than unaligned inductance  $L_{u1}$  and  $L_{u2}$  of asymmetrical SRM phases can be calculated:

$$L_{uj} = (N_j / N)^2 L_u, \quad (j=1,2) \quad (8)$$

### 3 Model Description

Simulation is based on appropriate SRM mathematical model described in [16]. The model provides torque and phase current waveforms, as well as the average torque, peak and rms phase current values. Phase current ( $i$ ) is defined as functions of flux linkage ( $\Psi$ ) and rotor position ( $\theta$ ):

$$i(\theta, \Psi) = i_o(\theta, \Psi) + i_{fe}(\Psi), \quad (9)$$

where

$$i_o = c_{o5} \left[ (1 - c_{o1}) \lambda \Psi - c_{o1} c_{o2} + c_{o1} \sqrt{(\Psi - c_{o3})^2 + c_{o4}^2} \right], \quad (10)$$

$$i_{fe} = c_{fe1} \Psi + c_{fe2} \Psi^\alpha. \quad (11)$$

The coefficients  $c_{ok}$  ( $k=1,2,\dots,5$ ) are the functions of the rotor position  $\theta$ , while the  $c_{fe1}$ ,  $c_{fe2}$  and  $\alpha$  are the constants. The above parameters can be determined by using the motor geometry data as well as some specific points of iron core  $B-H$  curve and unaligned phase inductance. The detailed procedure of model parameters

determination is explained in [16].

The flux linkage  $\Psi$  is calculated as:

$$\Psi = \int_0^t (v - Ri) dt, \quad (12)$$

where  $v$  is the applied phase voltage and  $R$  is phase resistance.

The produced torque  $T$  is determined as:

$$T = -\frac{\partial W_{mo}}{\partial \theta}, \quad (13)$$

where

$$W_{mo} = c_{o5} \left[ \frac{1-c_{o1}}{2} \Psi^2 - c_{o1}c_{o2}\Psi + \frac{c_{o1}}{2} (\Psi - c_{o3}) \sqrt{(\Psi - c_{o3})^2 + c_{o4}^2} + \frac{c_{o1}c_{o2}c_{o3}}{2} + \frac{c_{o1}c_{o4}^2}{2} \ln \left( \frac{\Psi - c_{o3} + \sqrt{(\Psi - c_{o3})^2 + c_{o4}^2}}{c_{o2} - c_{o3}} \right) \right], \quad (14)$$

is a part of magnetic field energy calculated as  $W_{mo} = \int i_o d\Psi$  with  $i_o$  defined by (10).

#### 4 Results

Parameters of the referent symmetrical motor are given in Table I. Magnetization curves of the referent motor for a number of rotor positions, obtained by computer simulation and by measurement, are shown in Fig. 1. Similarity of these results will provide the reliability of the simulation results in the time domain [1], [6], [17].

TABLE I  
MOTOR PARAMETERS OF THE REFERENT SYMMETRICAL SRM DRIVE

Parameter	Value
Stack length	48 mm
Shaft diameter	19 mm
Rotor yoke thickness	9 mm
Rotor pole height	10.5 mm
Stator yoke thickness	11 mm
Stator pole height	17.5 mm
Stator outer diameter	116 mm
Minimum airgap length	0.5 mm
Rotor pole arc, $\beta_r$	32°
Stator pole arc, $\beta_s$	30°
Number of rotor poles, $N_r$	4
Number of stator poles, $N_s$	6
Number of turns per phase, $N$	580
Phase resistance, $R$	6.9 $\Omega$
DC supply voltage, $V$	270 V
Unaligned inductance, $L_u$	48 mH

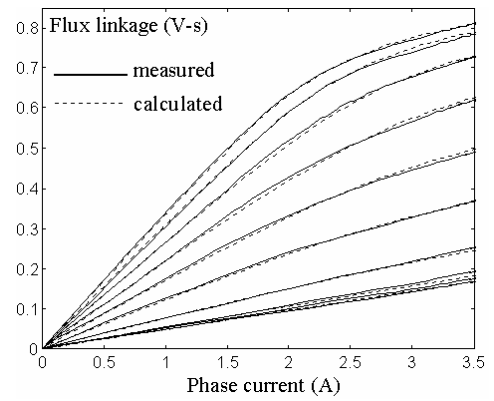


Fig. 1. Magnetization curves of the symmetrical motor.

Torque ripple characteristics of symmetrical and asymmetrical SRM drives are compared in the case of maximal output SRM drives power. In order to provide the maximal output power the optimal control parameters - turn on angle ( $\theta_{on}$ ), turn off angle ( $\theta_{off}$ ) and referent current ( $I_{ref}$ ), were precisely determined. The control parameters for asymmetrical SRM drives were determined for LS and HS phases separately.

