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Metal plastic straining processes with predictable mechanical and constitutive properties modeling

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Abstract: - theoretical bases of modeling of the processes plastic straining metal with predicting their mechanical and constitutive properties are considered in the paper present. The developed approach to modeling is built on decisions of the system of the main equations of the plastic yielding of solids, including features of damaging microdefect strain origins, values grain of polycrystal unit and internal energy of hardening. The example of metal damaging kinetic model in plastic straining processes with accompanying thermal operation is given.

Key-Words: - plasticity, stress, deformation, structure, microdefect, temperature, heat treatment, damaging, modeling

1 Inroduction

Many technological processes of metalware production are based on the joint use of plastic forming operations (plastic working) and heat treatment. The operating characteristics of the finished products substantially define mechanical and physico-constitutive properties of the materials.

A leading role in the technological creation of mechanical and constitutive properties of the finished products materials belongs to the operations of plastic working and heat treatment. The models of the plastic forming processes promotes a successful decision of the problems appearing under technology optimum design with criterial parameters satisfying conditions of the state-of-the-art production.

2 Problem Formulation

A combined physico-mechanical approach based on the study of both mechanical and constitutive properties amounting to criterial technological parameters of the strained metal evolves for the plastic forming processes modeling. This approach expects the determination of consistent stress and plastic yielding velocities fields under compound loading using the real work material models. It is particularly challenging in analysis and design of the nonstationary and high-speed strain processes with a strong stress and strain rate phase variation.

3 Problem Solution

A modeling of the plastic forming processes is based on solutions of the basic combined equations for mechanical and constitutive parameters of the strained material by attracting the experimental database for the edge conditions statement in plasticity theory problems.

3.1 Main equations

The basic combined equations describing a plastic yielding of solids with mechanical and constitutive properties in orthogonal curvilinear coordinates $x^i \, (i=1,2,3)$ is given by [1]:

$$\nabla_j \sigma^{ij} = \rho\left(j^i - F^i\right), \tag{1}$$

$$\nabla_i v^i = 0, \tag{2}$$

$$f\left(s^{ij}, e^{ij}, T, \chi_s, \mu_k\right) = 0, \tag{3}$$

$$\dot{e}_{ij} = \dot{\lambda}\frac{\partial f}{\partial s^{ij}}, \tag{4}$$

$$\frac{d\mu_k}{dt} = \dot{\mu}_k\left(\sigma^{ij}, e^{ij}, T, \chi_s, \mu_k\right), \tag{5}$$

where σ^{ij} - contravariant components of stress tensor; v^i, j^i, F^i - contravariant components of velocity, acceleration and external force density vectors, accordingly; e_{ij} - covariant components of strain tensor; ρ - density of the material; T - a thermodynamic temperature; χ_s - parameters related to deformations by nonholonomic correlations; μ_k - physico-constitutive parameters; ∇_j - a symbol signifying the covariant differentiation; $\dot{\lambda}$ - a positive scalar value proportionate to plastic strain force; s^{ij} - stress deviator components; t - time; $\dot{e}_{ij}$ - components of the strain rate tensor.

The system (1) - (5) consists of the motion equations (1), the incompressibility condition (2), the equations of yield surfaces (3), condition of strain rate gradience (4) and kinetic equations (5) for physico-constitutive parameters. In the combined equations (1) - (5):

$$\nabla_j \sigma^{ij} = \frac{\partial \sigma^{ij}}{\partial x^j} + \sigma^{kj}\Gamma^i_{kj} + \sigma^{ik}\Gamma^j_{kj} \ ,$$

$$\nabla_i v^i = \frac{\partial v^i}{\partial x^i} + v^k \Gamma^i_{ik} \, ,$$

$$j^i = \frac{\partial v^i}{\partial t} v^j \left(\frac{\partial v^i}{\partial x^j} + v^k \Gamma^i_{kj} \right),$$

$$\dot{\xi}_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x^j} + \frac{\partial v_j}{\partial x^i} - 2 v_k \Gamma^k_{ij} \right),$$

where Γ^i_{kj} - Kristoffel symbols.

