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# Optimization of spacing for rectangular not-unitary efficiency fin systems.

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*Abstract:* - Fin spacing is chosen to maximize heat transmission rate. Maximization is attained as a compromise between two opposite trends: fin surface maximization and adduction coefficient maximization. In this paper a new relation for the optimum spacing between rectangular fins is proposed. The relation has been obtained keeping into account non-unitary fin efficiency. Demonstration is achieved by a theoretical approach which regards only rectangular fins.

*Key-Words:* - heat flux, fin, spacing, efficiency.

## 1 Introduction

The operation of various systems (power electronics, CPU cooling) depends on natural convection cooling. Fins allow to increase heat flux rate. Spacing between fins is necessary; this may result in larger volumes. Hence, finding the optimum fin spacing is quite important; fin spacing is chosen to maximize heat transmission rate. Maximization is attained as a compromise between two opposite trends:

- fin surface maximization;
- adduction coefficient maximization.

Fin surface increase by diminishing spacing; adduction coefficient increase by increasing spacing. Optimal fin spacing has been found assuming the following approximations:

a) convection coefficient is uniform along the fin [1];
b) radiative heat transfer rate is negligible;
c) fin efficiency is assumed unitary.

In this paper it is demonstrated that spacing may be furthermore optimised if fin efficiency is kept into account; thus only approximations a) and b) are still taken into account. Demonstration is achieved by a theoretical approach which regards only rectangular fins. The optimised spacing allows to decrease thermal resistance maintaining the same fin system dimension. This result may be important for electronic applications where high heat transfer rates are produced by very small devices.

## 2 Optimal Rectangular Fin Spacing

Heat transfer rate transferred from an hot surface (a fin system) to the surrounding environment is given by the following relation:

$$q = q_c + q_r \tag{1}$$

When temperature difference is small the radiation term qr may be neglected [1]. In order to increase heat transfer rate it is possible to both increase the fin surface and, alternatively, increase convection coefficient *h*. A generic thin fin temperature distribution is ruled by the following relation if only convective heat transfer rate is considered

$$\frac{T - T_\infty}{T_p - T_\infty} = \frac{\cosh(m(L-x)) + \dfrac{h}{m\lambda}\sinh(m(L-x))}{\cosh(mL) + \dfrac{h}{m\lambda}\sinh(mL)} \tag{2}$$

Thus heat transfer rate is:

$$Q_d = \sqrt{hP\lambda A}(T_s - T_\infty)\frac{\sinh(mL) + \dfrac{h}{m\lambda}\cosh(m)}{\cosh(mL) + \dfrac{h}{m\lambda}\sinh(mL)} \tag{3}$$

The fin efficiency is defined as it follows:

$$\eta = \frac{Q_d}{Q_{id}} \tag{4}$$

Thus, if the only convective contribute is considered, fin heat flux rate may be found by the following relation:

$$Q_d = \eta \cdot h \cdot S \cdot (T_p - T_\infty) \tag{5}$$

When a rectangular fin is characterized by b much grater than t, it may be written:

$$m = \sqrt{\frac{hP}{\lambda A}} \cong \sqrt{\frac{2h}{t\lambda}} \tag{6}$$

Furthermore fin efficiency is well approximated by the following relation [2]:

$$\eta = \frac{tgh(mL)}{mL} \tag{7}$$

A rectangular fin system is constituted by an array of cavities as it is shown in Fig.1.

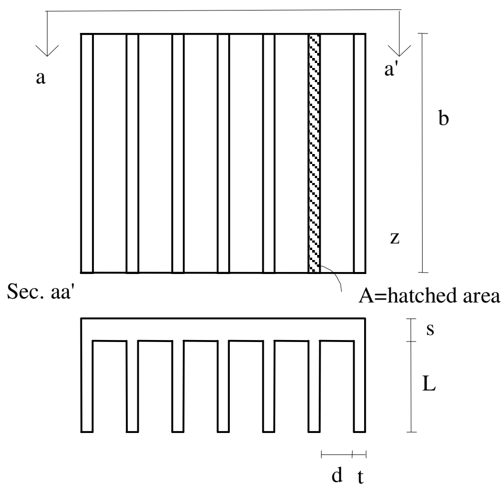


Fig.1: Rectangular fin system scheme.

On natural convection, convective coefficient depends on the dimension $d$ of each cavity according to the following laws [2],[3]:

$$h = \frac{Nu \cdot \lambda_a}{d} \tag{8}$$

where

$$Nu = \frac{1}{\sqrt{\frac{576}{(Ra')^2} + \frac{2.873}{(Ra')^{1/2}}}}, \; Ra' = Ra \cdot \frac{d}{b} \tag{9}$$

Eq. (8) states that convection coefficient $h$ increase when $d$ increase. Anyway global heat transfer rate increases by increasing total fin surface which means to increase the number of cavity corresponding to a reduction of d.

