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Unsteady MHD Stagnation-point Flow

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Abstract: The unsteady two-dimensional stagnation-point flow of a viscous fluid impinging on an infinite plate in the presence of a transverse magnetic field is studied. The plate is making harmonic oscillations in its own plane. A finite difference technique is employed and solutions for large frequencies of the oscillations are obtained for various values of the Hartman's number.

Keywords: unsteady, stagnation-point flow, magneto-hydrodynamics, oscillating plate.

1 Introduction

The flow of an incompressible viscous fluid over a moving plate is important in many industrial applications. Examples include the extrusion of plastic sheets, fabrication of adhesive tapes and application of coating layers onto rigid substrates, among others. If a magnetic field is present, viscous flows due to a moving plate in an electro-magnetic field, i.e. magnetohydrodynamic (MHD) flows, are relevant to many practical applications in the metallurgy industry, such as the cooling of continuous strips and filaments drawn through a quiescent fluid and the purification of molten metals from non-metallic inclusions.

The flow over a moving infinite plate is essentially a two-dimensional stagnation point flow which has received considerable attention for many years. This consists of a class of flows in the vicinity of a stagnation line that results from a two-dimensional flow impinging on a surface at right angles and flowing there after symmetrically about the stagnation point. Furthermore, time dependent or unsteady viscous flow near a stagnation point has also been widely investigated. Glauert [1] and Rott [2] studied the stagnation point flow of a Newtonian fluid when the plate performs harmonic oscillations in its own plane. Seshadri et al. [3] investigated the unsteady stagnation point flow on a heated vertical plate. The unsteady motion is caused by the impulsive change in the free stream velocity and by a sudden increase in the surface temperature. Kumari and Nath [4] examined the unsteady flow over an infinite disk rotating with a time dependent angular velocity in the presence of a magnetic field applied normally to the disk surface. The fluid is electrically conducting with constant properties. The Navier-Stokes equation and the energy equation governing the unsteady flow are reduced to a system of ordinary differential equations by using similarity transformations.

In this paper, we consider the unsteady two dimensional flow of a viscous fluid impinging on an infinite plate in the presence of a magnetic field. The plate is assumed to make harmonic oscillations in its own plane. The magnetic field is assumed to be transverse or perpendicular everywhere in the flow field. Solutions for large frequencies of the oscillations are obtained using a finite difference technique.

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2 Flow equations

The two-dimensional flow of a viscous incompressible in the presence of a magnetic field is governed by:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+\frac{1}{\rho}\frac{\partial p}{\partial x}=\nu\nabla^2 u-\frac{\sigma B_0}{\rho}u \qquad (2)$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+\frac{1}{\rho}\frac{\partial p}{\partial y}=\nu\nabla^2 v \qquad (3)$$

where $u=u(x,y)$, $v=v(x, y)$ are the velocity components, $p=p(x,y)$ is the pressure, $\nu=\dfrac{\mu}{\rho}$ is the kinematic viscosity, σ is the electrical conductivity and B_0 is the magnetic field. It is assumed that $\sigma B_0 \ll 1$, so that it is possible to neglect the effect of the induced magnetic field.

The continuity equation (1) implies the existence of a streamfunction $\psi=\psi(x,y,t)$ such that

$$u=\frac{\partial\psi}{\partial y},\ v=-\frac{\partial\psi}{\partial x} \qquad (4)$$

Substitution of (4) in (2) and (3) and elimination of pressure from the resulting equations using $p_{xy}=p_{yx}$ yields

$$\frac{\partial\left(\nabla^2\psi\right)}{\partial t}-\frac{\partial\left(\psi,\nabla^2\psi\right)}{\partial\left(x,y\right)}-\nu\nabla^4\psi+\frac{\sigma B_0}{\rho}\frac{\partial^2\psi}{\partial y^2}=0 \qquad (5)$$

Having obtained a solution of equation (5), the velocity components are given by (4) and the pressure can be found by integrating equations (2) and (3).