Non-zero symbols $\Gamma^i_{kj}\ (k = i, j)$ are calculated by formulas:

$$\Gamma^i_{ij} = \frac{1}{\mathrm{H}_i}\frac{\partial \mathrm{H}_i}{\partial x^j} \ , \qquad \Gamma^i_{jj} = -\frac{\mathrm{H}_j}{\mathrm{H}_i^2}\frac{\partial \mathrm{H}_j}{\partial x^i} \ ,$$

where H_i - Lame parameters.

Six equations (4) are not mutually independent and reduce to three coaxiality equations:

$$\frac{\dot{\xi}_{ij}}{\sigma_{ij}} = \frac{\dot{\xi}_{ii} - \dot{\xi}_{jj}}{\sigma_{ii} - \sigma_{jj}} \quad \langle i, j = 1,2,3, \quad i \neq j \rangle$$

and the resemblance condition $\omega_e = \omega_\sigma$ of the strain D_σ and stress rate $D_{\dot{\xi}}$ deviator (ω_e, ω_σ - deviator phase angles).

The Mizes generalised yield function is taken as load surface $f = 0$ [2]:

$$f\left(s^{ij}, e_{ij}, T, \chi_s, \mu_k\right) = \frac{1}{2}\left(s^{i\cdot}_{\cdot j} s^{j\cdot}_{\cdot i} - s^{i\cdot}_{\cdot i} s^{j\cdot}_{\cdot j}\right) - \\ - \tau_s^2\left(e_{ij}, T, \chi_s, \mu_k\right) = 0 \quad , \qquad (6)$$

where τ_s - a shear yield point; $s^{i\cdot}_{\cdot j}$ - mixed stress deviator components.

For parameters χ_s associated with strains e_{ij}, shear strain degree is taken (the Odkvist parameter):

$$\Lambda = \int_s \sqrt{2\left(de^{i\cdot}_{\cdot j} de^{j\cdot}_{\cdot i} - de^{i\cdot}_{\cdot i} de^{j\cdot}_{\cdot j}\right)} \ , \qquad (7)$$

where $de^{i\cdot}_{\cdot j}$ - strain incrementations deviator components D_{de} and intensity of the shear strain rates H bound by nonholonomic equation $\mathrm{d}\Lambda/\mathrm{dt} = \mathrm{H}$.

Values of the parameter Λ are determined by correlations (7) integration for each known strain s path when strain incrementations $de^{i\cdot}_{\cdot j}$ are known.

Structure microdefect damaging characteristics μ_k polycrystal unit grain size D [1, 2, 4], the crystal lattice inconvertible change energy characteristic (the internal energy of hardening $э^{(\mu)}$) [1, 4] are accepted as the work material constitutive parameters ω [1-3]. The enumerated constitutive properties of engineering material exercises a significant influence upon the finished product operating characteristics [2].

A calculation of the spatial plastic yield stress and velocities fields concerned with strained metal mechanical and constitutive properties is based on solutions of the combined basic equations (1) - (5) using the mapping of yield zone deviator stress space [1].

3.2 Associated law of the rapid metal plastic yielding (evolutional equations and the generalised Mizes yield function determination technique)

The load surface equation (3) and the strain rate gradience condition (4) determinate an associated law of the metal plastic yielding. The strained material plastic yielding is accompanied by the speed effect and the constitutive change substantially influencing the technological parameters and working characteristics of the finished products. The speed effects (inertial stress, thermal flux, speed hardening) and constitutive change describe the loading function (3) fully satisfactorily. The gradience condition (4) allows building the defining correlations between velocities $\dot{\xi}_{ij}$ (or incrementations de_{ij}), strain e_{ij} and stress s^{ij} for greater plastic deformations.

Structure of the correlations (4) satisfying the loading function (3) is determined by principle of the minimum true stress work on the plastic deformation incrementations [5]:

$$\partial\lambda = \dot{\lambda} dt = hd'f \, , \qquad (8)$$

where h > 0 - a variable parameter function defining physico-mechanical condition of the material; $d'f$ - a load surface under unchangeable deformation f = 0 differential.

