Proceedings of the 3rd IASME/WSEAS Int. Conf. on FLUID DYNAMICS & AERODYNAMICS, Corfu, Greece, August 20-22, 2005 (pp135-139)

On a certain class of singular solutions for power-law fluids

Josef MÁLEK and Milan POKORNÝ

Mathematical Institute, Charles University,
Sokolovská 83, CZ-186 75 Praha 8
Czech Republic
josef.malek@mff.cuni.cz http://www.karlin.mff.cuni.cz/˜malek,
milan.pokorny@mff.cuni.cz http://www.karlin.mff.cuni.cz/˜pokorny

Abstract: We consider selfsimilar solutions to the power-law model for the incompressible fluids. We construct a class of selfsimilar solutions that are singular on a line passing through the origin.

Key Words: power-law fluids, Navier–Stokes equations, selfsimilar solutions, Leray system, weak solution, singular solution

1 Introduction

We consider the following system of PDE's

$$\left.\begin{array}{l} \frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u}\cdot\nabla\boldsymbol{u} + \nabla\pi \\ -\nu \mathrm{div}\,(|\boldsymbol{Du}|^{p-2}\boldsymbol{Du}) = \boldsymbol{0} \\ \mathrm{div}\,\boldsymbol{u} = 0 \end{array}\right\} \text{in } \mathbb{R}\times\mathbb{R}^3, \qquad (1)$$

which describes the flow of a certain class of non-newtonian incompressible fluids. The model is usually called the power-law fluid. Here, $\boldsymbol{u}$ represents the velocity field, π is the pressure, $\boldsymbol{Du}$ with $(Du)_{ij} = \frac{1}{2}(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i})$ is the symmetric part of the velocity gradient, $\nu > 0$ and $p > 1$ are constants. Note that for $p = 2$ we get the well-known Navier–Stokes equations and the constant ν is then the reciprocal to the Reynolds number. Our system (1) is a special case of non-newtonian fluids with the stress tensor given by $\boldsymbol{S} = -\pi\boldsymbol{I} + \nu|\boldsymbol{Du}|^{p-2}\boldsymbol{Du}$. The reader can find further information about the derivation of the model in the framework of continuum mechanics as well as about the properties of fluids corresponding to the power-law models in [6].

If the fluid does not fill in the whole space, system (1) must be accompanied by the boundary conditions. Since we only deal with the Cauchy problem we will not touch this interesting and important problem here.

In general, the less p is, the less a priori estimates are available and thus the main challenge from the point of view of mathematical analysis is to prove the existence of a solution for p as low as possible. Let us note that there is a lot of physically interesting models when $p \in (1, \frac{3}{2}]$.

First mathematical results concerning the existence and the uniqueness of solutions to similar models go back to late sixties and are due to O.A. Ladyzhenskaya [2] and J.L. Lions [4]. In the nineties a series of results appeared which decreased the value for which the solution exists, see [5]. Nowadays, the global in time existence for a weak solution without any restriction on the size of the data is known for $p > \frac{8}{5}$, see [1]. Even though this result is proved for space periodic or no stick boundary conditions, it is not difficult to transform them for the Cauchy problem. The solution is known to be regular for $p \geq \frac{11}{5}$, see [5]. The Cauchy problem was studied in [9], however, model (1) only for $\frac{9}{5} < p \leq 2$. For $p > 2$, the stress tensor was considered in the form $\boldsymbol{S} = -\pi\boldsymbol{I} + \nu_0\boldsymbol{Du} + \nu_1|\boldsymbol{Du}|^{p-2}\boldsymbol{Du}$. In [7], using Nikolskii spaces, the authors considered also the case $p > 2$ with $\nu_0 = 0$. Again, even though the study is performed for space periodic boundary conditions, it is an easy matter to transform the result for the Cauchy problem.

Our aim is slightly different. We will study model (1) for rather small p and we will construct singular solutions in the selfsimilar form that we discuss next.