Fig. 2 shows the output power characteristics calculated for a number of drives with different values of coefficient  $k$  (1, 1.5, 1.9, 2, 2.5 and 3). It can be shown that the configuration with  $k = 1.9$  (i.e.  $N_1 \approx 305$ ,  $N_2 \approx 1054$ ) provides the widest constant power range.

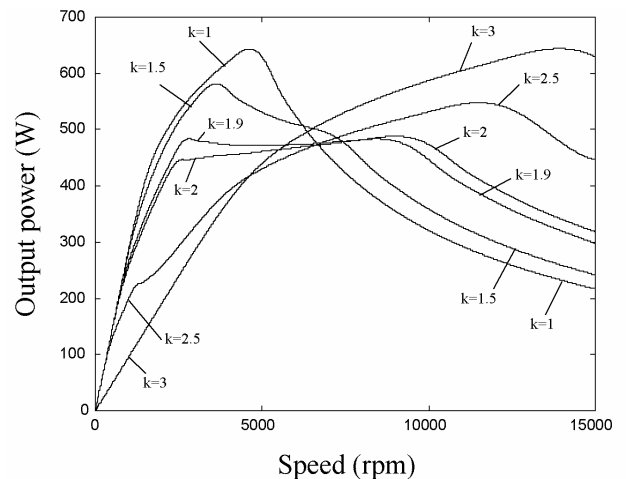


Fig. 2. Output power characteristics of SRM drives.

Optimal turn-on and turn-off angles, together with output torque characteristic for referent symmetrical drive are shown in Fig 3. Examples of phase currents and torque waveforms at 2000 rpm and 5000 rpm speeds are shown in Figs 4–7.

Optimal turn-on and turn-off angles for an asymmetrical drive ( $k=1.5$ ) are shown in fig 8. Examples of phase currents and torque waveforms at 2000 rpm and 5000 rpm speeds are shown in Figs 9–12.

The torque ripple ( $TR$ ) is defined as:

$$TR = (T_{max} - T_{min}) / T_{av} \quad (15)$$

where:  $T_{max}$  and  $T_{min}$  are maximal and minimal instantaneous torque values at the given rotor speed, and  $T_{av}$  is average torque. Calculated torque ripple versus rotor speed characteristics for four drives are presented in Fig 13. As it is expected, the lowest  $TR$  has symmetrical drive, and  $TR$  increases when  $k$  increases. It can be seen that the  $TR$  of symmetrical drive reaches maximum at the speed  $\omega_c$ , i.e. when turn-on angle cannot be further moved in advance. Maximum of  $TR$  in case of asymmetrical drive is at the speed  $\omega_{c1}$ , i.e. the speed  $\omega_c$  that corresponds the HS phase. Note, if we know  $\omega_c$  for symmetrical drive, then  $\omega_{c1}$  can be approximately computed as:  $\omega_{c1} = k \omega_c$ .

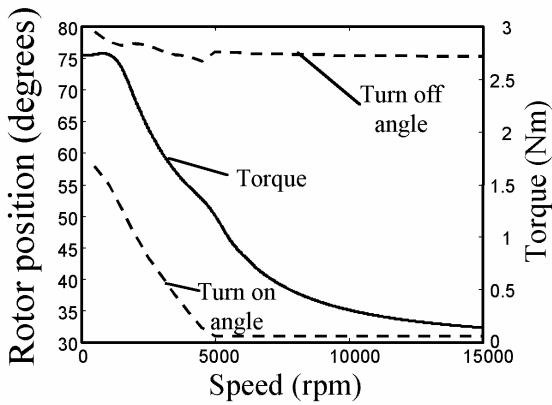


Fig. 3. Results for referent symmetrical SRM drive: Optimal turn on and turn off angles, and average output torque.

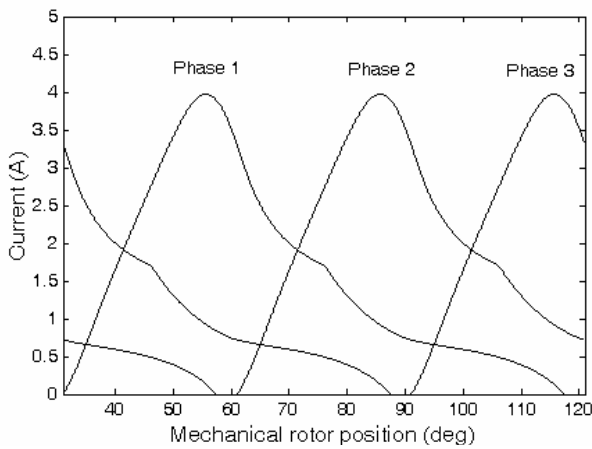


Fig. 4. Phase currents for referent symmetrical drive at 5000 rpm.

