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HYBRID REFERENCE MODEL-BASED AUTOTUNED PI CONTROL TESTING

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Abstract: - When testing controller features based on simulation modelling, it is important to establish the conditions for the controller operation as close to real functioning as possible. This paper presents both a suitable model of the controlled plant and a PI controller with autotuning features. The model of the plant takes into account not only the existing non-linear character of the physical phenomena in steady-state relations between inputs and outputs, but also some other phenomena such as saturation, accumulation stopping, technological limits etc. Such types of models, often referred to as engineering models, are suitable tools for controller testing, especially when their ability to master various operating conditions is examined. In the presented case, an evaluation of the selftuning capabilities of a PI controller with parameter adaptation via the continuous gradient method is the main subject of interest. In the usually used methodology, certain parameter changes in transfer function representation of the controlled object are proposed by the experimenters themselves without any correspondence to real causes. By contrast, in the presented non-linear model of a two-tank cascade, changes in physical and operating conditions can be made with all real consequences. The simple mathematical description of the physical laws is valid sufficiently exactly, and allows controller parameter tracking in the adaptation process. PI controller parameter adaptation uses the continuous gradient method combined with the unconventionally proposed reference model of the desired control error course, which universally covers the removal of disturbances and also the setting of a new operating point.

Keywords: - modelling, selftuning, PI control, two-tank cascade

1 Introduction

PI controllers in a continuous or discrete version are undoubtedly the most frequently used control strategy based on control actions and their velocity proportional to the control error. This strategy ensures that the required set point is kept without major problems, even when large changes of the controlled variable are caused by the disturbances and these are connected with changes in the properties of the controlled object.

The choice of a suitable method for determining the optimal parameters for the controller may present certain problems, because there are many methods [1], especially those developed for linear models of control loops [2]. On the other hand, industrial practice strongly prefers only those controller tuning methods that do not require the description of the controlled object to be in a mathematical form. This sometimes means that completely heuristic approaches are preferred. Their greatest disadvantage is the random character of the achieved quality, depending on the experience of the operator carrying out the controller tuning. If it is necessary to perform controller setting in tens of control loops, as is usual in the case of larger plants, then the need to engage experienced (and expensive) operators from companies producing or delivering control systems has strengthened the idea of introducing controllers capable of finding the optimal settings by themselves. This function of autotuning is sometimes directly required because it is the only solution when the controller has to manage different regimes of operation without pre-programmed parameter switching.

A very difficult situation in this attempt to set the optimal controller parameter arises when the controller has to keep the control variable at various required values under various load values. As the load is represented by the material or energy flows, there is a need for this control function in devices where the controlled variables are physical quantities expressing the state of the accumulation process influenced by external disturbances. A linear model representation, e.g., by means of transfer functions for the manipulated variable and the disturbances, is very inexact. This is due to the large parameter changes in dependence on the operating conditions. Thus, the only chance of success lies in finding the controller parameter setting supported by standard methods. To deal with this situation, one of the following procedures is usually applied:

- look for a so called compromise controller setting
- switch between the predetermined optimal controller settings
- adapt the controller parameters in the course of the control process.

The problem with the first procedure is to define the compromise; in the second procedure, it is not easy to define the proper instant for switching (regardless of searching for a suitable controller setting). The third approach is well prepared for mastering large changes in the dynamics of the controlled object, assuming careful preparation for using automatic procedures. In order to detect the weak points of autotuning and to design a new improved autotuning mechanism, a simulator has been developed in the MATLAB/SIMULINK program. This paper reports some results.

2 Model of a Two Tank Cascade

When testing autotuning features, the choice of the controlled object and the feasibility of modelling realistically play important roles. Experience from a nonlinear model of a tank has been used ([2], [3], [4]), but in order not exclude the potential loss of stability, two tanks are connected in a cascade. The tanks can be interconnected in several ways, which affects the mutual interaction and has an impact on the dynamics. An advantage of this choice is the fact that such cascades occur in industrial production (the first tank serves for preparing some parameters of the fluid medium, such as temperature, concentration, composition etc., while the second tank is used as an accumulator, reactor etc.) A similar kind of process dynamics can be found in devices where heat transfer takes place [3]. Thus, conclusions from control tests obtained on a water tank cascade model can be applied to other devices where the dynamics of the processes are analogous.