Correlations (4) subject to equality (8) become:

$$de_{ij} = h \frac{\partial f}{\partial s^{ij}} d'f \, .$$

Loading under unchangeable deformation functions differential [5]:

$$d'f = \frac{\partial f}{\partial s^{ij}} ds^{ij} + \frac{\partial f}{\partial T} d\left(T - \Delta T_s\right) + \frac{\partial f}{\partial \mu_k} d'\mu_k > 0 \, ,$$

where ΔT_s - a temperature incrementation related to dissipative function $w = s^{ij}\dot{\xi}_{ij}$; $d'\mu_k$ - physico-constitutive parameter differentials not related to deformations e_{ij}.

A technique of the generalised Mizes loading function determination (6) is given. The experimental data indicate a strong effect of the strain degree, temperature, rate and the material physico-constitutive properties on the yield point i.e.:

$$\sigma_s = \tau_s\sqrt{3} = \sigma_s\left(e_i, \dot{e}_i, T, \mu_k\right), \qquad (9)$$

where $e_i = \left(1/\sqrt{3}\right)\Lambda$ - a cumulative strain intensity; $\dot{e}_i = \left(1/\sqrt{3}\right)H$ - an intensity of the strain rates.

The dependence (9) is represented by hypersurface f = 0 for each material in the space $\sigma_s, e_i, \dot{e}_i, T, \mu_k$. The surface f = 0 can be specified for each material using supporting curves built on varying strain condition experience base. The following structure of supporting anisothermic hardening curves was used in terms of systematized experimental data:

$$\sigma_s = \sigma_s^{(из)} \exp\left[-\alpha\left(\frac{T - T_0}{T_{\max} - T_0}\right)^q\right], \qquad (10)$$

where $\sigma_s^{(из)} = \sigma_s^{(из)}\left(e_i, \dot{e}_0, T_0, \mu_k\right)$ - an isothermal hardening curve built under fixed strain rate $\dot{e}_0$ and initial temperature T_0; $T_{\max}$ - a maximum temperature of the processing; α, q - the mechanical properties temperature dependency parameters σ_s, σ_B (σ_B - a temporary breaking strength).

The anisothermic curve constructing for material with variable structure is a big problem. And it gets complicated by necessity to determinate values T, α, q which depend on initial condition and strain *s* path. The temperature T change relates to both deformation process thermal flux effect and work material heat abstraction (or penetration).

Determination of the values included into the dependency (10) is considered. Isothermal yield point variation relates to strain degree and rate and also constitutive parameters ω and *D*. The hardening internal energy value influences upon relationship between thermal flux effect and the forming energy dissipation and it is taken into account on the strain temperature calculation. Thereby

$$\sigma_s^{(из)} = \sigma_s^{(из)}\left(e_i, \dot{e}_i, T_0, \omega, D\right).$$

The yield point strain degree dependence can be aproximated by the power three-parameter dependence:

$$\sigma_s^{(из)} = \sigma_{0,2} + Be_i^{(n_0 - n_1 e_i)}, \qquad (11)$$

where $\sigma_{0,2}$ - an initial yield point; B, n_0, n_1 - strain hardening parameters which are determined using three supporting points of experienced curves.

During polycrystal deformation the initial material yield point depends on grain sizes. This dependence is conditioned by grain to grain strain transfer and defined by the Hall-Petch correlation [6]

$$\sigma_{0,2} = \sigma_0 + k_y D^{-\frac{1}{2}}, \qquad (12)$$

where D - an average grain diameter; σ_0 - an unrestricted dislocation motion resistance; k_y - a blocking factor (a measure).

The k_y value is proportional to stress σ_d required for unrestricted dislocation motion and depends on the metal dislocation structure [6] i.e.

$$k_y = \sigma_d l^{\frac{1}{2}}, \qquad (13)$$

where l - an average distance between the grain boundary and the nearest dislocation source.