2 Selfsimilar solutions

In the famous paper [3], J. Leray proposed the following construction of a weak solution to the Navier–Stokes equations, i.e. to system (1) with $p = 2$, which is not smooth. He considered the solution in the form ($T > 0$ a positive constant)

$$\begin{aligned} \boldsymbol{u} &= \frac{1}{\sqrt{T-t}}\boldsymbol{U}\Big(\frac{x}{\sqrt{T-t}}\Big) \\ \pi &= \frac{1}{T-t}P\Big(\frac{x}{\sqrt{T-t}}\Big). \end{aligned} \tag{2}$$

Under the assumption that there exists a weak solution to the Leray system

$$\begin{aligned} &\frac{y}{2}\cdot\nabla\boldsymbol{U} + \frac{\boldsymbol{U}}{2} + \boldsymbol{U}\cdot\nabla\boldsymbol{U} - \nu\Delta\boldsymbol{U} + \nabla P = \boldsymbol{0} \\ &\operatorname{div}\boldsymbol{U} = 0 \end{aligned} \tag{3}$$

such that $\boldsymbol{U}$ belongs to the Sobolev space $(W^{1,2}(\mathbb{R}^3))^3$, then $\boldsymbol{u}$ of the form $(2)_1$ is a weak solution to the Navier–Stokes equations such that $\lim_{t\to T^-}\|\boldsymbol{u}\|_2 = 0$ while $\lim_{t\to T^-}\|\nabla\boldsymbol{u}\|_2 = \infty$, i.e. $\boldsymbol{u}$ is a weak solution to the Navier–Stokes equations with the blow-up in finite time. However, J. Nečas, M. Růžička and V. Šverák showed in [8] that any solution to (3) such that $\boldsymbol{U} \in (L^3(\mathbb{R}^3))^3$ is identically zero. Later on, T.P. Tsai [11] extended this result to $\boldsymbol{U} \in (L^r(\mathbb{R}^3))^3$ for any $3 \le r < \infty$.

We would like to apply similar ideas to our model (1). Inspired by the selfsimilar scaling (cf. [6], Section B 1.4) we look for a solution to (1) in the form

$$\begin{aligned} \boldsymbol{u} &= (T-t)^{-\frac{p-1}{2}}\boldsymbol{U}\Big((T-t)^{-\frac{3-p}{2}}x\Big) \\ \pi &= (T-t)^{-(p-1)}P\Big((T-t)^{-\frac{3-p}{2}}x\Big). \end{aligned} \tag{4}$$

Then the case $p > \frac{11}{5}$ is subcritical (and thus relatively easily solvable), $p = \frac{11}{5}$ is critical and thus solvable with possibly more effort and $p < \frac{11}{5}$ is supercritical and thus any existence and regularity result requires considerably more effort than the former cases, see [6].

Inserting (4) into (1), one easily computes that $(\boldsymbol{U}, P)$ satisfies

$$\begin{aligned} &\frac{3-p}{2}y\cdot\nabla\boldsymbol{U} + \frac{p-1}{2}\boldsymbol{U} + \boldsymbol{U}\cdot\nabla\boldsymbol{U} \\ &-\nu\operatorname{div}\Big(|\boldsymbol{DU}|^{p-2}\boldsymbol{DU}\Big) + \nabla P = \boldsymbol{0} \\ &\operatorname{div}\boldsymbol{U} = 0, \end{aligned} \tag{5}$$

which reduces for $p = 2$ to the Leray system (3).