An important factor in the selection of the water tank cascade was that the nonlinearity and the variability in the behaviour of the two tanks originate from the dependence of the flow rates through the valves. The volume flow rate through the inlet and outlet valves can generally be described very precisely by the equation

$$Q = K_V f(u)\sqrt{\Delta P} \tag{1}$$

where the volume flow rate Q measured in l/min is proportional to the square root of the pressure difference ΔP in kPa on the valve, to the flow coefficient K_V and the dimensionless opening characterized by

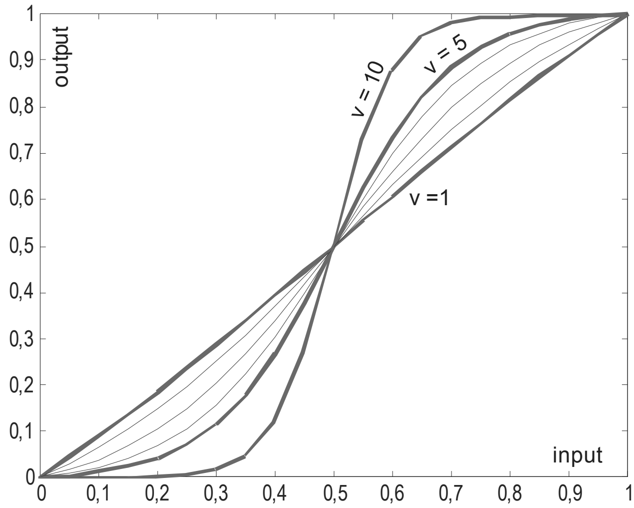

Fig. 1 Valve opening characteristics

the dependence on u in the form

$$f(u) = \frac{\tanh((u-0{,}5)\mathrm{v}) + \tanh(0{,}5\mathrm{v})}{2\tanh(0{,}5\mathrm{v})} \tag{2}$$

where v is an optional coefficient expressing a measure of valve non-linearity. The values of v may vary from 1 - when the opening characteristics of the valve are practically linear - to a value of about 10, when the valve flow characteristics approach a valve "closed – open" (*Fig. 1*).

The flow rate balance

$$\begin{aligned} F_1 \frac{\mathrm{d}}{\mathrm{d}t} h_1(t) &= K_{\mathrm{V1}} f(u_1)\sqrt{\frac{P_1}{\rho g} - h_1(t)} - K_{\mathrm{V2}} f(u_2)\sqrt{h_2(t)} \\ F_2 \frac{\mathrm{d}}{\mathrm{d}t} h_2(t) &= K_{\mathrm{V2}} f(u_2)\sqrt{h_2(t)} - K_{\mathrm{V3}} f(u_3)\sqrt{h_2(t)} \end{aligned} \tag{3}$$

determines the dynamics of the level changes in each tank. The right hand sides of (3), if they are put equal to zero, represent the relations for steady state computation. The results of such a computation are graphically depicted in *Fig. 7.*

3 Tested Controller Algorithm

The use of reference models and their responses as a paradigm for the required behaviour is a widely used method in autotuning. The required behaviour is usually expressed by the course of the controlled variable which is considered to be optimal. The suggested use of the control error instead of the control variable helps to remove a disadvantage of the existing specifications. The need to distinguish which input started the running process of control and which setting from the set of predefined parameter values should be applied are examples of the problems that can be avoided or reduced in the proposed solution.

The standard transfer function form method is used for the optimal controller setting [2]. If a differential equation for control error with damping coefficient ξ = √2/2 and two initial conditions

$$\tau^2\ddot{e}(t)+\tau\sqrt{2}\dot{e}(t)+e(t)=0 \quad e(t_0)=e_0,\ \dot{e}(t_0)=\dot{e}_0 \quad (4)$$

is considered instead of the closed loop transfer function in the standard form, the corresponding response

