Proceedings of the 5th WSEAS Int. Conf. on SIMULATION, MODELING AND OPTIMIZATION, Corfu, Greece, August 17-19, 2005 (pp341-346)

Newton Methods for the Steady State Solution of Nonlinear Electric Networks

A. MEDINA, A. RAMOS-PAZ
Facultad de Ingeniería Eléctrica, División de Estudios de Posgrado
Universidad Michoacana de San Nicolás de Hidalgo
Edificio de la División del Posgrado de Ingeniería Eléctrica, Ciudad Universitaria, 58030
Morelia, Michoacán, MEXICO
http://faraday.fie.umich.mx

Abstract: - In this contribution Newton techniques based on Numerical Differentiation (ND), Direct Approach (DA) and Difference Matrix (DM) procedures are applied for the computation of the periodic steady state solution of single phase nonlinear electric networks containing magnetizing branches of transformers and alternating current arc furnaces. The results obtained with the Newton methods are compared in terms of computational effort required to obtain the steady state solution and their related reliability towards convergence.

Key-Words: - Numerical Differentiation, Direct Approach, Difference Matrix, periodic steady state, magnetizing branches, arc furnaces.

1 Introduction

The dynamic behaviour of an practical electric power system can be represented by a state space equation. The solution process can be achieved through the application of a conventional Brute Force (BF) numerical solution method *e.g.* Euler's algorithm, Runge-Kutta or trapezoidal rule. However in cases where the electric power system components have an intrinsic poor damping the transient or steady state solution is obtained after considerable computation time. In [1] Newton methods are proposed, that allow a fast convergence of the state variables to the limit Cycle. These Newton techniques are based on Numerical Differentiation (ND), Direct Approach (DA) and Exponential Matrix (EM) procedures [1]. They have been applied to the efficient steady state solution of circuits containing TCRs [2], arc furnaces [2], TCSC [3] among other commutated devices. Besides, they have been used with parallel processing techniques based on Multithreading [4] and PVM [5]. A recently developed Newton technique based on a Difference matrix [6] allows the computation of nonlinear electric circuits. In this contribution, two nonlinear electric circuits are solved using the ND, DA and DM methods in order to compare its robustness and efficiency to obtain the steady state solution.

2 Newton Methods

The behavior of nonlinear components or loads can be described in the time domain by the nonlinear Ordinary Differential Equation (ODE),

$$\dot{x} = f(x,t) \tag{1}$$

if (1) has periodic time-varying parameters, then

$$f(x,t+mT) = f(x,t) \tag{2}$$

The integration of (2) is carried-out over time periods T. After m periods the state x_n is obtained, an additional period results in x_{m+1}, this is called the Base Cycle [7]. In [7] $m = 7$ is suggested for poorly damped systems and $m = 4$ for the opposite case. If a disturbance on the trajectory is assumed so that x0 is perturbed as $x_0+\Delta x_0$, or more generally $x(t) \rightarrow x(t)+\Delta x(t)$, then equation (1) takes the form

$$\dot{x}+\Delta\dot{x} \approx f(x+\Delta x,t) \tag{3}$$

The linearization of (3) with respect to x taking the first order terms in the Taylor series results in

$$\underline{\dot{x}}+\Delta\dot{x} \approx \underline{f(x,t)}+D_x f(x,t)\Delta x \tag{4}$$

Since the underline part of equation (4) is satisfied by (1) then (4) can be written as

$$\Delta\dot{x} \approx J(t)\Delta x \tag{5}$$

where the Jacobian is given by

$$J(t) = D_x f(x,t) \tag{6}$$

Proceedings of the 5th WSEAS Int. Conf. on SIMULATION, MODELING AND OPTIMIZATION, Corfu, Greece, August 17-19, 2005 (pp341-346)

where D_x determine the partial derivatives of $f(x,t)$ with respect to x. Thus, a time-varying Jacobian $J(t)$ has been obtained.

The solution of (5) has the form

$$\Delta x(t) = \Phi(t)\Delta x_0 \tag{7}$$

where

$$\Phi(t) = e^{\int_0^T J(t)dt} \tag{8}$$

2.1 Extrapolation of the Limit Cycle

The extrapolation of the solution to the Limit Cycle [9] is achieved with the recursive equation [1],

$$x^{\infty} = x^i + C\left(x^{i+1} - x^i\right) \tag{9}$$

where,

$$C = (I - \Phi)^{-1} \tag{10}$$

x_{∞} state variables at limit cycle.
x_i state variables at the beginning of the base cycle [1].
x_{i+1} state variables at the end of the base cycle.
C, Φ, I, iteration, identification and unit matrix respectively.