Definition 1 *We say that $\boldsymbol{u} \in (L^2_{loc}(\mathbb{R}^3))^3$ with $|\boldsymbol{DU}|^{p-1} \in L^1_{loc}(\mathbb{R}^3)$ is a weak solution to (5) if*
(i) $div\boldsymbol{U} = 0$ in $\mathcal{D}'(\mathbb{R}^3)$
(ii)

$$\begin{aligned} \int_{\mathbb{R}^3} &\Big((2p-5)\boldsymbol{U}\cdot\boldsymbol{\varphi} - \frac{3-p}{2}(\boldsymbol{U}\otimes y) : \nabla\boldsymbol{\varphi} \\ &-(\boldsymbol{U}\otimes\boldsymbol{U}) : \nabla\boldsymbol{\varphi} + \nu|\boldsymbol{DU}|^{p-2}\boldsymbol{DU} : \boldsymbol{D\varphi}\Big)dy \\ &= 0 \end{aligned}$$

for all $\boldsymbol{\varphi} \in \mathcal{V} = \{\boldsymbol{u} \in ((\mathcal{D}(\mathbb{R}^3))^3; div\boldsymbol{u} = 0\}$.

Assume that, for $p < 3$, the velocity field $\boldsymbol{U}$ belongs to $(L^2(\mathbb{R}^3))^3 \bigcap (L^{\max\{\frac{2p}{p-1},\frac{3p}{3-p}\}}(\mathbb{R}^3))^3$, $\nabla\boldsymbol{U} \in (L^p(\mathbb{R}^3))^3$ or, for $p \ge 3$, $\boldsymbol{U} \in (L^2(\mathbb{R}^3))^3$, $\nabla\boldsymbol{U} \in (L^p(\mathbb{R}^3))^3$. Taking as test function $\boldsymbol{\varphi} = \boldsymbol{U}^\varepsilon\eta_R$, where $\boldsymbol{U}^\varepsilon$ is a divergence-free approximation of $\boldsymbol{U}$ in the spaces mentioned above and $\eta_R(y) = \eta(\frac{y}{R})$ is the standard cut-off function, $\eta(y) = 1$ in $B_1(0)$, $\eta = 0$ outside $B_2(0)$, η smooth, we get, after passing with $\varepsilon \to 0$ and $R \to \infty$ that

$$\frac{5p-11}{4}\int_{\mathbb{R}^3}|\boldsymbol{U}|^2dy = -\nu\int_{\mathbb{R}_3}|\boldsymbol{DU}|^p dy.$$

Thus, such a solution may exist only for $p < \frac{11}{5}$.

Note that for $p \ge \frac{9}{5}$ we have $2 \le \frac{2p}{p-1} \le \frac{3p}{3-p}$ and thus there is no additional regularity assumption.

If $\boldsymbol{U} \in (W^{1,2}(\mathbb{R}^3))^3$, we may set $\boldsymbol{U}(y)$ as initial value for system (1) and for $p > \frac{5}{3}$, there exists a local-in-time solution to (1) such that $\nabla\boldsymbol{u} \in (L^p((0,t^*); L^{3p}(\mathbb{R}^3))^9$, see [6]. Thus $\nabla\boldsymbol{U} \in (L^{3p}(\mathbb{R}^3))^9$ and in particular, $\boldsymbol{U}$ is bounded. We have the following "regularity" result:

Proposition 2 *Let $p > \frac{5}{3}$ and $\boldsymbol{U} \in (W^{1,2}(\mathbb{R}^3))^3$ be a weak solution to (5). Then $\nabla\boldsymbol{U}$ belongs to $(L^{3p}(\mathbb{R}^3))^9$ and thus $\boldsymbol{U}$ to $(L^\infty(\mathbb{R}^3))^3$.*

Moreover, provided there is a nontrivial solution $(\boldsymbol{U}, P)$ to (5) such that $\boldsymbol{U} \in (L^2(\mathbb{R}^3))^3$ with $\nabla\boldsymbol{U} \in (L^p(\mathbb{R}^3))^3$, then for $\boldsymbol{u}$ defined by $(4)_1$

$$\|\boldsymbol{u}(t)\|_{(L^2(\mathbb{R}^3))^3} = (T-t)^{\frac{11-5p}{2}}\|\boldsymbol{U}\|_{(L^2(\mathbb{R}^3))^3}$$