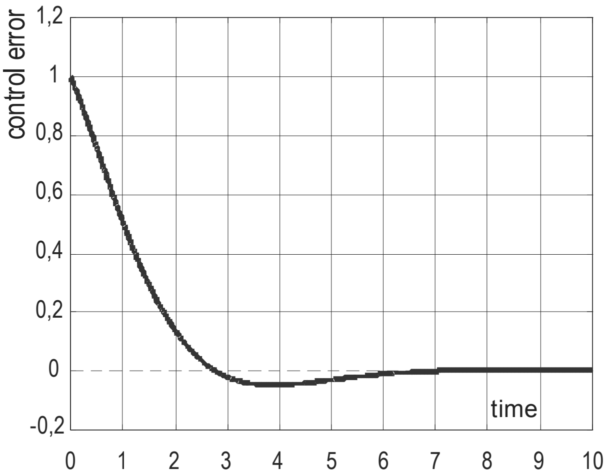

Fig. 3 Control error response with damping factor $\xi = \sqrt{2}/2$

is depicted in *Fig. 3*.

The control error response in *Fig. 3* can represent the desired behaviour when there is a step change of either the set point or the disturbance on the output. If this reference output is compared with the real control error $e_s(t)$ and the difference between them is evaluated by means of the quadratic criterion, an equation for parameter changes (movement) can be formulated, according to which the velocity of the

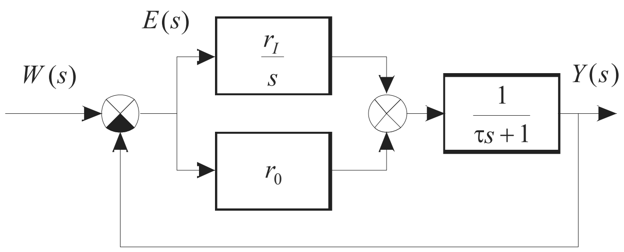

Fig. 2 Block scheme of the linear system approximation used for obtaining the sensitivity functions

parameter changes follows the gradient direction of the surface representing the quadratic criterion values in the space of the adapted parameters.

Instead the model (4) other ways to define the model of f required behaviour can be used, e.g., models from [5] offer a broad spectrum of possibilities .

The following equation of parameter adaptation (5), including all necessary rearrangements, can be notated as follows:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{r}(t) &= -\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{K}\boldsymbol{grad}Q(\boldsymbol{r}(t),t) = \\ &= -\boldsymbol{K}\frac{\mathrm{d}}{\mathrm{d}\boldsymbol{r}}\frac{\mathrm{d}}{\mathrm{d}t}\int_0^{\tau}\left(e(t)-e_s(\boldsymbol{r}(t),t)\right)^2 dt = \quad (5)\\ &= -2\boldsymbol{K}\left(e(t)-e_s(\boldsymbol{r}(t),t)\right)\frac{\mathrm{d}}{\mathrm{d}\boldsymbol{r}}e(\boldsymbol{r}(t),t)\end{aligned}$$

where $\boldsymbol{r}(t)$ represents the vector of adapted parameters, matrix $\boldsymbol{K}$ represents the optional gains to be set in the adaptation loops. The last term in Equation (5), if this is specified for the case of a PI controller parameter adaptation

$$\frac{\mathrm{d}}{\mathrm{d}\boldsymbol{r}}e(\boldsymbol{r}(t),t)=\begin{bmatrix}\dfrac{\partial e}{\partial r_0}\\ \dfrac{\partial e}{\partial r_I}\end{bmatrix}=\begin{bmatrix}c_{r_0}(t)\\ c_{r_I}(t)\end{bmatrix} \quad (6)$$

represents the sensitivity functions $c_{r_0}(t), c_{r_I}(t)$.

These functions serve as dynamical multiplying factors in the loops of parameter adaptation. In the parameter movement they determine the direction of this movement in the parameter plane. It is not easy to obtain these from a non-linear model of the control circuit, especially if a continuous search process is intended.