In order to maximize convective heat transfer it is needed to get an optimal fin spacing. Let introduce the following statements:

$$y = \frac{d}{b}$$
$$R = \frac{g \cdot \beta \cdot \Delta T \cdot b^3}{\nu^2} \Pr \tag{10}$$

Eq. (10) yield that:

$$Ra' = R \cdot y^4 \tag{11}$$

For obtaining optimal spacing it is assumed in literature [3] that the fin efficiency is 1; such an assumption allows to write fin convective heat transfer rate as follows:

$$H = \cdot \frac{2 \cdot (z+d) \cdot L \cdot \lambda_a \cdot (T_p - T_\infty)}{b \cdot \sqrt{\frac{576}{R^2 \cdot y^4} + y^2 \frac{2.873}{\sqrt{R}}}} \tag{12}$$

It may be shown that maximum value of qc is attained when [3]:

$$y = 2.71 \cdot R^{-1/4} \rightarrow d_{ott} = 2.71 \cdot b \cdot R^{-1/4} \tag{13}$$

## 3 Further Convective Heat Transfer Optimization

Relation (13) was found by considering a unitary fin efficiency. If efficiency is introduced a further optimization of fin spacing may be attained. Fin efficiency can be conveniently approximated by the following relation:

$$\eta = \frac{1}{1 + \frac{1}{3}(mL)^2} \tag{14}$$

It may be shown that Eq. (14) approximates fin efficiency better than Taylor series truncated to the third term. Error committed using (14) on behalf of (7) is below 10% when $mL$ is below 1.5 (appendix A).

It may be simply shown that convective fin heat rate is:

$$q_c = 2 \cdot \left( \frac{z}{d} + 1 \right) \cdot (L \cdot b) \cdot h \cdot \eta \cdot (T_p - T_\infty) \tag{15}$$

Eq. (15) may be rewritten introducing equation (14) and (8) as follows:

$$q_c = \frac{2 \cdot (z+d) \cdot L \cdot \lambda_a \cdot (T_p - T_\infty)}{b \cdot \sqrt{\frac{576}{R^2 y^4} + y^2 \frac{2.873}{\sqrt{R}}} + \frac{2\lambda_a L^2}{3\lambda_w t} y} \tag{16}$$

Maximizing $q_c$ means to solve the following equation in terms of $y$:

$$\frac{d}{dy}\left( b \cdot \sqrt{\frac{576}{R^2 y^4} + y^2 \frac{2.873}{\sqrt{R}}} + \frac{2\lambda_a L^2}{3\lambda_w t} \right) = 0 \tag{17}$$

It yields to the following parabolic equation on variable $u = y^6$:

$$\left(\frac{16\cdot 2.873\cdot \lambda_a^2 L^4}{9\cdot t^2\sqrt{R}\cdot \lambda_w^2}-\frac{4\cdot b^2\cdot 2.873^2}{R}\right)\cdot u^2 + \left(\frac{16\cdot 576\cdot \lambda_a^2 L^4}{9\cdot t^2\cdot R^2\cdot \lambda_w^2}+\frac{16\cdot b^2\cdot 2.873\cdot 576}{R^{5/2}}\right)u - \left(\frac{16\cdot b^2\cdot 576^2}{R^4}\right) \quad (18)$$

Thus optimal spacing is given by:

$$d_{opt} = b\cdot \sqrt[6]{u_{opt}} \quad (19)$$

# 4 Application

CPU's fin dissipaters may have a relevant importance both on CPU processing performance and CPU lifetime. Heat flux rate of two base like dimension CPU dissipater have been compared; dissipaters fin spacing has been determined by two different methods:

a) unitary fin efficiency method;
b) the proposed true fin efficiency method (Eq. (14)).

The following geometrical and thermal parameter have been considered:

$\Delta T$ = 80 K
$\lambda_w$ = 100 [W m-1 K]
$\lambda_a$ = 0,0261 [W m-1 K]
$v^2$ = 2,531x10-10 [m4 s-2]
$g\beta$ = 0,027 [m s-2 K-1]
$Pr$ = 0,701 (biatomic gas)
$t$ = 0,001 m
$L$ = 0,14 m
$b$ = 0,08 m
$z$ = 0,09 m

Fin spacing obtained for case a) method is:

$$d_A = 5{,}18\ mm \quad (20)$$

By eq. (19), fin optimal spacing for case b) is:

$$d_B = 4{,}43\ mm \quad (21)$$

The optimal spacing dB produces a 20 cavities dissipater; on the contrary fin spacing attained by unitary efficiency method (case a)) yields 17 cavities.

Heat flux increase due to case b) fin spacing optimization can be calculated by the following ratio:

$$\frac{q_B}{q_A} = \frac{2\cdot\left(\frac{z}{d_B}+1\right)\cdot(L\cdot b)\cdot(h\cdot\eta)_B}{2\cdot\left(\frac{z}{d_A}+1\right)\cdot(L\cdot b)\cdot(h\cdot\eta)_A} \quad (22)$$

Using fin spacing values in Eq. (20) and (21), ratio (22) results:

$$\frac{q_B}{q_A} = \frac{(z+d_B)\cdot d_A\cdot(h\cdot\eta)_B}{(z+d_A)\cdot d_B\cdot(h\cdot\eta)_A} = 1{,}03 \quad (23)$$

The proposed fin spacing brings to an approximately 3% dissipated heat flux increase.