The solution of (9) implies a quadratic convergence process if Φ and C are updated for each evaluation of and linear for their single or partial iterative evaluation [1]. The matrix Φ is of $n \times n$ order, where n is the number of state variables. Usually $J(t)$ can be analytically obtained, but this is not always the case, in special with highly nonlinear or commutated components. Alternatively Φ can be obtained by columns by the sequential perturbation of state variables

2.1 Numerical Differentiation (ND)

In this method a sequential perturbation of the form $x^i + \varepsilon e^i$ is applied where ε is a small number, *e.g.* 10^{-6} *p.u.* and e^i is the column i of the unit matrix I. Figure 1 illustrates the identification process of Φ used by the Numerical Differentiation (ND) method [1].

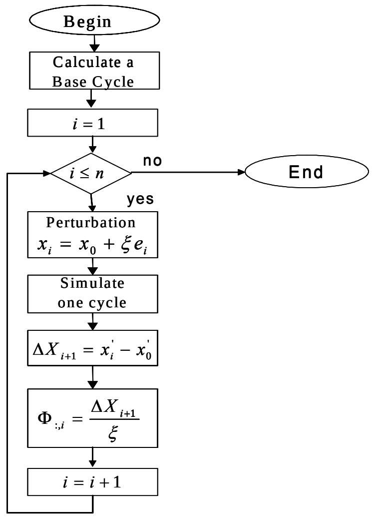


Figure 1. Numerical Differentiation Method

2.2 Direct Approach (DA)

An alternative to the ND procedure is the use of the Direct Approach methodology; it consist of the integration of $\Delta x = J(t)\Delta x$ with initial vectors being sequentially the columns of the identity matrix I, generating as a result the matrix Φ. The fundamental steps on which the DA method is based are detailed in Figure 2.

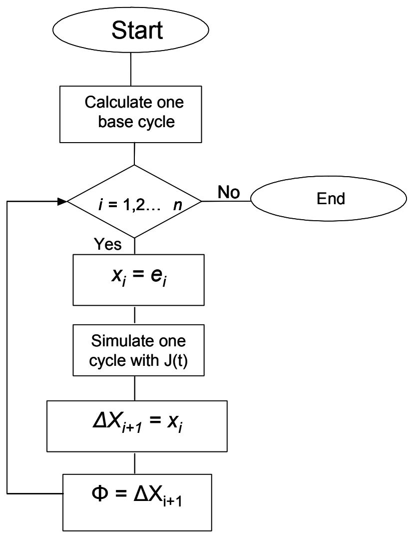


Figure 2. Direct Approach Method

2.3 Difference Matrix Technique (DM)

A set of Ordinary Differential Equations (ODEs) of the form given by equation (1), can be represented into a N-dimensional representation as follows [6]

$$D_{jk} x_k = f(x_k, t_k) \tag{11}$$

where N is an odd value and D_{jk} is calculates as

$$D_{kj} = \begin{cases} 0 & j = k \\ \dfrac{(-1)}{2\sin\left(\dfrac{\pi}{N}(j-k)\right)} & j \neq k \end{cases} \quad (12)$$

This *N*-dimensional representation allows a discretization of Equation (1) generating a set of algebraic equations.

Equation (11) can be represented in compact form as:

$$\mathbf{Dx} = \mathbf{f} \quad (13)$$

The solution of (11) can be obtained with the application of the Newton-Raphson algorithm, using the initial conditions vector x_0, obtained with the application of the Fourth Order Runge-Kutta method.

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} \quad (14)$$

The vector of initial conditions required by the Runge-Kutta method is

$$x_k = [0 \quad 0 \quad 0 \quad \cdots \quad 0]^T \quad (15)$$

whereas the time vector is

$$t_k = [T/N \quad 2T/N \quad 3T/N \quad \cdots \quad kT/N]^T \quad (16)$$

Figure 3 illustrates the solution process associated with the application of the DM technique.

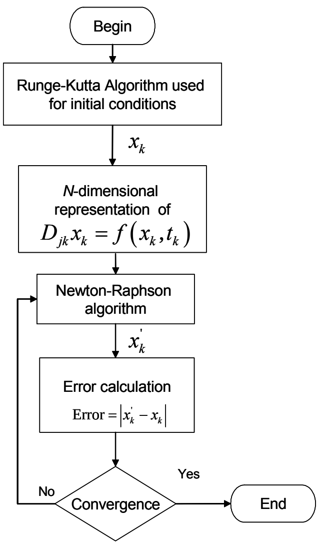


Figure 3. Difference Matrix Method

Sparsity Techniques Applied to the Difference Matrix Technique

Because the sparse characteristic associated with the formulation of (13), sparsity techniques are incorporated to increase the computational efficiency of the iterative solution process. Since there is no change of topology during the iterative process of solution of (13), the ordering and factorization of (13) are carried-out only once, in the first iteration, and the resulting factors are used throughout the iterative solution process.