However, it is easy to formulate a sensitivity model for a linear system. If a linear system having the same step response as the introduced reference course of the control error can be used for generating the sensitivity functions, then there is no problem in deriving a linear model of the control loop (*Fig. 2*)

The sensitivity functions determined for the model in *Fig. 2* are expressed by equations (7), with the use of Laplace transforms

$$\begin{aligned}\tau s^2\frac{\partial E(s)}{\partial r_I}+(1+r_0)s\frac{\partial E(s)}{\partial r_I}+r_I\frac{\partial E(s)}{\partial r_I}&=-E(s)\\ \tau s^2\frac{\partial E(s)}{\partial r_0}+(1+r_0)s\frac{\partial E(s)}{\partial r_0}+r_I\frac{\partial E(s)}{\partial r_0}&=-sE(s)\end{aligned} \quad (7)$$

It is evident that in the simulation model the sensitivity function can be obtained from the Frobenius state space model, where the state variable $c_{r_0}(t)$ is the derivative of $c_{r_I}(t)$.

Fig. 5 shows two examples of the PI parameter evolution. They were stored when a two-tank cascade level control was carried out as a response to the same step changes of the set point value. These steps were repeatedly applied, always after achieving a new steady state. The new run always started from the original state but with the starting values of the PI controller parameters that had been achieved by the adaptation during the previous response. The achieved results, in spite of the strongly nonlinear

properties of the plant and the used linear approximation of the sensitivity model, are of good quality, but are conditioned by a careful choice of the gains in the adaptation loops and the parameter limit settings.
In the real applications, it is not possible to carry out adaptation in such a way. The reference course of the control error (*Fig. 3*) which is good for tuning performed during responses to step changes of the set point or disturbances may not suit the standard oper-

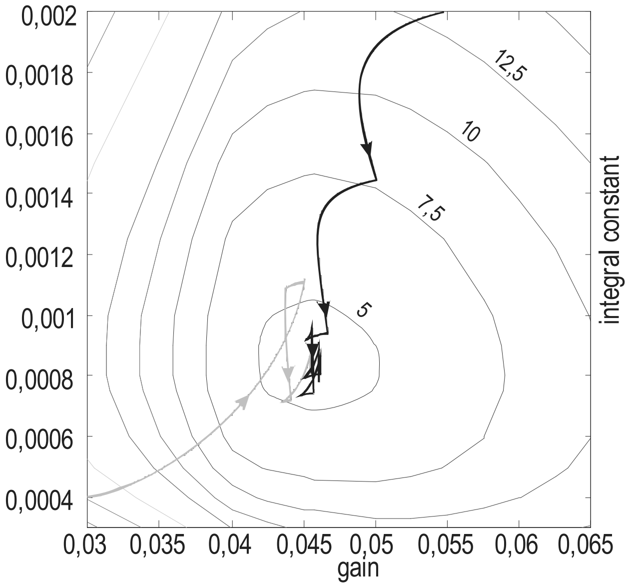

Fig. 5 PI parameter movement towards the minimum of criterion in several steps of repeated simulation

ating situation, when the input changes have a general character. Therefore a new modification has been introduced. The basic idea is to use the reference course of the control error only as a predictor of the desired course. The future control error reference development is based on our knowledge of the control error state evaluated in the elective time instants. Because of the second order of the linear reference model, this involves computing the first derivative of the control error from the recorded control error values taken from the (simulated) real control loop.
If the control error course is described by the second order equation (4), its Laplace transform is

$$E(s)=\frac{\tau^2(se_0+\dot e_0)+\sqrt{2}\tau e_0}{\tau^2 s^2+\sqrt{2}\tau s+1}$$

$$=\frac{se_0+\dot e_0+\frac{\sqrt{2}}{\tau}e_0}{s^2+\frac{\sqrt{2}}{\tau}s+\frac{1}{\tau^2}}= \qquad (8)$$

$$(se_0+\dot e_0+\frac{\sqrt{2}}{\tau}e_0)\frac{\frac{\sqrt{2}}{2\tau}}{\frac{\sqrt{2}}{2\tau}\tau\left[(s+\frac{\sqrt{2}}{2\tau})^2+\frac{1}{2\tau^2}\right]}$$

and the corresponding original in the time domain

$$e(t)=\mathrm{L}^{-1}\left\{(se_0+\dot e_0+\frac{\sqrt{2}}{\tau}e_0)\frac{2\tau}{\sqrt{2}}\frac{\frac{\sqrt{2}}{2\tau}}{(s+\frac{\sqrt{2}}{2\tau})^2+\frac{1}{2\tau^2}}\right\}=$$

