Proceedings of the 3rd IASME/WSEAS Int. Conf. on FLUID DYNAMICS & AERODYNAMICS, Corfu, Greece, August 20-22, 2005 (pp18-23)

Stokes flow and Oseen flow in R^3 with the Coriolis force

Nečasová Šárka
Mathematical Institute of the Academy of Sciences
Žitná 25
11567 Prague 1
Czech Republic
matus@math.cas.cz

Abstract: We deal with the problem of the Stokes and Oseen flow in R^3 when the Coriolis force is presented.

Key-Words: Stokes problem, Oseen problem, strong solution, Coriolis force

1 Introduction

The problem of the motion of a rigid body through a liquid has attracted the attention of several scientists over a century ago. The first systematic study on this subject initiated with the pioneering work of Kirchhoff [K], Lord Kelvin [T] regarding the motion of one or more bodies in a frictionless liquid. After that many mathematicians have furnished significant contributions to this fascinating field under different assumptions on the body and on the fluid. We wish to quote the work of Brenner [B] concerning the steady motion of one or more bodies in a linear viscous liquid in the Stokes approximation. Weinberger [W1]-[W2] and Serre [S] regarding the fall of a body in a incompressible Navier-Stokes fluid under the action of gravity. Further we can refer to the work of Farwig, Hisdida, Muller [FHM], Galdi [G2]-[G3], Gunther, Hudspeth, Thomann [GHT], etc.

Before describing the main results, we would like to introduce some basic problemes of practical interest. The orientation of a long bodies in liquid of different nature is a fundamental issue in many practical interest. A first, fundamental step in modelling and the orientation of long bodies in liquids is to investigate experimantally their free fall behavior (sedimentation), both in Newtonian and viscoelastic liquids see [Le], [PC].

In our paper we consider the Stokes and Oseen problem with Coriolis force $\omega \times u$ when $\omega = \lambda g$ in the whole space R^3. Our plan is the following. First we deal with the problem

$$-\mu\Delta u + \lambda g \times u = \nabla p + F, \tag{1}$$

$$\text{div} u = 0. \tag{2}$$

For simplicity we consider $g = e_2, \lambda = 1$. This happens, for example, when characteristic velocity is small (slow motion). Secondly, we consider the linearization of the Navier-Stokes equations Oseen problem

$$v \cdot \nabla u + \lambda g \times u - \mu\Delta u = \nabla p + F, \tag{3}$$

$$\text{div} u = 0. \tag{4}$$

We assume $g = e_2$, $v = e_2$.

2 Preliminaries

The Lebesgue spaces are denoted by $L^p(\mathbf{R}^n)$, $1 \leq p \leq \infty$, and equipped the norms $\| \cdot \|_{0,p}$. By $W^{k,p}(\mathbf{R}^N)$, $k \geq 0$ an integer, $1 \leq p \leq \infty$, we denote the usual Sobolev spaces with the norms

$$\| \cdot \|_{k,p} = \sum_{|\alpha|=0}^{k} \|D^\alpha \cdot \|_{0,p} \tag{5}$$

where $\alpha = (\alpha_1, \alpha_2, \alpha_3)$ denotes the standard multi-index. Further, we define the homogeneous Sobolev spaces $D^{m,q}(\mathbf{R}^n)$ as

$$D^{m,q}(\Omega) = \overline{C_0^\infty(\Omega)}^{\|\nabla \cdot \|_{m-1,q}} \tag{6}$$

equipped with the norm $\|\nabla \cdot \|_{m-1,q}$. Denote by $S(\mathbf{R}^n)$ the space of functions of rapid decrease consisting of element u from $C^\infty(\mathbf{R}^n)$ such that

$$\sup_{x\in\mathbf{R}^n} (|x_1|^{\alpha_1} \dots |x_n|^{\alpha_n} |D^\beta u(x)|) < \infty \tag{7}$$

for all $\alpha_1, \dots, \alpha_n > 0$ and $|\beta| \geq 0$. For $u \in S(\mathbf{R}^n)$ we denote by $\widehat{u}$ its Fourier transform:

$$\widehat{u}(\xi) = \frac{1}{(2\pi)^{n/2}} \int_{\mathbf{R}^n} \bar{e}^{ix\cdot\xi} u(x) dx, \tag{8}$$

where i stands for the imaginary unit. It is well-known that $\widehat{u} \in S(\mathbf{R}^n)$ and that moreover,

$$u(x) = \frac{1}{(2\pi)^{n/2}} \int_{\mathbf{R}^n} e^{ix\cdot\xi} \widehat{u}(\xi) d\xi. \tag{9}$$

Given a function $\Phi : \mathbf{R}^n \to \mathbf{R}$, let us consider the integral transform

$$Tu \equiv h(x) = \frac{1}{(2\pi)^{n/2}} \int_{\mathbf{R}^n} e^{ix\cdot\xi} \phi(\xi) \widehat{u}(\xi) dx, \tag{10}$$

where $u \in S(\mathbf{R}^n)$.

Lemma 1 *Let $\Phi : \mathbf{R}^n \to \mathbf{R}$ be continuous together with the derivative $\frac{\partial^n \phi}{\partial\xi_1 \dots \partial\xi_n}$ and all preceding derivatives for $|\xi_i| > 0$, $i = 1, \dots, n$. Then, if for some $\beta \in [0, 1)$ and $M > 0$*

$$|\xi_1|^{k_1+\beta} \dots |\xi_n|^{k_n+\beta} \left| \frac{\partial^k \phi}{\partial\xi_1^{k_1} \dots \partial\xi_n^{k_n}} \right| \leq M \tag{11}$$

where k_i is zero or one and $K = \sum_{i=1}^n k_i = 0, 1, \dots, n$, the integral transform (10) defines a bounded linear operator from $L^q(\mathbf{R}^n)$ into $L^r(\mathbf{R}^n)$, $1 < q < \infty$, $1/r = 1/q - \beta$ and we have $\|Tu\|_r \leq c\|u\|_q$, with a constant c depending only on M, r and q.