3 Test Cases

The described Newton methodologies are now applied to obtain the periodic steady state solution of the nonlinear electric circuits of the case studies to be presented. The results are compared in terms of the computational effort required by each Newton technique to obtain the steady state solution.

Case 1:

The circuit illustrated in Figure 4 consists of an electric arc furnace connected to a voltage source across a transmission line represented by a simplified r-l branch. The circuit can be modeled by means of four ODE's. The arc furnace is represented by two ODE's detailed in Appendix A. The source is represented by a sinusoidal function of *1.0 p.u.* of amplitude. In all cases a convergence criterium of 10^{-10} *p.u.* is used to obtain the steady state solution.

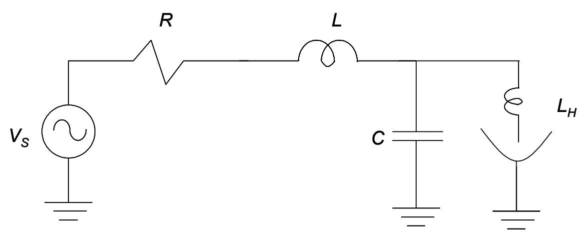


Figure 4. Test case 1

Table 1 illustrates the application of the four methodologies to obtain steady state solution of electric system illustrated by Fig 3. Column 1 gives the number of full cycles needed to obtain the steady state and columns 2-5 the absolute errors between successive estimations of the unknown state variables. Note that 188 full cycles (periods of time) are needed to reach the steady state solution. With the BF method, whereas the ND and DA methods require 48. This represents 25 % of the total number of cycles needed by the BF method. To obtain the numerical solution with the four methods 512 time steps per period were used. Four iterations are required by the Newton methods to meet the selected convergence criterium.

Table 1. Maximum errors during the convergence process using BF, ND, DA and DM techniques, Test Case I

Number of full cycles	Maximum Mistake			
	BF	ND	DA	DM
1	7.6816E+00	7.6816E+00	7.6816E+00	7.6816E+00
2	1.0513E+00	1.0513E+00	1.0513E+00	1.0513E+00
3	3.4924E-01	3.4924E-01	3.4924E-01	3.4924E-01
⋮	⋮	⋮	⋮	⋮
8	2.6290E-03	2.6290E-03	2.6290E-03	2.6290E-03
18	2.8827E-03	4.8388E-03	4.8388E-03	1.7702E-01
28	4.5487E-04	1.0429E-03	1.0429E-03	7.8717E-04
38	7.2121E-05	1.2488E-06	1.2479E-06	1.9815E-06
48	1.1439E-05	2.6645E-14	1.7764E-14	7.3300E-12
⋮	⋮			
188	9.5487E-11			

Case 2:

The electrical system shown in Figure 5 contains 3 transmission lines, two magnetizing branches, one generator, two capacitors banks and one electric arc furnace. The dynamics of the electric system is represented by a set of nine ODE's in which fluxes through lines, flux in the arc furnace, capacitor voltages and arc furnace radius have been selected as state variables.

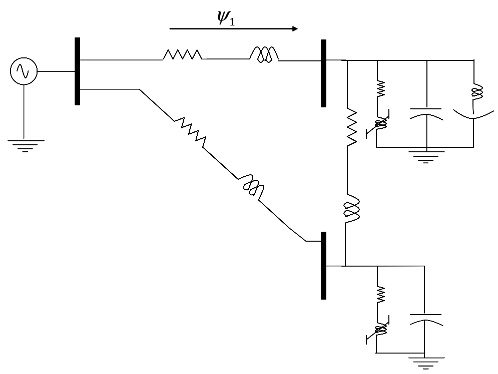


Figure 5. Test case 2

Table 2 illustrates the number of full cycles required by the BF, ND and DA methods to reach the steady state solution. The number of cycles required by the ND and DA methods represent the 42.85% of the those needed by the BF method. An additional iteration is required by the DM method.