$$=\frac{1}{\frac{\sqrt{2}}{2\tau}}\left[\mathrm{e}^{-\frac{\sqrt{2}}{2\tau}(t-t_0)}\sin\left(\frac{\sqrt{2}}{2\tau}(t-t_0)\right)\right]' e_0$$

$$+\frac{1}{\frac{\sqrt{2}}{2\tau}}\left[\mathrm{e}^{-\frac{\sqrt{2}}{2\tau}(t-t_0)}\sin\left(\frac{\sqrt{2}}{2\tau}(t-t_0)\right)\right](\dot e_0+\frac{\sqrt{2}}{\tau}e_0)$$

$$=\mathrm{e}^{-\frac{\sqrt{2}}{2\tau}(t-t_0)}\left[\sin\left(\frac{\sqrt{2}}{2\tau}(t-t_0)\right)+\cos\left(\frac{\sqrt{2}}{2\tau}(t-t_0)\right)\right]e_0$$

$$+\frac{2\tau}{\sqrt{2}}\mathrm{e}^{-\frac{\sqrt{2}}{2\tau}(t-t_0)}\sin\left(\frac{\sqrt{2}}{2\tau}(t-t_0)\right)\dot e_0 \qquad (9)$$

Result (9) can be interpreted as a predictor of the desired behaviour depending on the initial conditions e_0 a $\dot e_0$ in a shifted starting time instant t_0. The shifting of instant t_0 is discontinuous and can be performed periodically or non-periodically. Difficulties with the control error derivative estimation $\dot e_0$ can be avoided

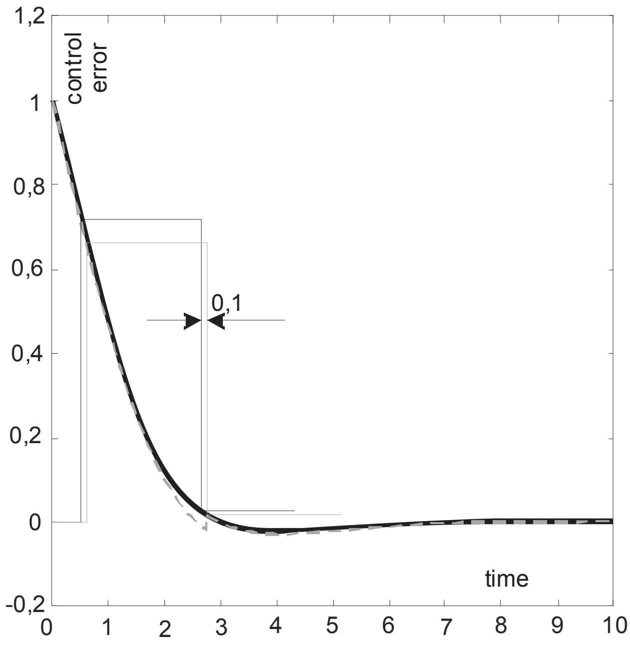

Fig. 4 Actualization in reference response

by calculating from the last saved and topical real value of the control error.

Let e_0 represent a value of the control error saved in time t_0 that follows the instant t_p of the start of a topical prediction by a time interval *Δt,* i.e. $t_p = t_0 + \Delta t$. From the value of the control error e_p sampled by Δt later, i.e., at the time instant $t_p = t_0 + \Delta t$, we can start predicting $e(t)$ due to model (4). The prediction starts by Δt after the instant t_0, when the signal for a new actualization was issued. Up to this time, the predictor operates with the sliding initial conditions from the previous actualization, or with the starting conditions.

A graphical presentation of the described actualization can be found in *Fig. 4* with $\Delta t = 0.1$ s.

