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Wear Identification in Misaligned Journal Bearings

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Abstract: Journal bearings are among of the most common components in the industry due to their simple construction, low cost and high efficiency. Rotating shafts are supported by them, and during the transient periods of sequentially start-ups and shut-downs and extreme operation conditions also that occurred through their life time, provoke wear on the bearing pads. The wear produced in misaligned journal bearings is more intense than the wear produced in aligned ones. Thus in this paper the coupled phenomenon of journal bearing wear under misalignment angles in the two main directions is studied. The question is, if it is possible to identify the magnitude of the wear in the material of the bearings under misalignment conditions using response or other measurements. The second question under investigation is to find the dynamic coefficients, stiffness and damping, that are used to calculate the dynamic behavior or the stability of the rotor bearing system.

The method presented in this paper describes an identification procedure of the wear of the bearings. The response should be measured at a particular point (the midpoint of the rotor) at two different speeds with different wear at each bearing. Least square method between the measured and the computed responses at the above particular point and for two different speeds is used to minimize the objective function and thus to find out the two different generally bearing wear percentages.

Then the dynamic coefficients of the bearings are calculated by solving the Reynolds equation, obtaining thus the pressure distribution of the oil film, and by finding the equilibrium position. The 4x4 stiffness and damping matrices including the force-moment and displacement-rotation relations (the existence of moment-rotation relations is the characteristic difference between the aligned and misaligned cases) with all non-diagonal coupling terms as a function of the wear depth is taken into account for the analysis. Some of these dynamic coefficients are presented in this paper in the form of diagrams as functions of the Sommerfeld number.

This method stand in good stead the early diagnosis of the potentially bearing surface damage due to worn effects, forwarding the safe operation of the rotating system and the cost savings of the future shut-downs.

Key-Words: Wear, radial clearance, journal bearings, misalignment, rotor, rotating systems

1 Introduction

In rotor-bearing systems, it is of highly interest the knowledge of the bearing condition, which affects the dynamic behaviour, as well as the stability of the system, and the possibility of the control. The wear of the bearing material during the operation of the system causes the change of the bearings clearance and therefore the change of the other dynamic characteristics of both the bearings and the rotor.

Hydrodynamic journal bearing, which support misalignment rotating shafts, for a long period of time is the cause of significant wear in the bearing surfaces. So, the identification of the new operating radial clearances is of great importance.

Dufrane et al [4] have investigated the worn journal bearing and establish a model of wear geometry for use in the analysis. This worn model is not of circular type.

Vaidyanathan and Keith [5] have evaluated the performance characteristics of non–circular journal bearing for different bearing profiles namely circular, worn-circular, two lobe and elliptical. Hasimoto et al [6] have examined theoretically the effects of wear under normal operating conditions in both laminar and turbulence regimes.

Anjani Kumar and SS. Mishra [7] have examined the effects of wear in journal bearing on the stability of a rigid shaft on two journal bearings, in turbulent flow and with not circular wear model.

Nikolakopoulos and Papadopoulos [8] have examined the effects of the misalignments on the stability of a linear rotor-bearing system in laminar flow, via the direct method of Lyapunov.

The inverse problem of structural damping of a Timoshenko beam in Wavy fluids is solved by Gounaris et al [3] using the so called predictor-corrector method. This method is used to locate the minimum of a multi-parametric function. This method could also be applied here to give a more efficient and fast solution for the minimum of the objective function.

Fillon and Bouyer [12] present a thermodynamic analysis of a worn plain journal bearing. They conclude that the worn bearings present not only some disadvantages but also some advantages , such as lower temperature , since in certain cases of significant defects due to wear the geometry approaches that of a lobe bearing.

Feng and Hahn [13] present a vibration analysis of statically indeterminate rotors with hydrodynamic bearings, taking in account the relative lateral alignment between the journal and the bearing housing

In this paper pre-calculated responses are used instead of the measurements of the eccentricity of the rotor midpoint. These responses corresponds in the two different radial clearances (which later have to be determined), and depend on the angular velocity of the rotor as well as of the wear depth and misalignment angles. This calculation presupposes the numerical solution of the Reynolds equation which gives the pressure distribution of the oil film [2]. The misalignment angles are also taken here into consideration introducing at the same time the flexibility of the rotor. This point will be explained in detail in latter paragraph. The applied loads at each bearing are computed solving the static problem.