Proceedings of the 3rd IASME/WSEAS Int. Conf. on FLUID DYNAMICS & AERODYNAMICS, Corfu, Greece, August 20-22, 2005 (pp135-139)

while

$$\|\nabla \boldsymbol{u}\|_{(L^p(\mathbb{R}^3))^9} = (T-t)^{\frac{9-5p}{2}} \|\nabla \boldsymbol{U}\|_{(L^p(\mathbb{R}^3))^9}$$

and thus it is a nonsmooth weak solution to system (1) if $p \in (\frac{9}{5}, \frac{11}{5})$. Since for $p = 2$ such a solution cannot exist, one may expect that at least for $p \in (2, \frac{11}{5})$, the same could hold true. However, for $p = 2$, the proof is based on the fact that the quantity $\frac{|\boldsymbol{U}|^2}{2} + P + y \cdot \boldsymbol{U}$ satisfies the maximum principle. The same fact for $p \neq 2$ is far from being evident and we thus leave the existence/nonexistence of weak solutions to (5) with the above given regularity as an interesting open problem.

3 Singular solutions

We come to the main result of this short note. We would like to construct singular solutions to (5) in a special form. As a matter of fact, singular solutions to (5) are actually singular solutions to (1) and thus they might be of a special interest.

Definition 3 *Let $A \subset \mathbb{R}^3$ be of zero three-dimensional Lebesgue measure. We say that $\boldsymbol{U} \in (C^2(\mathbb{R}^3 \setminus A))^3$, $P \in C^1(\mathbb{R}^3 \setminus A)$ is a singular solution to (5) provided (5) holds for $(\boldsymbol{U}, P)$ pointwise in $\mathbb{R}^3 \setminus A$.*

Let us denote by

$$\psi(y) = (y_3 - y_2)^2 + (y_1 - y_3)^2 + (y_2 - y_1)^2.$$

We will look for singular solutions to (5) in the form

$$\begin{aligned} \boldsymbol{U}(y) &= \Big(\frac{y_3 - y_2}{\psi^\alpha}, \frac{y_1 - y_3}{\psi^\alpha}, \frac{y_2 - y_1}{\psi^\alpha}\Big) \\ P(y) &= Q(\psi(y)) \end{aligned} \qquad (6)$$

for some $\alpha > 0$ and a suitable smooth function Q. Note that $\boldsymbol{U}$ is smooth outside the line $y_1 = y_2 = y_3$, P is smooth outside the same line provided Q is smooth. Moreover, outside this line

$$\operatorname{div} \boldsymbol{U} = 0.$$

Assume for a moment that $(\boldsymbol{U}, P)$ of the form above is a singular solution to (5). Let us consider for a moment just $\boldsymbol{U}$; we would like to find conditions on α which would imply that $\boldsymbol{U}$ is a weak solution to (5) in the sense of Definition 1. First of all,

$$\begin{aligned} \alpha(p-1) < 1 \quad &\text{i.e.} \quad \alpha < \frac{1}{p-1} \\ 2\alpha - 1 < 1 \quad &\text{i.e.} \quad \alpha < \frac{3}{4}, \end{aligned}$$

in order to make sense for all integrals appearing in the weak formulation. Since $(\boldsymbol{U}, P)$ satisfies the equation pointwise outside one line, in order to get the integral identity, we must be able to perform the corresponding integration by parts. Thus we get additionally

$$\begin{aligned} \alpha(p-1) < \frac{1}{2} \quad &\text{i.e.} \quad \alpha < \frac{1}{2(p-1)} \\ 2\alpha - 1 < \frac{1}{2} \quad &\text{i.e.} \quad \alpha < \frac{1}{2}. \end{aligned}$$

Unfortunately, as will be seen below, our singular solutions will not be weak solutions in the sense of Definition 1.