The equation of the new course beginning at t_p is

$$e(t) = \mathrm{e}^{-\frac{\sqrt{2}}{2\tau}(t-t_p)} \left[\sin\left(\frac{\sqrt{2}}{2\tau}(t-t_p) \right) + \cos\left(\frac{\sqrt{2}}{2\tau}(t-t_p) \right) \right] e_p + \frac{2\tau}{\sqrt{2}} \mathrm{e}^{-\frac{\sqrt{2}}{2\tau}(t-t_p)} \sin\left(\frac{\sqrt{2}}{2\tau}(t-t_p) \right) \dot{e}_p \qquad (10)$$

where $t_p = t_0 + \Delta t$, $t - t_p = t_0 + \Delta t + \vartheta - t_0 + \Delta t = \vartheta$ and the starting values e_p and $\dot{e}_p$ are defined by the formula

$$e_p = e(t_p)$$

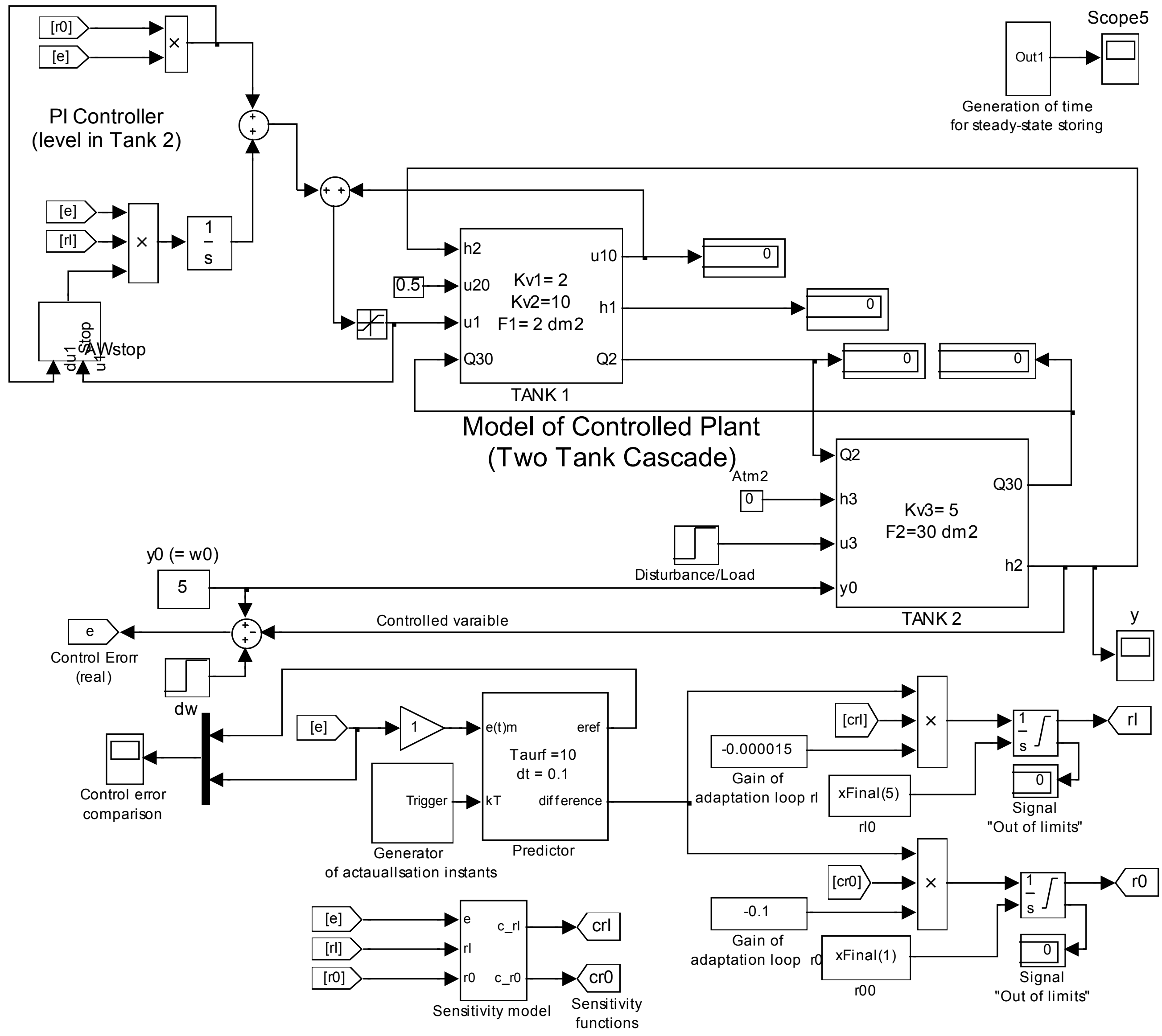

Fig. 6 Simulink program block scheme for PI autotuned level control testing

$$\dot{e}_p = \frac{\left\{ \mathrm{e}^{\frac{\sqrt{2}}{2\tau}(-\Delta t)} e(t_p - \Delta t) - \left[\sin\left(\frac{\sqrt{2}}{2\tau}(-\Delta t)\right) + \cos\left(\frac{\sqrt{2}}{2\tau}(-\Delta t)\right) \right] e_p \right\}}{\frac{2\tau}{\sqrt{2}} \sin(\frac{\sqrt{2}}{2\tau}(-\Delta t))}$$

In the equations for controller parameter adaptation, the differences between the control error from the control loop and the topical predicted

$$\dot{r}_I(t) = -k_I c_{r_I}(t)\big(e(t) - e_s(t)\big)$$

$$\dot{r}_0(t) = -k_0 c_{r_0}(t)\big(e(t) - e_s(t)\big)$$