The dependency (12) is in accord with experimental data and can be defined by dislocation density basic correlations as is shown by T. Ekobori [6].

The two-phase example straining experiments under intermediate heat treatment have shown that the damaging accumulation threshold $\omega = \omega_*$ obtains 0,2...0,3. So there is a part of microdefects not hidden. Under further strain development threshold $\omega = \omega_*$ obtains 0,6...0,7 and physico-constitutive defects start influence upon finished product working (mechanical) properties greatly. The hiding and residual damage variation diagrams processing and its correlation to the dependence $\sigma_d\left(\rho^*\right)$, where ρ^* - a dislocation density, allows to determine relationship to the stress required for blocked dislocation motion (Fig. 1)

$$\sigma_d = \sigma_{d/\omega=0} + A\omega^m, \qquad (14)$$

where *A* and *m* - power function parameters.

The isothermal yield point dependence at prescribed temperature-speed strain conditions subject to correlations (11) - (13) takes on form

$$\sigma_s^{(из)} = \sigma_0 + \left(\sigma_{d/\omega=0} + A\omega^m\right)D^{-\frac{1}{2}}l^{\frac{1}{2}} + Be_i^{(n_0 - n_1 e_i)}. \qquad (15)$$

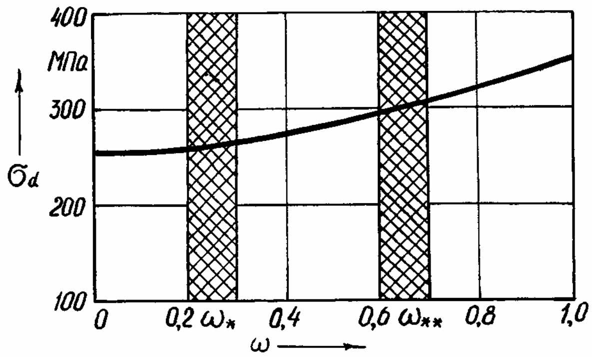

Fig. 1. The relationship between the blocked dislocation motion stress (σ_d) and damaging (ω) during low-carbon low-alloy steel straining (C 0,08... 0,20 %, Cr 0,15... 0,30 %)

The material temperature incrementation related to the forming energy dissipation is determined from the energy conservation equation

$$da_i = c\rho dT + dэ^{(\mu)}, \qquad (16)$$

where a_i - a specific forming work; c – a material specific heat; $э^{(\mu)}$ - an internal dissipation energy related to constitutive parameters variation μ_k.

The equation (16) validity is confirmed by experimental data. According to this data not all plastic deformation work transfers to heat. A part of it is spent on the material constitution change. So the internal hardening energy was taken as internal dissipation energy i.e. $э^{(\mu)} = э_n$.

The load surfaces equation (6), subject to dependencies (10) and transition to contravariant stress deviator components results in:

$$\frac{1}{2}\left(s^{ij}s^{ij}g_{jj}^2 - s^{ii}s^{jj}g_{ii}g_{jj}\right) - \left(\tau_s^{(из)}\right)^2 \exp\left[-2\alpha\left(\frac{T-T_0}{T_{\max}-T_0}\right)^q\right] = 0,$$

where g_{ij} - components of the metric tensor.

The researches carried out on steel straining show that thermal flux effect lead to noticeable temperature increase (300...400 K) and strain resistance reduction in the field of the heavy final strain (100...150 MPA). Specific forming work decreases accordingly in comparison with the process isothermal condition. The stress, specific effort and task tool local load determination error can reach 20...30% if the thermal flux effect is not taken into account.

3.3 Kinetic equations for constitutive parameter

The type of the kinetic equations (5) and functions included is defined by constitutive parameter study and accumulated experimental data on their change under different straining mode systematization. The critical product operating experience shows that microdefect damaging, polycrystal unit grain size and internal hardening energy are those constitutive properties influencing greatly upon working characteristics.