Easily we get that

$$y \cdot \nabla \boldsymbol{U} = (1 - 2\alpha)\boldsymbol{U}$$

and thus

$$\begin{aligned} &\frac{3-p}{2} y \cdot \boldsymbol{U} - \frac{1-p}{2}\boldsymbol{U} \\ &= \frac{(1-2\alpha)(3-p) - (1-p)}{2}\boldsymbol{U}. \end{aligned}$$

Next, for the convective term,

$$\begin{aligned} \boldsymbol{U} \cdot \nabla \boldsymbol{U} &= \frac{1}{\psi^{2\alpha}}\Big(-2y_1 + y_2 + y_3, \\ &\qquad -2y_2 + y_1 + y_3, -2y_3 + y_1 + y_2\Big) \\ &= \frac{1}{2(2\alpha - 1)} \nabla\Big(\frac{1}{\psi^{2\alpha-1}}\Big). \end{aligned}$$

Thus this term can be compensated by the pressure; it would be a weak solution provided $4\alpha - 1 < 1$, i.e. $\alpha < \frac{1}{2}$.

Finally, after some tedious calculations, we get

$$|\boldsymbol{D}\boldsymbol{U}|^2 = \frac{6\alpha^2}{\psi^{2\alpha}}$$

and

$$-\operatorname{div}(|\boldsymbol{D}\boldsymbol{U}|^{p-2}\boldsymbol{D}\boldsymbol{U}) = \frac{6^{\frac{p}{2}}\alpha^{p-1}(1-(p-1)\alpha)}{\psi^{(p-2)\alpha+1}}\boldsymbol{U}.$$

Altogether, we have

$$\begin{aligned} &\frac{(1-2\alpha)(3-p) - (1-p)}{2}\boldsymbol{U} \\ &+ \frac{1}{2(2\alpha-1)}\nabla\Big(\frac{1}{\psi^{2\alpha-1}}\Big) \\ &+ \nu\frac{6^{\frac{p}{2}}\alpha^{p-1}(1-(p-1)\alpha)}{\psi^{(p-2)\alpha+1}}\boldsymbol{U} + \nabla P = \boldsymbol{0}. \end{aligned}$$

Now, two cases lead to the fact that functions of the type (6) solve (5).

Case 1:

$$\begin{aligned}(3-p)(1-2\alpha) &= 1-p\\ (p-1)\alpha &= 1\end{aligned}$$

and thus $p = 2$ and $\alpha = 1$. Therefore, for any $A \in \mathbb{R}$,

$$\begin{aligned}\boldsymbol{U} &= \frac{A}{\psi}(y_3 - y_2, y_1 - y_3, y_2 - y_1)\\ P &= -\frac{A^2}{2}\frac{1}{\psi}\end{aligned}$$

is a singular solution to the Leray system (3), which is not a weak solution. It provides, via (2), a singular solution to the Navier–Stokes equations. Note that the pressure is unbounded from below. The reader may compare this with the fact that weak solutions to the Navier–Stokes equations are smooth provided the pressure is bounded from below, see [10].

Case 2:

$$(p-2)\alpha + 1 = 0$$

i.e. $\alpha = \frac{1}{2-p}$. Now, the singular solution will be of the form

$$\begin{aligned}\boldsymbol{U}(y) &= \frac{\beta}{\psi^\alpha}(y_3 - y_2, y_1 - y_3, y_2 - y_1)\\ P(y) &= -\frac{\beta^2}{2(2\alpha-1)}\frac{1}{\psi^{2\alpha-1}}\end{aligned}$$

with $\alpha = \frac{1}{2-p}$ and $\beta \in \mathbb{R}$ properly chosen in such a way that

$$\begin{aligned}&\beta\frac{(3-p)(1-2\alpha)-(1-p)}{2}\\ &= \Big((p-1)\alpha - 1\Big)6^{\frac{p}{2}}\alpha^{p-1}|\beta|^{p-2}\beta.\end{aligned}$$