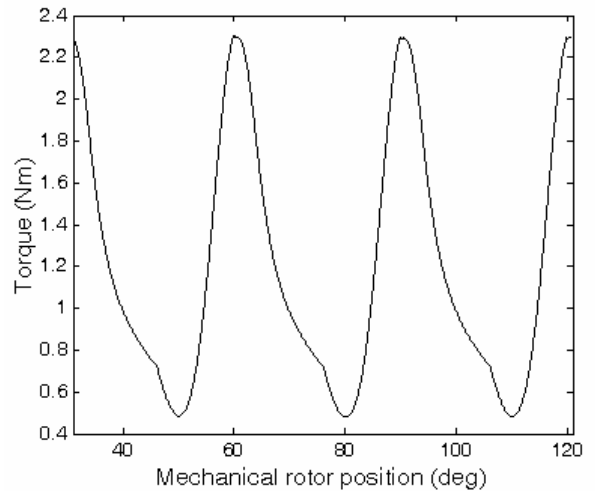


Fig. 5. Torque waveform for referent symmetrical drive at 5000 rpm.

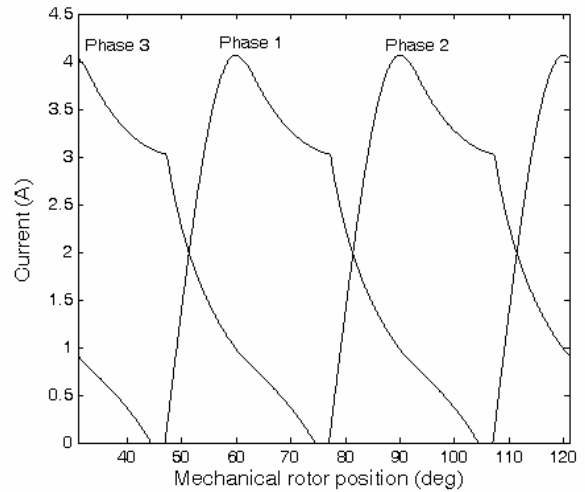


Fig. 6. Phase currents for referent symmetrical drive at 2000 rpm.

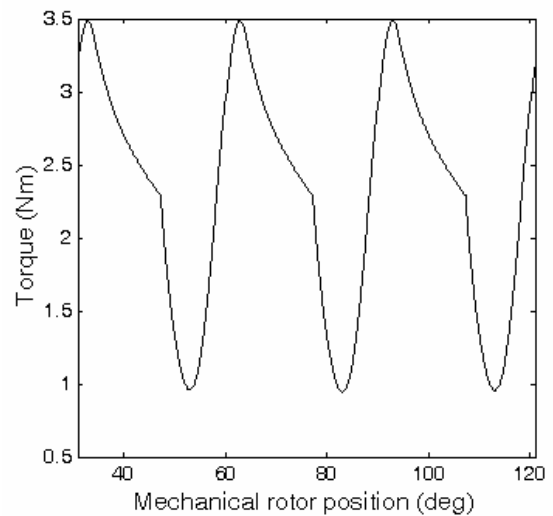


Fig. 7. Torque waveform for referent symmetrical drive at 2000 rpm.

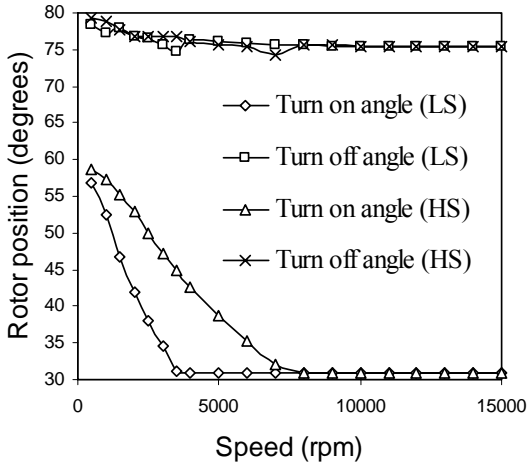


Fig. 8. Results for asymmetrical SRM drive ( $k=1.5$ ): Optimal turn on and turn off angles.

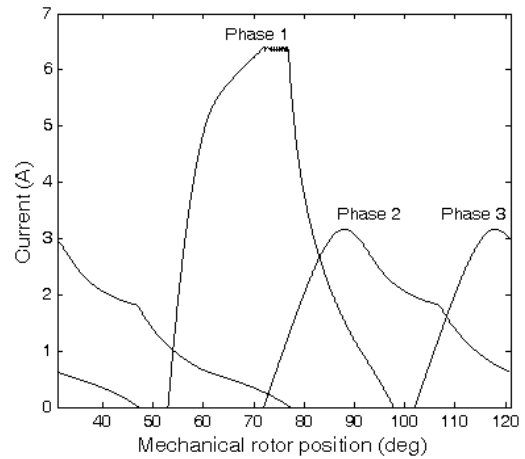


Fig. 11. Phase currents for asymmetrical drive ( $k=1.5$ ) at 2000 rpm.