Table 2. Maximum errors during the convergence process of the BF, ND, DA and DM techniques, Test Case II

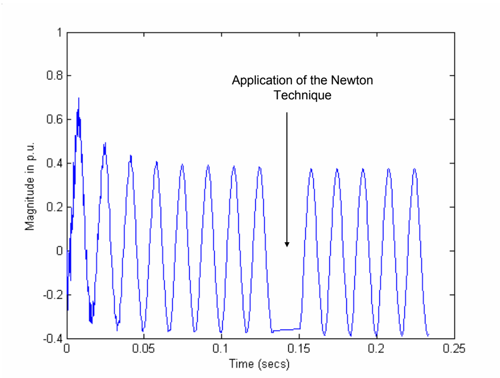
Cycles	BF	ND	DA	DM
1	8.6655E+00	8.6655E+00	8.6655E+00	8.6655E+00
2	9.8720E-01	9.8720E-01	9.8720E-01	9.8720E-01
3	3.3283E-01	3.3283E-01	3.3283E-01	3.3283E-01
⋮	⋮	⋮	⋮	⋮
8	1.9843E-02	1.9843E-02	1.9843E-02	1.9843E-02
18	2.8624E-03	4.6431E-03	4.6430E-03	4.2029E-01
28	4.5231E-04	7.0651E-05	7.0648E-05	6.1948E-02
38	7.1745E-05	1.3098E-10	1.2453E-10	6.3007E-04
48	1.1384E-05	3.5388E-15	3.1560E-15	6.5624E-07
58	1.8065E-06			1.2000E-13
⋮	⋮			
112	8.7051E-11			

The Brute Force procedure requires 1.05 seconds to reach the steady state whereas the DN and DA methods 0.19 and 0.2 seconds, respectively, representing the 18% and 19 % respectively, of the time consuming by the BF. The DM requires 1421 seconds without sparsity techniques and 398 using sparsity techniques. These values represent 1353 and 379 times the computing time required by the BF. The advantage of applying is clearly evident since the computation time is reduced to 28% of the needed by the DM method without sparsity techniques used.

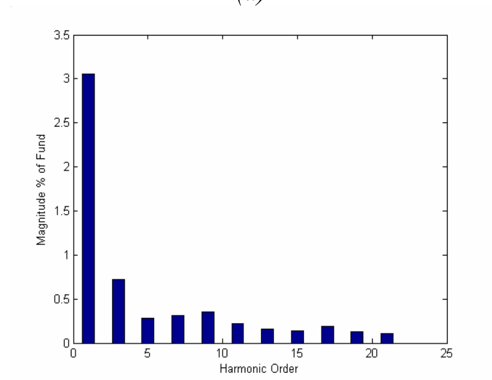
Figure 6 illustrates selected waveforms and their harmonic spectra. These results were obtained using the DM technique. The waveforms obtained by the application of the ND and DA methods are identical in steady state.

Figures 6(a) and 6(b) show the flux waveform and its harmonic spectrum for the flux 1. It can be noticed that the third harmonic represents the 3.0 % of the fundamental and the higher order harmonics are smaller than 0.7% of the fundamental. Figures 6(c) and 6(d) illustrate the waveform and the harmonic spectrum of the arc furnace flux respectively. The third harmonic is 1.8% of the fundamental whereas the fifth harmonic the 0.8%. Finally Figure 6(e) illustrates the electric arc furnace radius *vs* time. Figures 6(a) and 6(c) indicate the moment when a Newton method is applied. The harmonic components are obtained with the Discrete Fast Fourier Transformation algorithm [9].

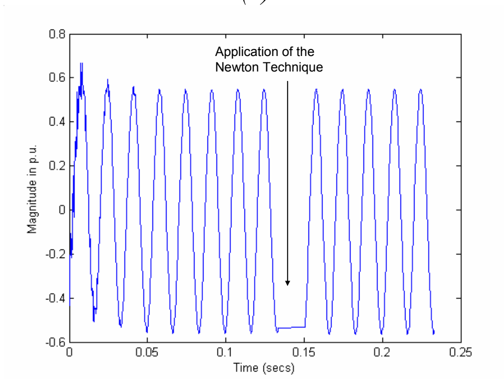
Proceedings of the 5th WSEAS Int. Conf. on SIMULATION, MODELING AND OPTIMIZATION, Corfu, Greece, August 17-19, 2005 (pp341-346)



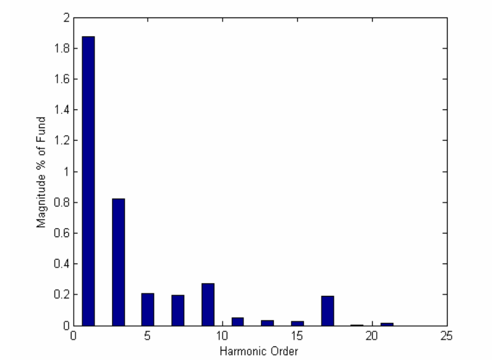
(a)