Many controlled plants behave like the proportional systems, the dominant part of which corresponds to the behaviour of the first order system characterized by its time constant T_1. Then a conversion can be made into dimensionless time t/T_1, with the advantage that we can work with a dimensionless defined reference model allowing simplified starting adjustment of the controller parameters $\tilde{r}_0 = \sqrt{2} - 1$ and $\tilde{r}_I = 1$.

4 Simulation results

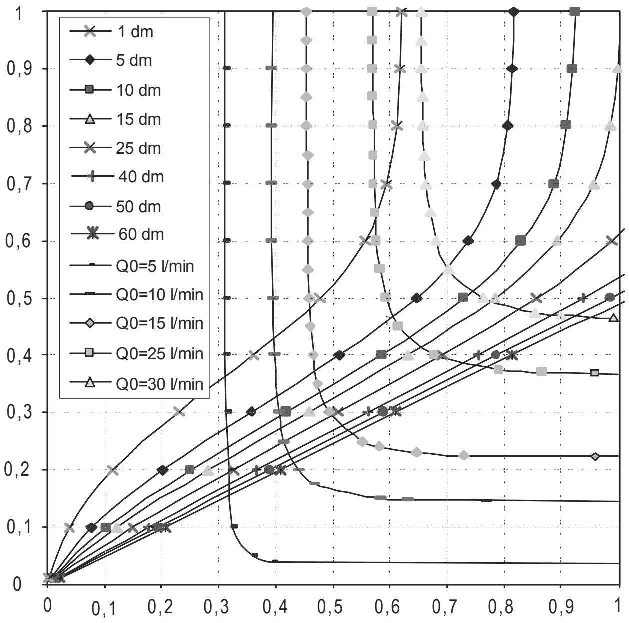

Fig. 7 Setting and load characteristics

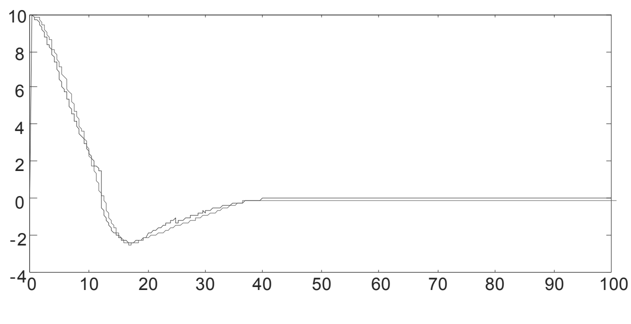

Fig. 8 Real and actualized reference control error

The simulation focused on testing how changes of the operating points influence the performance of the controller. The steady-state operating points create the curves depicted in the characteristics (*Fig. 7*).
A resultant illustrative example of the courses of the control errors is shown in Fig. 8.

5 Conclusion

Simulation tests have shown that the quality of control results still depends on setting the auxiliary parameters, especially the actualization interval. The desired fully automatic tuning function has not been achieved to an extent that would enable it to be used directly in practical applications. Basically, our results confirm that the use of the control error as a reference is commonly applicable both for disturbances and for the desired value changes.

Acknowledgement

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