Inserting the value of α we find that

$$|\beta| = \frac{(2-p)^{\frac{p-1}{p-2}}}{6^{\frac{p}{2(p-2)}}\Big(\nu(3-2p)\Big)^{\frac{1}{p-2}}}$$

provided $3 - 2p > 0$, i.e. $p < \frac{3}{2}$. Thus

$$\begin{aligned}\boldsymbol{U}(y) &= \pm\frac{(2-p)^{\frac{p-1}{p-2}}}{6^{\frac{p}{2(p-2}}\Big(\nu(3-2p)\Big)^{\frac{1}{p-2}}}\frac{1}{\psi^{\frac{1}{2-p}}}\\ &\quad (y_3 - y_2, y_1 - y_3, y_2 - y_1)\\ P(y) &= -\frac{(2-p)^{\frac{2(p-1)}{p-2}}}{6^{\frac{p}{(p-2}}\Big(\nu(3-2p)\Big)^{\frac{2}{p-2}}}\frac{2-p}{2p}\frac{1}{\psi^{\frac{p}{2-p}}}\end{aligned}$$

is for $1 < p < \frac{3}{2}$ a singular solution (but not a weak solution) to system (5).

We have proved

Theorem 4 *Let $(\boldsymbol{U}, P)$ be of the form (6). Then the pair is a singular solution to system (5) if:*
a) $p = 2$, $\alpha = 1$, $A \in \mathbb{R}$ arbitrary

$$\begin{aligned}\boldsymbol{U} &= \frac{A}{\psi}(y_3 - y_2, y_1 - y_3, y_2 - y_1)\\ P &= -\frac{A^2}{2}\frac{1}{\psi}\end{aligned}$$

b) $p \in (1, \frac{3}{2})$, $\alpha = \frac{1}{2-p}$

$$\begin{aligned}\boldsymbol{U}(y) &= \frac{\beta}{\psi^{\frac{1}{2-p}}}(y_3 - y_2, y_1 - y_3, y_2 - y_1)\\ P(y) &= -\frac{\beta^2(2-p)}{2p}\frac{1}{\psi^{\frac{p}{2-p}}},\end{aligned}$$

where

$$|\beta| = \frac{(2-p)^{\frac{p-1}{p-2}}}{6^{\frac{p}{2(p-2)}}\Big(\nu(3-2p)\Big)^{\frac{1}{p-2}}}.$$

Let us complete the result with several remarks. We may also study singular solutions to the steady power-law model. Formally it means that we do not take the time derivative and thus we have system (5) without the first two terms, i.e.

$$\begin{aligned}&\boldsymbol{U}\cdot\nabla\boldsymbol{U} - \nu\mathrm{div}\Big(|\boldsymbol{DU}|^{p-2}\boldsymbol{DU}) + \nabla P = \boldsymbol{0}\\ &\mathrm{div}\,\boldsymbol{U} = 0.\end{aligned}$$

Thus we get that for any $A \in R$

$$\begin{aligned}\boldsymbol{U}(y) &= \frac{A}{\psi^{\frac{1}{p-1}}}(y_3 - y_2, y_1 - y_3, y_2 - y_1)\\ P(y) &= -\frac{A^2(p-1)}{2(3-p)}\frac{1}{\psi^{\frac{3-p}{p-1}}}\end{aligned}$$

is a singular solution (but not a weak one) to the steady power-law model with any $p > 1$.

Another possibility (and in some sense more natural) is to look for a solution in the form

$$\begin{aligned}\boldsymbol{V}(y) &= \Big(\frac{y_3 - y_2}{|y|^\alpha}, \frac{y_1 - y_3}{|y|^\alpha}, \frac{y_2 - y_1}{|y|^\alpha}\Big)\\ P(y) &= Q(|y|).\end{aligned} \tag{7}$$

Again, $\operatorname{div} \boldsymbol{V} = 0$ outside the origin. Proceeding as above we get

$$\frac{(1-\alpha)(3-p)-(1-p)}{2}\boldsymbol{U} - \frac{1}{2}\frac{\nabla\psi}{|y|^{2\alpha}} + \nu\frac{\alpha^{p-1}(\alpha+3-p\alpha)\psi^{\frac{p-2}{2}}}{|y|^{(p-2)\alpha+2}}\boldsymbol{U} + \nabla P = \boldsymbol{0}.$$