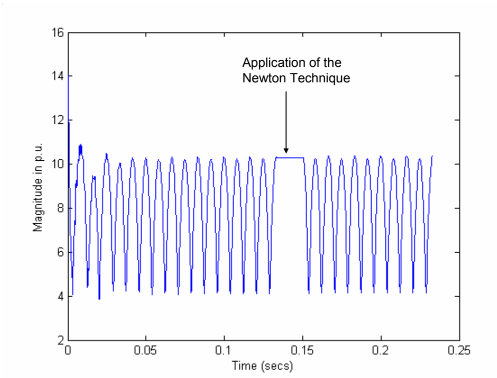
(b)



(c)



(d)



(e)

Figure 6. State variables vs. time and harmonic spectrum (a) ψ_l *vs* time, (b) harmonic spectrum (c) ψ_{arc} *vs* time, (d) harmonic spectrum (e) arc furnace radius.

The algorithms were coded in C language and the digital simulations executed in a Pentium IV, 3.06 GHz personal computer. GNUPLOT [10] was used for the graphical representation of the state variables and their harmonic spectrum.

5 Conclusion

This investigation has detailed three Newton methods, Numerical Differentiation, Direct Approach and Difference Matrix methods for the periodic steady state solution of nonlinear electric systems.

It has been observed that the methods are robust to obtain the steady state solution under non-sinusoidal conditions. On average, the ND and DA methods require less than 43% of the total number of cycles required by the BF method.

Regarding the computation effort the ND and DA methods are 5.52 and 5.25 times respectively, faster than the BF method and 7105 times than the DM method. The high dimension resulting in the DM method resulted in the iterative solution being on average 7100 times slower than the ND and DA methods and 1353 times slower than BF method. Using sparsity techniques the DM is 2094 and 1990 and 379 timer slower than the ND, DA and BF methods, respectively.

References:

[1] Semlyen, A. and Medina, A. "Computation of the periodic steady state in systems with nonlinear components using a hybrid time and frequency domain methodology", *Transactions on Power*

Systems, Vol. 10, No. 3 , August 1995, pp. 1498-504.
[2] A. Medina and N. García, “Newton Methods for the fast computation of the periodic steady state solution of systems with nonlinear and time varying elements”, in *Proc. Conf. IEEE Power Eng. Soc. Summer Meeting*, Vol. 2, Edmonton, AB, Canada, July 18-22, 1999, pp. 664-669.
[3] A. Medina, A. Ramos-Paz, and C. R. Fuerte-Esquivel, Swift computation of the periodic steady state solution of power systems containing TCSCs, 25 (2003) 689-694. 2003 Elsevier Science Ltd.
[4] N. Garcia, E. Acha, and A. Medina, “Swift Time Domain Solutions of Electric Systems Using Parallel Processing”, in Proc. of the Sixth IASTED International Conference, Rhodes, Greece, July 2001, pp. 172-177.
[5] A. Medina, A. Ramos-Paz, C.R. Fuerte-Esquivel, Periodic Steady State Solution of Electric Systems With Nonlinear Components Using Parallel Processing, Transactions on Power Systems, Vol. 8, No. 2, May 2003, pp 963-965.
[6] R.G. Campos and L.O. Pimentel, “A Finite-dimensional representation of the quantum angular momentum operator”, Nuevo Cimento B116, pp.31-45, 2001.
[7] T.S. Parker and L.O. Chua, “Practical Numerical Algorithms of Chaotic Systems”, Springer-Verlag, New York, 1989.
[8] E. Acha, A. Semlyen, N. Rajakovic, “A Harmonic Domain Computational Package for Nonlinear Problems and its Application to Electric Arcs”, IEEE Trans. on Power Delivery, vol. 5, No. 3, July, pp. 1390-1397.
[9] J. Arrillaga, B. Smith, N. Watson, A. Wood, “Power Systems Harmonic Analysis”, John Wiley and Sons.
[10] [online]. Avaliable: www.gnuplot.org

Appendix A. Arc Furnace Model

Based on the model the arc furnace presented in [8], the arc furnace is represented by two differential equations; the first one represents the arc voltage as,

$$\frac{dv}{dt} = k_3 r^{-(m+2)} \left(\frac{\psi_h}{L_h} \right)^2 \qquad (17)$$

and the electric arc furnace radius is obtained as,

$$\frac{dr}{dt} = \frac{k_3}{k_2} r^{-(m+3)} \left(\frac{\psi_{arc}}{L_{arc}} \right) - \frac{k_1}{k_2} r^{n-1} \qquad (18)$$

where constants k_1, k_2 and k_3 are associated with power balance equation for the arc.