An idea about damaging as quantity ω describing defect accumulation during deformation process is disseminated in mechanics of strained rigid body [1-3, 7-10]:

$$\frac{d\omega}{dt} = \dot{\omega}(\lambda_i) \qquad \text{(i=1,2,...)}, \qquad (17)$$

where λ_i - parameters related to load path.

The damaging value varies over the range $0 \le \omega \le 1$, where the value $\omega = 1$ corresponds to the macrodestruction moment. Damaging according to state-of-the-art conception is related to metal plastic loosening represented initially by dislocation structure development and following separate nucleating submicroscopic crack and submicropore diffused formation. Microcrack formation, growth, coagulation and finally main macro crack formation, meaning metal macrodestruction are observed under the further deformation. The residual relative volume increase value ($\varepsilon_{\cdot i}^{i\cdot}$ - the linear plastic strain tensor invariant) was taken for plastic loosening measure. A plastic loosening point $\varepsilon_{\cdot ikp}^{i\cdot}$ critical value relates to the macrocrack formation point. The kinetic equation (17), subject to relationship between plastic loosening and accumulated deformation is given by

$$\frac{d\omega}{dt} = \frac{\left[\varepsilon_{\cdot i}^{i\cdot}(\Lambda)\right]' H}{\varepsilon_{\cdot i}^{i\cdot}(\Lambda_{np})}, \qquad (18)$$

where Λ_{np} - a limiting extent of shear deformation corresponding to the destruction point; the stroke means the parameter Λ differentiation.

An essential constitutive parameter of the strained metal is a grain value (diameter D). Experimental data [4] show that extent, temperature and strain rate influence upon a grain size of the metal processed by pressure i.e.

$$\frac{dD}{dt} = \dot{D}(\Lambda, H, T, \mu_k). \qquad (19)$$

Three-dimensional diagrams representing grain size and deformation extent and temperature dependencies (recrystallization diagrams) can be considered as the equation (19) integrals for material with given initial structure μ_{ko} and strain rate H_0:

$$D(\Lambda, T, H_0, \mu_{k0}) = \int_0^{\Lambda}\int_{T_0}^{T} \dot{D}(\Lambda, T, H_0, \mu_{k0})\, d\Lambda\, dT,$$

presenting a family of hypersurface in phase space D, Λ, H, T, μ_k. It is convenient to assign the surface $D = D(\Lambda, T, H_0, \mu_{k0})$ by a set of supporting plane curves $D = D(\Lambda, T_0, H_0, \mu_{k0})$. The study on differential geometry of experimental curves $D = D(\Lambda, T_0, H_0, \mu_{k0})$ for a number of steels has allowed to define the following kinetic equation (19):

$$\frac{dD}{dt} = -\gamma(\Lambda - \Lambda_0)^{p-1} H D_0 \exp\left[-\gamma(\Lambda - \Lambda_0)^p\right],$$

where D_0 - a grain size matched the deformation Λ_0; γ, p - parameters determined by supporting experimental points.

Recrystallization annealing is carried out to recover plastic properties of processed half-finished products. In this case it is reasonable to predict half-finished item material grain using recrystallization supporting curves $D = D(\Lambda_0, T, H_0, \mu_{k0})$ (fig.2).

The kinetic equations (19) integral surfaces built for prescribed plastic straining conditions are comfortable to use when modeling metal plastic forming processes with predictable mechanical and constitutive properties.

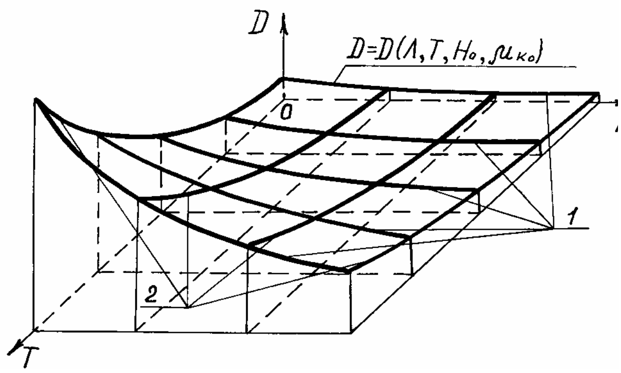

Fig. 2. The Integral surface of the kinetic equation (19) in phase subspace D, Λ , T :
1 - supporting curves D = D($\Lambda, T_0, H_0, \mu_{k0}$);
2 - supporting curves D = D$\left(\Lambda_0, T, H_0, \mu_{k0}\right)$.