The problem of the clearances identification of the two bearings is reduced of the minimization of the described objective function. All the known methods of minimum detection can be used in order to solve the problem for those clearances that correspond to the minimum value of the objective function.

The eigenfrequencies are presented in both table and diagram form (Cambell diagrams).

2 Rotor model formulation using FEM

The following assumptions are used in this work,

- A rigid rotor supported by rigid bearings is assumed.
- A steady state operation is assumed.
- The rotor is rigid with circular cross section and the unbalance is represented as a concentrated mass on the shaft.
- The bearings are anisotropic and rigid and are modeled as a set of linear spring and damping coefficients.
- The wear is produced by misalignment forces.
- The film thickness is described by the equation 13 according to figure 1
- The wear model follows the model introduced by Dufranne et. al at ref. [4]
- The external vertical load is considered constant

The finite element formulation of a rotating shaft with disks on two or more journal bearings is given by the following equations in matrix form [10]:

$$[M]\{\ddot{X}\}+[C]\{\dot{X}\}+[K]\{X\}=\{F\} \tag{1.}$$

Where $[M], [C]\ and\ [K]$ are the total mass, damping and stiffness matrices and $\{X\}$ and $\{F\}$ the arrays of displacement-rotation and forces-moment at any degree of freedom. The total mass, damping and stiffness matrices are assembled using the respective 8x8 matrices of all elements of the rotating shaft.

The element mass matrix is

$$[M]^e=[M_T]^e+[M_R]^e \tag{2.}$$

Where $[M_T]^e$ and $[M_R]^e$ are the translational and rotational mass matrices respectively given by [10],

$$[M_T]^e=\int_0^\ell \mu[\Psi]^T[\Psi]ds\,,\ [M_R]^e=\int_0^\ell J_D[\Phi]^T[\Phi]ds \tag{3.}$$

$[\Psi]=\begin{bmatrix}\psi_1 & 0 & 0 & \psi_2 & \psi_3 & 0 & 0 & \psi_4\\ 0 & \psi_1 & -\psi_2 & 0 & 0 & \psi_3 & -\psi_4 & 0\end{bmatrix}$ and

$[\Phi]=\begin{bmatrix}0 & -\psi_1' & \psi_2' & 0 & 0 & -\psi_3' & \psi_4' & 0\\ \psi_1' & 0 & 0 & \psi_2' & \psi_3' & 0 & 0 & \psi_4'\end{bmatrix}$ are the translational and rotational shape functions respectively:

$\psi_1=1-3(s/\ell)^2+2(s/\ell)^3$, $\quad \psi_2=s\left[1-2(s/\ell)+(s/\ell)^2\right]$,

$\psi_3=3(s/\ell)^2-2(s/\ell)^3$, $\psi_4=\ell\left[-(s/\ell)^2+(s/\ell)^3\right]$

The element damping matrix based on the gyroscopic phenomenon and neglecting the internal damping effect is:

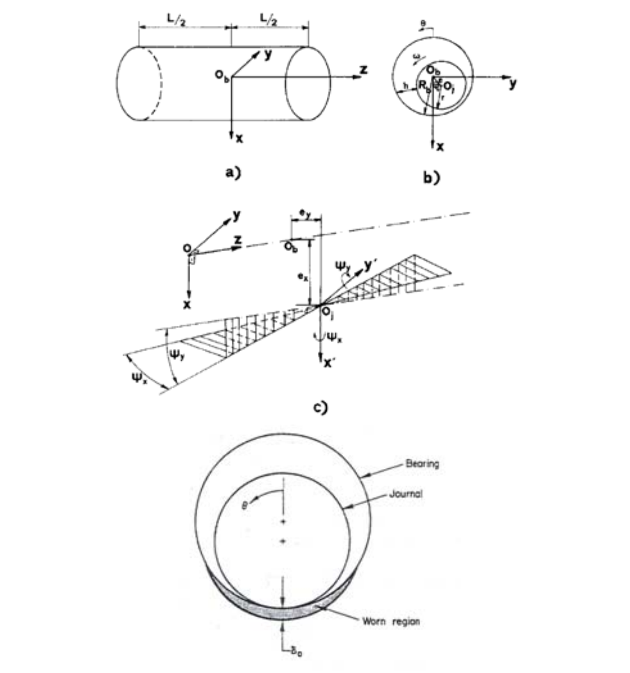
$$[C]^e=-\omega[C_G]^e \tag{4.}$$

where ω is the angular velocity of the rotor and $[C_G]^e$ is the gyroscopic matrix of the element, given by:

$$[G]^e=\left([N]^e-\left([N]^e\right)^T\right),\ \text{with}\ [N]^e=\int_0^\ell J_P[\Phi_T]^T[\Phi_B]ds \tag{5.}$$

The stiffness matrix is:

$$[K]^e=[K_C]^e \tag{6.}$$



(d)

Figure 1: (a) Geometry of bearing (b) of journal bearing, (c) of misalignment angles and (d) of the wear region [4].