# 5 Conclusion

In this paper a new relation for the optimum spacing between rectangular fins is proposed. The relation has been obtained keeping into account non-unitary fin efficiency.

The difference between heat dissipation rate calculated by literature method (unitary fin efficiency) and the heat dissipation rate evaluated by the proposed method increases as fin height grows. Application on CPU dissipater shows heat flux increase up to 3%.

Considering Arrhenius theory [7], electronic component life decreases exponentially with temperature, thus even a low increase on heat flux rate maybe significant for CPU lifetime.

# 6 Appendix A: Fin Efficiency Approximation

In paragraph 1 a fin efficiency approximation is proposed to ease optimal fin spacing calculation. Considering the dimensions sketched in Fig.1, fin efficiency is expressed by the following relation:

$$\eta = \frac{tgh(mL)}{mL} \quad (24)$$

Using Eq. (24) Taylor series expansion, truncated to the third component, fin efficiency can be approximated as follows:

$$\eta = \frac{1}{1+\frac{1}{2}(mL)^2} \quad (25)$$

Comparing (24) and (25) equations behaviour, it is possible to show (Fig. 2) that a better efficiency approximation can be obtained by means of the following relation [2]:

$$\eta = \frac{1}{1+\frac{1}{3}(mL)^2} \tag{26}$$

Analyzing Fig. 2, error committed using (26) instead of (24) is inferior to 10% when $mL$ value is less than 1.5.

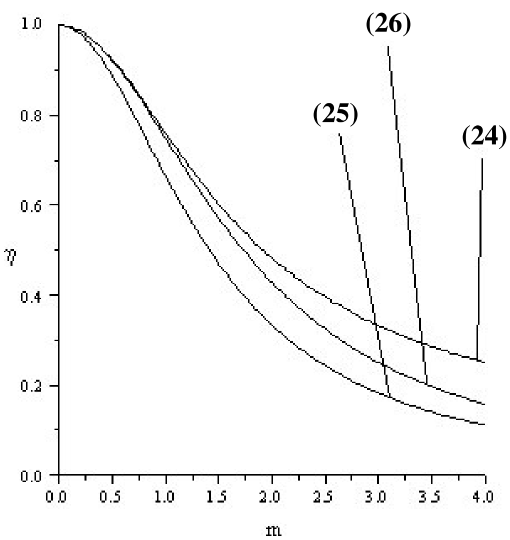


Fig. 2: Comparison between real and its approximations fin efficiency behavior.

## 7 Symbols

| Symbol | Units | Description |
|---|---|---|
| $A$ | $m^2$ | single fin bar final surface area |
| $b$ | m | fin length |
| $\beta$ | $K^{-1}$ | thermal dilatation coefficient |
| $d$ | m | single fin cavity width |
| $d_A$ | m | optimal fin spacing obtained by using a unitary fin efficiency |
| $d_B$ | m | optimal fin spacing obtained with fin efficiency given by Eq. (13) |
| $d_{opt}$ | m | optimal fin spacing |
| $\Delta T$ | K | difference between fin and air temperatures |
| $g$ | $m \cdot s^{-2}$ | gravity acceleration |
| $\gamma$ | adimensional | division between fin length and fin spacing |
| $h$ | $W \cdot m^{-2} \cdot K^{-1}$ | fin-air convection coefficient |
| $\eta$ | adimensional | fin efficiency |
| $L$ | m | fin height |
| $\lambda_w$ | $W \cdot m^{-1} \cdot K^{-1}$ | fin thermal conductivity |
| $\lambda_a$ | $W \cdot m^{-1} \cdot K^{-1}$ | air thermal conductivity |
| $Nu$ | adimensional | Nusselt number |
| $\nu$ | $m^2 \cdot s^{-1}$ | cinematic viscosity |
| $P$ | m | single fin bar perimeter |
| $Pr$ | adimensional | Prandtl number |
| $Q_d$ | W | single fin bar heat transfer rate |
| $Q_{id}$ | W | maximum exchanged heat flux |
| $q$ | W | heat flux |
| $q_A$ | W | convective heat flux obtained with known optimization method |
| $q_B$ | W | convective heat flux obtained with the proposed optimization method |
| $q_c$ | W | convective heat flux |
| $q_r$ | W | radiative heat flux |
| $R$ | adimensional | Rayleigh number referred to $b$ |
| $Ra$ | adimensional | Rayleigh number |
| $Ra'$ | adimensional | modified Rayleigh number |
| $S$ | $m^2$ | single fin bar area |
| $T$ | K | fin temperature distribution |
| $T_p$ | K | fin wall temperature |
| $T_s$ | K | fin cavity surface temperature when x=0 |
| $T_\infty$ | K | air temperature |
| $t$ | m | single fin bar width |
| $u$ | adimensional | substitution variable |
| $u_{ott}$ | adimensional | optimum u value |
| $y$ | adimensional | division between fin cavity width and fin length |
| $z$ | m | fin width |

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