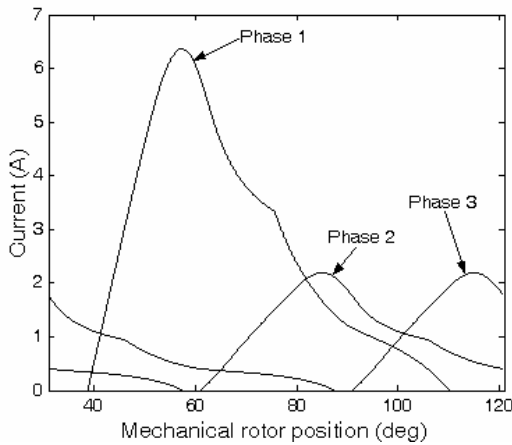


Fig. 9. Phase currents for asymmetrical drive ( $k=1.5$ ) at 5000 rpm.

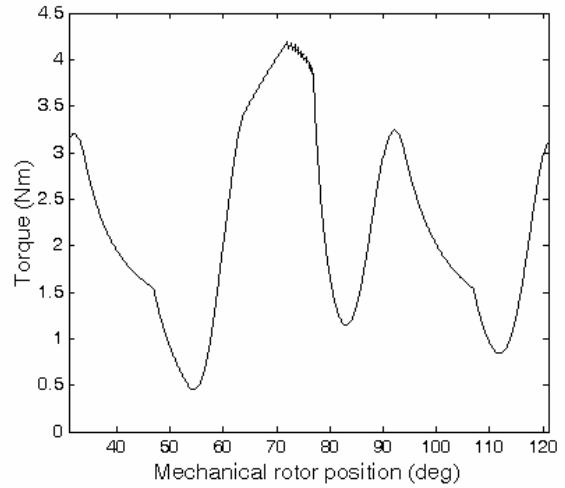


Fig. 12. Torque waveform for asymmetrical drive ( $k=1.5$ ) at 2000 rpm.

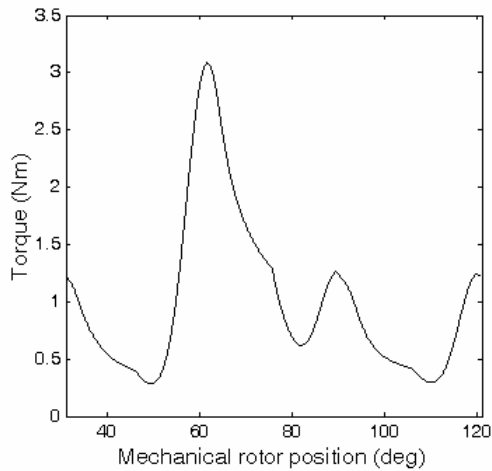


Fig. 10. Torque waveform for asymmetrical drive ( $k=1.5$ ) at 5000 rpm.

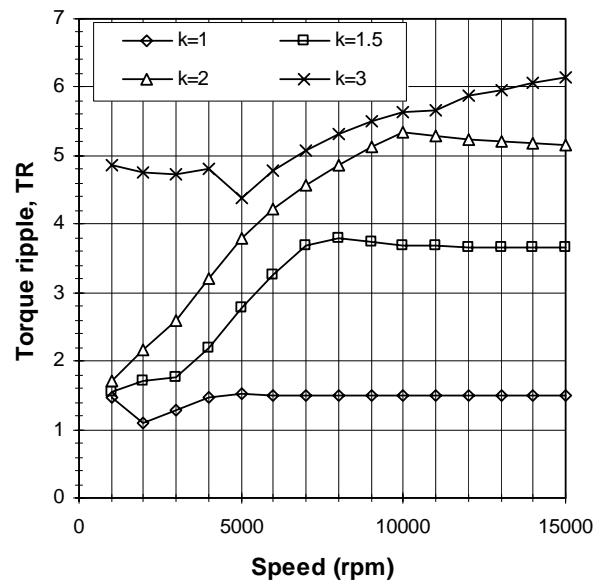


Fig. 13. Torque ripples vs. rotor speed characteristics for four SRM drives.

## 6 Conclusion

In this paper the torque ripple in the asymmetrical SRM drive has been considered. The obtained results refer to the case when maximal output power is developed. Torque ripple has been determined as a function of speed for various asymmetrical drives. The relation between arrangement of windings and the speed when torque ripple reaches maximum, is determined.

The proposed asymmetrical SRM drive can provide wider constant power range than symmetrical one. The disadvantages of the asymmetrical drives, however, are lower maximal output power and higher torque ripples than it is case in the symmetrical drives.

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