$[K_C]^e$ is the conventional stiffness matrix of the beam element, based on the Euler-Bernoulli theory given by,

$$[K_B]^e=\int_0^\ell EI[\Psi'']^T[\Psi'']ds \tag{7.}$$

If disks exist on the shaft at the DOF's of the respective nodes the 4x4 mass matrix of the disk $[M_D]^n$ and the 4x4 gyroscopic matrix of the disk $[C_G]^n$ must be added to the global mass and gyroscopic matrices.

Furthermore if the rotor is supported on journal bearings the stiffness and damping matrices have to be calculated, and the global stiffness and damping matrices must include the ones due to bearings. Although, in many cases in the literature only the four coefficients of the upper left sub-matrix (2x2) are included, here the full 4x4 matrix is considered for the calculation of the stability,

$$[C]^e=[C_G]^e+[C_B]^n\ \text{and}\ [K]^e=[K_C]^e+[K_B]^n \tag{8.}$$

$[C_B]^n$ and $[K_B]^e$ are the damping and stiffness matrices respectively corresponding to the bearing at node *n*. These matrices are presented in the section 3 where the Reynolds equation is solved. All other matrices are given in explicit form in Appendix A. For Timoshenko beam the corresponding matrices are given in ref. [1].

3 Stiffness and damping matrices of the bearing

In this chapter the bearing stiffness and damping matrices are calculated. A system of a rigid rotor on two identical journal bearings at both ends, enforced by a vertical constant force is considered here.

In the case that the bearing is perfectly aligned with respect to rotor, only hydrodynamic forces $F_{x\ or\ y} = F_{x\ or\ y}(x,\ y,\ \dot{x}, \dot{y})$, as functions of the displacements x, y and the velocities $\dot{x}, \dot{y}$ in the two main directions of the journal center O_j, are present. In the misaligned case due to misalignment angles ψ_x and ψ_y, hydrodynamic moments are introduced. Both forces and moments in the misaligned case are functions of the displacements x, y, of the misalignment angles ψ_x and ψ_y and their corresponding velocities with respect to the journal center O_j. These forces and moments can be expressed in general form as,

$$F = F\left(x, y, \psi_x, \psi_y, \dot{x}, \dot{y}, \dot{\psi}_x, \dot{\psi}_y\right) \tag{9.}$$

Where F stands for any of F_x, F_y, M_x, M_y.
Under the assumption of small perturbations, the Taylor expansion of the above equation may be written, neglecting the terms of higher order,

$$\begin{aligned} F = F_o + K_{Rx}\delta x + K_{Ry}\delta y + K_{R\psi_x}\delta\psi_x + K_{R\psi_y}\delta\psi_y + \\ + C_{R\dot{x}}\delta\dot{x} + C_{R\dot{y}}\delta\dot{y} + C_{R\dot{\psi}_x}\delta\dot{\psi}_x + C_{R\dot{\psi}_y}\delta\dot{\psi}_y \end{aligned} \tag{10.}$$

Obviously there are four elastic and four damping linear coefficients. Generally, including the coupling terms, there are 16 elastic and 16 damping coefficients which may be written as,

$$K_{R_i X_j} = \frac{\partial F_i}{\partial X_j} \text{ and } C_{R_i \dot{X}_j} = \frac{\partial F_i}{\partial \dot{X}_j} \tag{11.}$$

where $X_j = x,\ y,\ \psi_x,\ \psi_y$, $\dot{X}_j = \dot{x},\ \dot{y},\ \dot{\psi}_x,\ \dot{\psi}_y$ and F_i stands for force or moment, and i and j denote the directions x or y.
All these are analytically calculated with the method, presented by Nikolakopoulos and Papadopoulos in [2]. In that reference the Reynolds equation is solved by FEM for laminar and incompressible flow with constant lubricant viscosity in order to obtain:
a. The equilibrium positions of the rotor center in both bearings. These equilibrium positions will be used then to find the rotor's position at the point where the measurements are taken, when solving the inverse problem.
b. The bearings hydrodynamics coefficients (stiffness and damping) at the actual found bearing clearances to study the dynamics of rotor bearing system.
Then the stiffness and the damping coefficients are calculated for both journal bearings, and are used for the assembly of the bearings in the global stage for the dynamic problem formulation.