3 Solutions

We consider the two-dimensional flow of an incompressible fluid against an infinite plate normal to the flow. We assume that the plate is making harmonic oscillations on its own plane with velocity in the x-direction equal to $a\,e^{i\omega t}$ where a and ω are constants.

The boundary conditions are then given by

$$\frac{\partial\psi}{\partial y}=a\,e^{i\omega t},\ \frac{\partial\psi}{\partial x}=0 \quad \text{at} \quad y=0,$$
$$\frac{\partial\psi}{\partial y}=cx \quad \text{as} \quad y\to\infty \qquad (7)$$

Following Glauert [1], we assume that

$$\psi=cx\,f(y)+a\,e^{i\omega t}\,g(y) \qquad (8)$$

The boundary conditions take the form

$$f(0)=f'(0)=0, \quad g'(0)=1$$
$$f'(\infty)=1, \ \ g'(\infty)=0 \qquad (9)$$

Using (8) in (5), we obtain

$$\nu\,f^{(iv)}+c\,f\,f'''-c\,f'\,f''-\frac{\sigma B_0}{\rho}f''=0$$
$$\nu\,g^{(iv)}+c\,f\,g'''-c\,f''\,g'-i\,\omega\,g''-\frac{\sigma B_0}{\rho}g''=0 \qquad (10)$$

We non-dimensionalize system (10) and the boundary conditions (9) using

$$\eta = \sqrt{\frac{c}{\nu}}\ y, \quad f(y) = \sqrt{\frac{\nu}{c}}\ F(\eta), \qquad g(y) = \sqrt{\frac{\nu}{c}}\ G(\eta), \tag{11}$$

and upon integration of the resulting equations once with respect to η, we get

$$F''' + F\,F'' - F'^2 - M\,F' = -1 - M \qquad F(0) = F'(0) = 0, \quad F'(\infty) = 1 \tag{16}$$

and

$$G''' + F\,G'' - F'G' - M\,G' - \frac{i\omega}{c}G' = 0 \qquad G'(0) = 1, \;\; G'(\infty) = 0 \tag{17}$$

where M is the Hartman's number.

When $M = 0$, the solution to system (16) corresponds to the well-known Hiemenz flow. System (16) is solved numerically using the shooting method with a finite difference technique for different values of M. We found that for $M = 0$, $F''(0) = 1.23259$ which is in good agreement with the Hiemenz solution. Numerical values of $F''(0)$ for different values of M are given in Table 1.

Letting $\phi(\eta) = G'(\eta)$, then system (17) becomes

$$\phi'' + F\,\phi' - F'\,\phi - M\,\phi - \frac{i\omega}{c}\phi = 0 \qquad \phi(0) = 1, \;\; \phi(\infty) = 0 \tag{18}$$

For large values of the parameter $\frac{\omega}{c}$, we let

$$Y = \sqrt{\frac{i\omega}{c}}\,\eta = \sqrt{\frac{i\omega}{\nu}}\,y \tag{19}$$

Letting $\alpha = \sqrt{c/i\omega}$, then $\frac{\mathrm{d}}{\mathrm{d}\eta} = \frac{1}{\alpha}\frac{\mathrm{d}}{\mathrm{dY}}$

and system (18) takes the form

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}Y^2} + \alpha\left(F\frac{\mathrm{d}\phi}{\mathrm{d}Y} - \frac{\mathrm{d}F}{\mathrm{d}Y}\phi \right) - \phi - \alpha^2 M\,\phi = 0 \qquad \phi(0) = 1, \quad \phi(\infty) = 0 \tag{20}$$

The expansion for $F(Y)$ near the wall $Y = 0$ is

$$F(Y) = \frac{1}{2}A\alpha^2Y^2 - \frac{1}{6}\left(1+M\right)\alpha^3Y^3 + \frac{1}{24}MA\alpha^4Y^4 + \frac{1}{120}\left(A^2 - M^2 - M\right)\alpha^5Y^5 + \cdots \tag{21}$$

which is valid for small Hartmann numbers. In this expansion, $A = F''(0)$.