The internal hardening energy E_n or its specific quantity $\mathit{э}_n = dE_n / d\Omega$ is one of the main factors defining inconvertible strained metal crystalline structure change, where Ω - a volume of the strain zone. According to experience electronic microscope data the internal hardening energy value is close to the full dislocation energy value [4]. Its change is described by following kinetic equation:

$$\frac{d\mathit{э}_n}{dt} = \mu^{(\mathit{э}_n)} \sqrt{I_2(D_\sigma)} H \text{ ,}$$

where $\mu^{(\mathit{э}_n)}$ - a parameter related to deformation.

For a number of engineering material $\mu^{(\mathit{э}_n)} = \mu_0^{(\mathit{э}_n)} \exp(-\Lambda / \Lambda_{np})$, where $\mu_0^{(\mathit{э}_n)}$ - initial parameter value.

3.4 The models realization and further studies aspects

A plastic work processes modeling including the spatial stress and strain fields and according work material mechanical'n'constitutive properties determination [1] was realized in terms of the stated approach. Let's consider the work metal damaging change kinetics in multiple-function plastic straining process with intermediate recrystallization annealing thermal operation which is convenient to present by the diagram (the fig. 3), where Λ_i, ω_i - an extent of shear deforming and a damaging measure on operation i (i = 1,2,… , n); ω_{ocmi} - a residual damaging measure after heat treatment before plastic forming operation (i + 1). Dependence determined by experience is used to calculate residual damaging after heat treatment:

$$\omega_{ocmi} = \omega_* (1 - \exp(-i)) . \qquad (20)$$

The predictable material damaging measure after *i* forming operation is $\omega_i = \omega_{ocmi} + \Delta\omega_i$, where $\Delta\omega_i$ - a damaging incrementation on *i* operation.

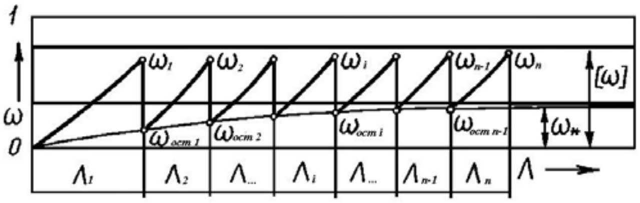

Fig. 3. Diagram of the work metal damaging change in multiple-function plastic straining process with intermediate recrystallization annealing

The dependence (20) allows to forecast metal structure damaging using multiple-function plastic forming technologies with final heat treatment aimed at both strain microdefect damaging level reduction and high strength properties invariance. Similar low-carbon low-alloy steel (carbon 0,08 ...0,20 % and chromium 0,15... 0,30 %) heat treatment consists in heating to the temperature intercritical interval with the following quench mode cooling. After plastic straining such a heat treatment mode allows to get the high strength and visco-plastic properties of low-carbon steel due to generating low amount of strengthening phase (4…6 % martensite) in plastic ferritic matrix. Besides austenization heating hide the part of microdefects and provides the minimum structure damaging (ω = 0, 40... 0,45).

The perspective aim of the further studies is the metal plastic strain processes modeling subject to dilatancy [11], as well as nano-material producing under intensive plastic deformation conditions including the superplasticity conditions [12].

4 Conclusion

The system of the basic equations is given. It describes plastic forming of the metal with predictable mechanical and constitutive properties. The defining correlations for constitutive parameters – microdefect damaging deformation characteristic, polycrystal unit grain size and internal hardening energy – are developed. As example the strained material damaging model is given for multiple-function plastic forming technologies with intermediate recrystallization annealing.

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