$$\left[K_B\right]^n = \begin{bmatrix} k_{F_x x} & k_{F_x y} & k_{F_x \psi_x} & k_{F_x \psi_y} \\ k_{F_y x} & k_{F_y y} & k_{F_y \psi_x} & k_{F_y \psi_y} \\ k_{M_x x} & k_{M_x y} & k_{M_x \psi_x} & k_{M_x \psi_y} \\ k_{M_y x} & k_{M_y y} & k_{M_y \psi_x} & k_{M_y \psi_y} \end{bmatrix} \quad \text{and}$$

$$\left[C_B\right]^n = \begin{bmatrix} c_{F_x x} & c_{F_x y} & c_{F_x \psi_x} & c_{F_x \psi_y} \\ c_{F_y x} & c_{F_y y} & c_{F_y \psi_x} & c_{F_y \psi_y} \\ c_{M_x x} & c_{M_x y} & c_{M_x \psi_x} & c_{M_x \psi_y} \\ c_{M_y x} & c_{M_y y} & c_{M_y \psi_x} & c_{M_y \psi_y} \end{bmatrix} \tag{12.}$$

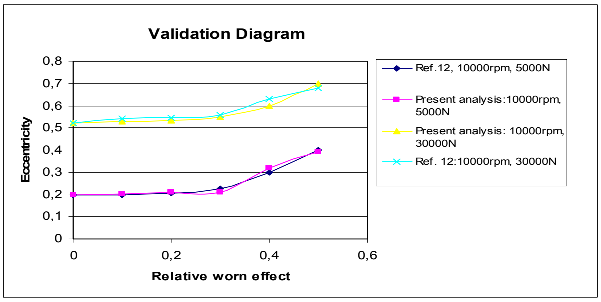


Figure 2. Validation results of present paper and Ref [12]

3.3 Wear Model

The wear model used in the present analysis is the well known model presented in [4] by Dufranne at. al.
The film thickness is given by superposition of film thicknesses as it is mentioned by Nikolakopoulos and Papadopoulos [8] and Dufranne at. al [4], for the abrasive bearing wear.

$$h(\theta, z) = c + e_0 \cos\theta + z\left[\psi_y \cos(\theta + \phi_0) + \psi_x \sin(\theta + \phi_0)\right] + \delta h \tag{13.}$$

where,

$$\delta h = c(\delta 0 - 1 - \cos\theta) \tag{14.}$$

The worn zone is supposed to be centered to the vertical load direction and is estimated by the equation, $\cos\theta = \delta 0 - 1$ given at [7].

3.4 Case study

In figure 2 some validation results are illustrated between this analysis and these of reference [12], which are in a good agreement.
Suppose a full (360°) journal bearing with bearing length l =50.8 *mm,* diameter d = 50.8 *mm,* radial clearance c = 65 μm, oil viscosity μ = 0.012 *Pa.s* rotating at n = 150 rad/sec and loaded by four different vertical loads, 150 Nt, 250 Nt , 550 Nt and 1250Nt. It is also supposed that the misalignment take place in two different angles, same on both of misalignment planes (figure 1), which is 0.0002559rad and 0.0005118rad respectively. Then the stiffness and damping coefficients for the full misaligned case are calculated and presented.
As it is obvious that, besides the usual four terms of stiffness ($k_{F_x x}, k_{F_x y}, k_{F_y y}, k_{F_y x}$) and four of damping ($c_{F_x x}, c_{F_x y}, c_{F_y x}, c_{F_y y}$), there are also the stiffness terms of rotational degrees of freedom in the two main directions, as well as all the coupling terms for all degrees of freedom translational and rotational.
In Figure 3 the variation of the sixteen stiffness and sixteen damping coefficients are depicted as a function of the dimensionless misalignment angles, Sommerfeld number and the wear depth.
The dynamic coefficients coming from these figures could be used for the stability analysis, dynamic and control of a worn misaligned rotor - journal bearing systems.