Since for large values of ω/c the parameter α is small, we let

$$\phi(Y) = \sum_{n=0}^{\infty}\alpha^n\phi_n(Y) = \phi_0(Y) + \alpha\phi_1(Y) + \alpha^2\phi_2(Y) + \cdots \tag{22}$$

The boundary conditions are

$$\phi_0(0) = 1, \quad \phi_n(0) = 0 \;\text{ if }\; n \geq 1, \qquad \phi_n(\infty) = 0 \;\; \forall n \tag{23}$$

Substituting (22) in (18) and equating the coefficients of different powers of α to zero, we find that the boundary value problem for $\phi_0(Y)$ is

$$\frac{\mathrm{d}^2\phi_0}{\mathrm{d}Y^2} - \phi_0 = 0, \;\; \phi_0(0) = 1, \;\; \phi_0(\infty) = 0$$

with solution $\phi_0(Y) = \exp(-Y)$.

The second equation gives that ϕ_1 is zero. The next three equations for $\phi_2(Y)$, $\phi_3(Y)$, and $\phi_4(Y)$ are

$$\frac{d^2\phi_2}{dY^2} - \phi_2 = M\,\phi_0$$

$$\frac{d^2\phi_3}{dY^2} - \phi_3 = M\,\phi_1 - \frac{1}{2}AY^2\phi_0' + AY\,\phi_0$$

$$\frac{d^2\phi_4}{dY^2} - \phi_4 = M\,\phi_2 + \frac{1}{6}(1+M)Y^3\phi_0' - \frac{1}{2}AY^2\phi_1' + AY\phi_1 - \frac{1}{2}(1+M)Y^2\phi_0$$

Solving these equations and using the boundary conditions, we obtain

$$\phi_2(Y) = -\frac{M}{2}Y\,e^{-Y}$$

$$\phi_3(Y) = -A\,e^{-Y}\left(\frac{1}{12}Y^3 + \frac{3}{8}Y^2 + \frac{3}{8}Y\right)$$

$$\phi_4(Y) = e^{-Y}\left\{\left[\frac{3(1+M)}{16} + \frac{M^2}{2}\right]Y + \left[\frac{3(1+M)}{16} + \frac{M^2}{2}\right]Y^2 + \frac{M+1}{8}Y^3 + \frac{M+1}{48}Y^4\right\}$$

If $M = 0$, results obtained by Glauert [1] are recovered.

4 Conclusion

Results for this flow are obtained for various values of the Hartman's number M. At the higher frequencies, the perturbation is a shear layer, exactly as on a plate oscillating in a fluid at rest. Figure 1 shows the variation of $F'(\eta)$ for various M. An effect of the Hartman's number is to increase the velocity near the wall as it increases. Also, from Table 1, $F''(0)$ increases with the magnetic parameter M. The reason for this behaviour is that the magnetic field B induces a force along the surface which supports the motion. As a result, the velocity along the surface is increased everywhere and hence that the shear stress on the wall increases with increasing Hartman's number.

M	$F''(0)$
0	1.23259
0.25	1.32898
0.5	1.41886
1.0	1.58333
1.5	1.73210
2.0	1.86885
2.25	1.93351
3.0	2.11532

Table 1. Numerical values of $F''(0)$ for different values of M.

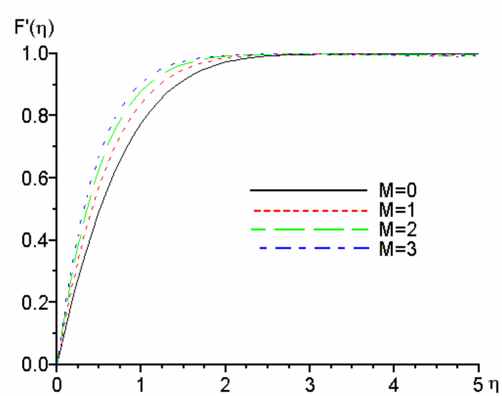

Fig. 1. Variation of $F'(\eta)$ for various M.

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