Unlike the previous situation, the convective term cannot be absorbed into the pressure and thus we may get selfsimilar singular solutions only without the convective term. Thus, solving

$$\begin{aligned} (3-p)(1-\alpha) &= 1-p \\ \alpha + 3 &= p\alpha \end{aligned}$$

gives $\alpha = \frac{5}{2}$, $p = \frac{11}{5}$ and

$$\begin{aligned} \boldsymbol{V}(y) &= \frac{A}{|y|^{\frac{5}{2}}}(y_3 - y_2, y_1 - y_3, y_2 - y_1) \\ P(y) &= const \end{aligned}$$

is a singular (not a weak) solution to system (5) without the convective term and it provides a singular solution to (1) without the convective term, both for $p = \frac{11}{5}$.

Finally, considering steady power-law model without the convective term, we observe that

$$\begin{aligned} \boldsymbol{V}(y) &= \frac{A}{|y|^{\frac{3}{p-1}}}(y_3 - y_2, y_1 - y_3, y_2 - y_1) \\ P(y) &= const \end{aligned}$$

is a singular (and not a weak) solution to

$$\begin{aligned} -\operatorname{div}(|\boldsymbol{DV}|^{p-2}\boldsymbol{DV}) + \nabla P &= \boldsymbol{0} \\ \operatorname{div}\boldsymbol{V} &= 0 \end{aligned}$$

for any $p > 1$.

Acknowledgments: The work is a part of the research project MSM 0021620839 financed by MSMT and partly supported by the grant of the Czech Science Foundation No. 201/03/0934.

References:

[1] J. Frehse, J. Málek, M. Steinhauer, On existence results for fluids with shear dependent viscosities, in: *Partial differential equations, theory and numerical solutions*, eds. W. Jäger, J. Nečas, O. John, K. Najzar, J. Stará, Chapman & Hall, 2000, pp. 121–129.

[2] O.A. Ladyzhenskaya, *The Mathematical Theory of Viscous Incompressible Fluid*, Gordon and Breach, 1969

[3] J. Leray, Sur le mouvement d'un liquide visqueux emplissant l'espace, *Acta Math.*, Vol. 63, 1934, pp. 193–248.

[4] J.L. Lions, *Quelques méthodes de résolution des problèmes aux limites non linéaires*, Dunod, Paris 1969

[5] J. Málek, J. Nečas, M. Rokyta, M. Růžička, *Weak and measure-valued solutions to evolutionary PDEs*, Chapman & Hall, London, 1996

[6] J. Málek, K.R. Rajagopal, Mathematical issues concerning the Navier–Stokes equations and some of their generalizations, in: *Handbook of evolutionary PDE's, Vol. II*, eds.: C. Dafermos, E. Feireisl, 2005 (in print).

[7] J. Málek, D. Pražák, M. Steinhauer, On the existence and regularity of solutions for degenerate power-law fluids, preprint of the Charles University, Prague.

[8] J. Nečas, M. Růžička, V. Šverák, On self–similar solutions of the Navier–Stokes equations, *Acta Math.*, Vol. 176, 1997, pp. 283–294.

[9] M. Pokorný, Cauchy problem for non-newtonian viscous compressible fluid, *Appl. Math.*, Vol. 41, No. 3, 1996, pp. 169–201.

[10] G. Seregin, V. Šverák, Navier-Stokes with lower bounds on the pressure, *Arch. Rational Mech. Anal.*, Vol. 163 No. 1, 2002, pp. 65–86.

[11] T.P. Tsai, On Leray's self-similar solutions of the Navier–Stokes equations satisfying local energy estimates, *Arch. Rational Mech. Anal.*, Vol. 143, No. 1, 1998, pp. 29–51.