4 Inverse problem solution

As it is well known journal bearing after a long period of running, depending on many factors (overloads, quality of lubricants, water in lubricants, misalignments effects etc), causes geometric changes due to wear with the most significant change, the change of the clearances. In this work it is assumed that the geometric changes due to wear follows the wear model

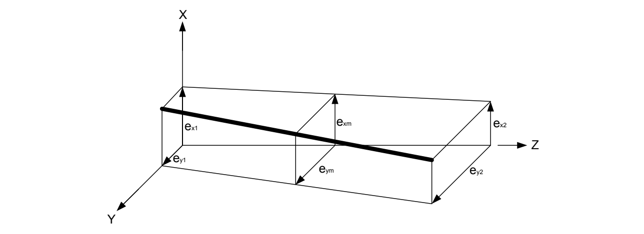


Figure 4: The midpoint response of the rotor as function of the eccentricities of the journal bearings.

introduced at ref. [4] by Dufrane et al. The identification of these clearances is the main task of the present work.
The position of the rotor midpoint (Figure 4), as it is obvious, is determined from the relative positions of the rotor ends that correspond to the axial position of the bearings, i.e. from the journal bearings eccentricities. In Figure 4 a rigid rotor is presented.
However the method can also be applied in flexible rotors using similar relations, ref [13]:

$$e_{xm} = \frac{e_{x1} + e_{x2}}{2} \text{ and } e_{ym} = \frac{e_{y1} + e_{y2}}{2} \qquad (15.)$$

The flexibility of the rotor is taken into consideration by assuming the mentioned misalignment angles Ψy, Ψx. These angles are arbitrarily assumed as a result of statically deflection of the rotor and a given misalignment of the bearing base. Surely if some unbalance has to be taken into consideration then the deflection of the rotor has to be calculated by equations (1).

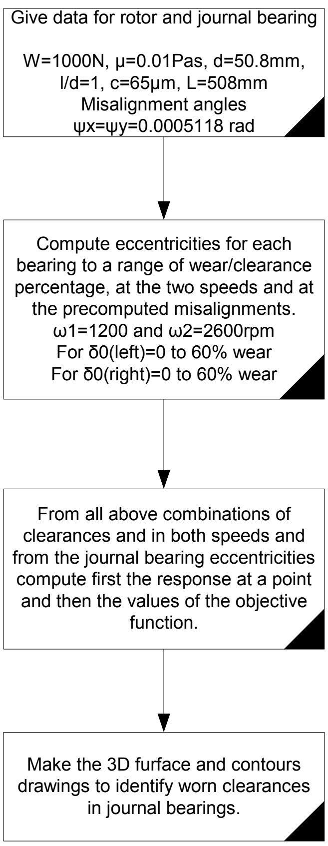


Figure 5: Flow chart of the algorithm used.

The eccentricity at a journal-bearing is determined as the equilibrium position, where the external forces are opposite and equal to the forces coming from the oil film. This eccentricity depends on various parameters, including the angular velocity and the radial clearance as well as from the effects of the wear depth in respect to radial clearance and is determined solving the Reynolds equation as per ref. [2].
By maintaining all other parameters in the journal bearing constants and assuming various combinations of worn effects as a function of radial clearances in both journal bearings, eccentricities are computed and thus the rotor midpoint eccentricities are determined as functions of the various worn effects combinations.
Since the unknown which must be determined are the two clearances, then two equations have to be used to solve iteratively the problem. This is achieved, if for each combination of wear depth, the journal bearing eccentricities and thus the rotor midpoint response (also supposed to be measured) is computed at two different rotor angular velocities.
The above is obtained by assuming as "objective function" to be minimized, the sum of the squares of the differences between the measured and computed responses to various worn effects combinations at the two different speeds.

$$\begin{aligned} F &= Objective\ Function = \\ &= \left(e_{xm}^{(1)} - e_{x_{meas}}^{(1)}\right)^2 + \left(e_{ym}^{(1)} - e_{y_{meas}}^{(1)}\right)^2 + \left(e_{xm}^{(2)} - e_{x_{meas}}^{(2)}\right)^2 + \left(e_{ym}^{(2)} - e_{y_{meas}}^{(2)}\right)^2 \end{aligned} \qquad (16.)$$

The above computed responses are those eccentricities calculated when equilibrium is obtained in the journal bearing. After that by applying equation (23) the response is found at the midpoint of the rotor.
The measuring point could be in any possible position of the rotor shaft and the same could be for the applied load.
Here, both are assumed in the mid point for simplicity reasons, but this does not affect on the generality of the presented method for the wear identification.
The described method is a general method. It could be used for asymmetric loads, as well as for dissimilar bearings since there is not restriction for the calculation of the stiffness and damping bearings coefficients as a function of wear depth. These bearing properties could be calculated for any combination of external load, viscosity, bearing geometry and misalignment angles.
Finally, the clearances requested as will be the ones which minimize the above objective function will be the surface minimum in a 3D drawing which indicates in the plane axes the clearances of the journal bearing and vertical to the plan the values of the above objective function computed.
Regarding to the above equilibrium computations for eccentricities, an assumed misalignment coming from the initial misalignment of the two bearings axes as well as from the static deflection of the shaft is taken into consideration.

4.1 Case study of clearances identification

The current case study is created according to algorithm illustrated at flow chart of figure 5. Two measurements are taken (virtually) at the middle of a shaft supported by two full 360° journal-bearings at two different angular velocities (1200 and 2600 rpm):

$$e_{x_{meas}}^{(1)} = 2.48 \times 10^{-5} m, \quad e_{y_{meas}}^{(1)} = 3.72 \times 10^{-5} m \quad \text{at 1200 rpm}$$

and

$$e_{x_{meas}}^{(2)} = 1.275 \times 10^{-5} m, \quad e_{y_{meas}}^{(2)} = 3.13 \times 10^{-5} m \quad \text{at 2600 rpm}$$

The radius of the bearings is 25.4 mm,the bearing length is 50.8mm,the radial clearance is 65μm. and the oil viscosity is 0.012Pa.s
The wear percentage of the left and the right bearing is **20% and 30%** of the radial clearance, respectively and the misalignment angle at both bearing positions is $\psi_x=\psi_y=0.0005118$rad.Then the objective function described by the equation (24).
In the picture 6.a the ***–Log(F)*** is presented and the picks correspond to the minimum of the function ***F***. Two minimum can be identified: 30% wear of the left and 20% of the right or vise versa.
To construct the surface of Figure 6.a, the wear depth of both bearings are changing from 0% to 60% μm of the radial clearance and the "measurements" are taken at 20% and 30%. Two rotational speeds are considered: 1200 and 2600 rpm.

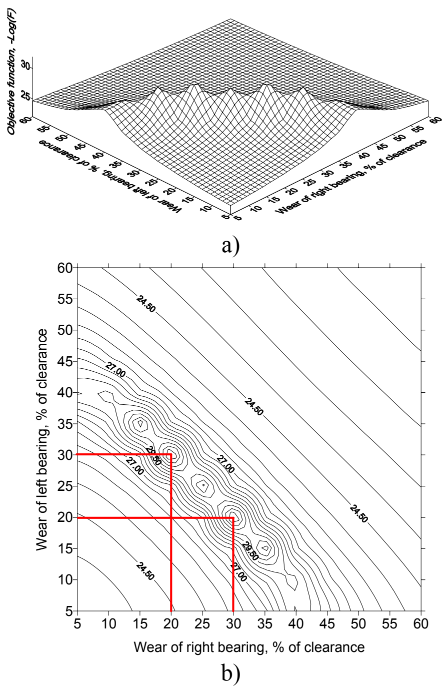


Figure 6: a) Surface and b) contour diagrams of the objective function. The minimum are found at 20% (or (30%) wear of the left bearing and 30% (or 20%) wear of the right one.

Since the differences of the terms of equation **16** are quite small, in order to obtain the surface of Figure 6, the function $-Ln(F)$ is drawn instead of function F . The two picks indicates the worn effect combinations of **(20%, 30%)** and **(30%, 20%)** that make the objective function minimum and give the identified clearances.

5 Conclusions

A method for identification of wear in journal bearings is presented. The method lays on defect identification methods using numerical approach. It is a general method, identifying operating clearances without to be necessary the disassembly of the rotor-bearing systems in order to measure the new clearance due to wear. The journal bearing and the rotor are modeled using the finite element method. Bearing stiffness and damping matrices are obtained solving the Reynolds equation. The full 4x4 matrices are used to model the dynamic characteristics of rotor-bearing system.

Misalignments as well as non diagonal coupling terms are taken into consideration to complete the model introducing at the same time the flexibility of the rotor.

A numerical method of clearance defect identification due to wear in a rotating system of a flexible rotor supported on two misaligned journal bearing, by response measurement at any arbitrally point is developed and presented.

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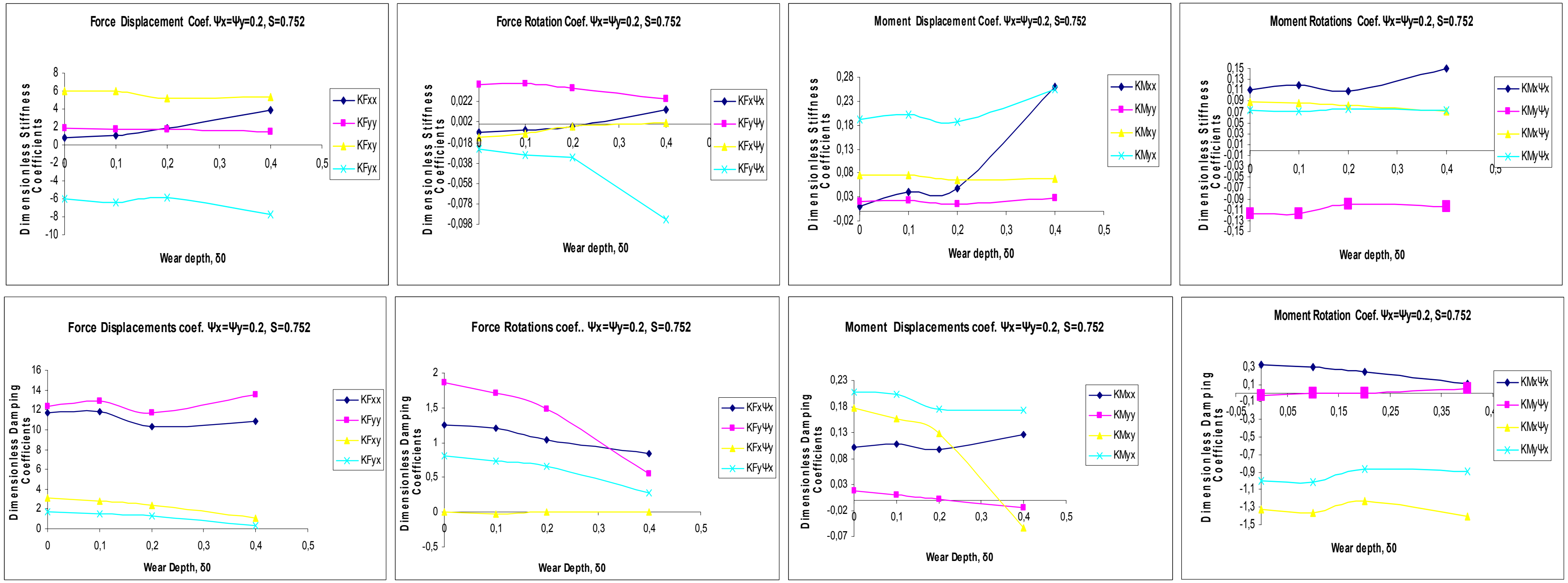
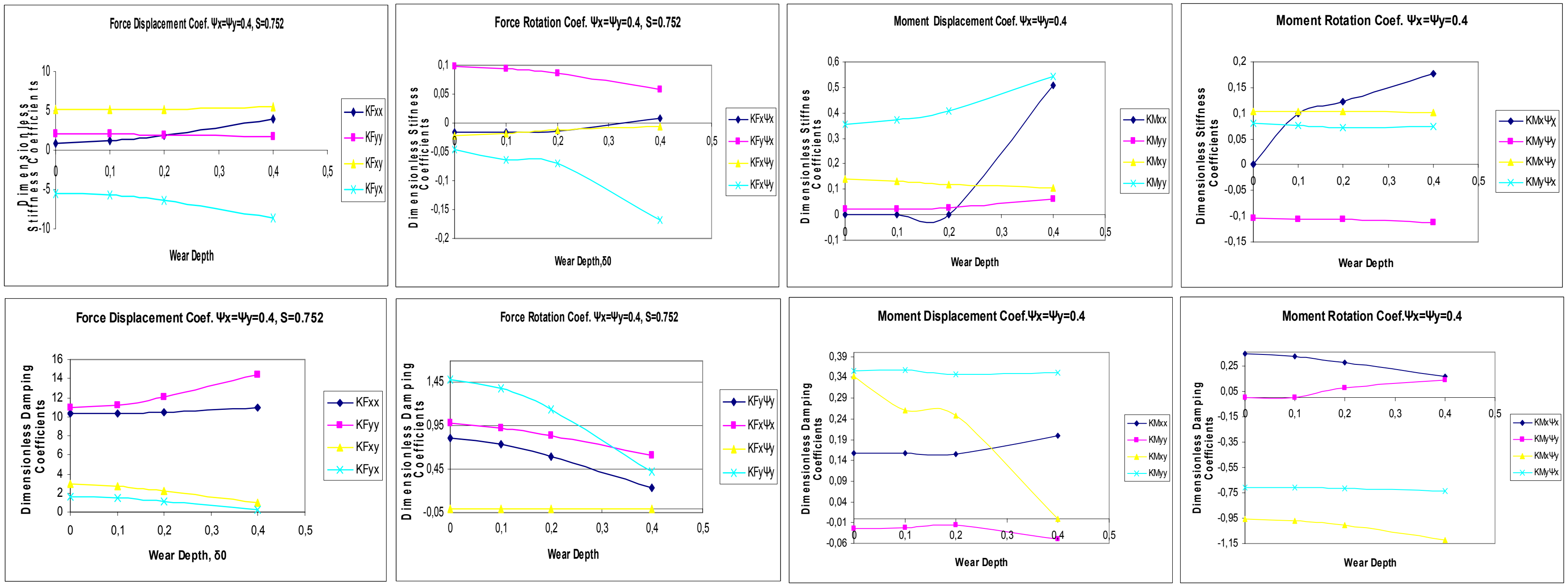
Nomenclature

A :	Area
a, b, c :	Interpolation coef's
c :	Radial clearance
D or d :	Journal diameter
e_0:	Eccentricity
$\varepsilon=e/c$:	Eccentricity ratio
$\bar{K}_{F_i x_j} = K_{F_i x_j}\frac{c}{W}$:	Dimensionless Stiffness coefficients due to force displacement
$\bar{K}_{F_i \psi_j} = K_{F_i \psi_j}\frac{c}{WL}$:	Dimensionless Stiffness coefficients due to force rotations
$\bar{K}_{M_i x_j} = K_{M_i x_j}\frac{c}{WL}$:	Dimensionless Stiffness coefficients due to moment displacement
$\bar{K}_{M_i \psi_j} = K_{M_i \psi_j}\frac{c}{WL^2}$:	Dimensionless Stiffness coefficients due to moment rotation
$\bar{C}_{F_i \dot{X}_j} = C_{F_i \dot{X}_j}\frac{c\omega}{W}$:	Dimensionless Damping coefficients due to force displacement
$\bar{C}_{F_i \dot{\psi}_j} = C_{F_i \dot{\psi}_j}\frac{c\omega}{WL}$:	Dimensionless Damping coefficients due to force rotation
$\bar{C}_{M_i \dot{X}_j} = C_{M_i \dot{X}_j}\frac{c\omega}{WL}$:	Dimensionless Damping coefficients due to moment displacement
$\bar{C}_{M_i \dot{\psi}_j} = C_{M_i \dot{\psi}_j}\frac{c\omega}{WL^2}$:	Dimensionless Damping coefficients due to moment rotation
N :	Revolutions
W, F_{ext} :	External loads
h :	Film thickness
l:	Bearing length
M_x, M_y:	Hydrodynamic moments
μ:	Lubricant viscosity
P:	Fluid film pressure
r:	Journal diameter
R_b:	Bearing diameter
ω:	Angular velocity
S:	Sommerfeld number
U :	Velocity of Journal Surface parallel to the film
L :	Rotor length
V :	Squeeze film velocity
ψ_x, ψ_y:	Misalignment angles
φ_0 :	Attitude angle
x, y :	Spatial coordinates
$\dot{} = \frac{\partial}{\partial t}$:	Time derivative
q_i, :	Generalized coordinate
λ_n :	Eigenvalues
F_{hydr}, F_x, F_y:	Hydrodynamic forces
$\bar{\psi}_i = \psi_i \frac{c}{L}$:	Dimensionless Misalignment angle
$\delta 0$:	Dimensionless wear depth



a) Sommerfeld Number S=0.752, Misalignment angles $\bar{\Psi}x = \bar{\Psi}y = 0.2$



b) Sommerfeld Number S=0.752, Misalignment angles $\bar{\Psi}x = \bar{\Psi}y = 0.4$

Figure 3: The stiffness and damping coefficients vs. wear depth, Sommerfel number and misalignment angles