For more details see [L].

3 Stokes problem in the whole space $\mathbf{R}^3$

We are interested in the Stokes problem with the Coriolis forces in the whole space.

$$-\mu\Delta u + \lambda g \times u = \nabla p + \mathcal{R} f, \tag{12}$$
$$\text{div } u = 0. \tag{13}$$

Let $g = e_2$.
In establishing estimates for (12), (13) it is important to single out the dependence of the constants entering the estimates on the dimensionless parametr $\mathcal{R}$. We will consider the problem

$$-\mu\Delta u + \lambda e_2 \times u = \nabla p + f, \tag{14}$$
$$\text{div } u = 0. \tag{15}$$

and we prove corresponding estimates for its solutions. The estimates for (12), (13) will be obtained if we make replacements

$$f \to f/\mathcal{R}, \tag{16}$$
$$p \to p/\mathcal{R}, \tag{17}$$
$$x_i \to \mathcal{R} x_i. \tag{18}$$

Main theorem 1
Given $f \in L^q(\mathbf{R}^n)$, $1 < q < \infty$, there exists a pair of functions u, p with $u_1, u_3 \in L^q$, $u \in D^{2,q}(\mathbf{R}^n)$, $\nabla p \in L^q$ satisfying the Stokes problem (12), (13) and moreover

$$|u|_{2,q} + |p|_{1,q} + \|u_1\|_q + \|u_3\|_q \leq c_1 \mathcal{R} \|f\|_q. \tag{19}$$

Also, if $1 < q < 3$ and $i = 1, 3$

$$|u_i|_{1,q} + \mathcal{R}^{1/3} |u_2|_{1,\frac{3q}{3-q}} + |u|_{2,q} + |p|_{1,q} \leq c_2 \mathcal{R} \|f\|_q \tag{20}$$

and if $1 < q < \frac{3}{2}$, $i = 1, 3$

$$|u_i|_{1,q} + |u_i|_q + \mathcal{R}^{3/2} \|u_2\|_{\frac{3q}{3-2q}} + \mathcal{R}^{1/3} |u_2|_{1,\frac{3q}{3-q}} \tag{21}$$
$$+ |u|_{2,q} + |p|_{1,q} \leq c_3 \mathcal{R} \|f\|_q,$$

where c_j, $j = 1, 2, 3$ depend on n, q.

Proof. We shall look for a solution to (12), (13) corresponding to $f \in C_0^\infty(\mathbf{R}^n)$ of the form

$$u(x) = \frac{1}{(2\pi)^{n/2}} \int_{\mathbf{R}^n} e^{ix\cdot\xi} U(\xi) d\xi, \tag{22}$$
$$p(x) = \frac{1}{(2\pi)^{n/2}} \int_{\mathbf{R}^n} e^{ix\cdot\xi} P(\xi) d\xi. \tag{23}$$

Replacing (22), (23) into (14), (15) furnishes the following algebraic system for U and P:

$$\xi^2 U_1 + i\xi_1 P(\xi) + \lambda U_3 = \widehat{f}_1, \tag{24}$$
$$\xi^2 U_2 + i\xi_2 P(\xi) = \widehat{f}_2, \tag{25}$$
$$\xi^2 U_3 + i\xi_3 P(\xi) - \lambda U_1 = \widehat{f}_3, \tag{26}$$
$$i\xi_m U_m = 0, \; m = 1, \dots, 3. \tag{27}$$

Proceedings of the 3rd IASME/WSEAS Int. Conf. on FLUID DYNAMICS & AERODYNAMICS, Corfu, Greece, August 20-22, 2005 (pp18-23)

Solving (24)-(27) for U and P delivers

$$U_1 = \frac{(\xi^4 + \lambda\xi_1\xi_3)(\widehat{f}_1\xi^2 - \xi_1\xi_m\widehat{f}_m)}{H} \quad (28)$$
$$-\frac{\lambda(\xi_2^2 + \xi_3^2)(\widehat{f}_3\xi^2 - \xi_3\xi_m\widehat{f}_m)}{H} \quad (29)$$
$$H = \xi^8 + \lambda^2\xi_1^2\xi_3^2 + \lambda^2(\xi_1^2 + \xi_2^2)(\xi_2^2 + \xi_3^2), \quad (30)$$

$$U_3 = \frac{\lambda(\xi^2\widehat{f}_1 - \xi_1\xi_m\widehat{f}_m)(\xi_1^2 + \xi_2^2)}{G} \quad (31)$$
$$+\frac{(\xi^4 + \lambda\xi_1\xi_3)(\xi^2\widehat{f}_3 - \xi_3\xi_m\widehat{f}_m)}{G} \quad (32)$$
$$G = \xi^8 + \lambda^2\xi_1^2\xi_3^2 + \lambda^2(\xi_1^2 + \xi_2^2)(\xi_2^2 + \xi_3^2), \quad (33)$$

$$P = \frac{\xi_m\widehat{f}_m}{i\xi^2} - \frac{\lambda(\xi_1U_3 - \xi_3U_1)}{i\xi^2}, \quad (34)$$

$$U_2 = \frac{\widehat{f}_2}{\xi^2} - \frac{i\xi_2}{\xi^2}\Big(\frac{\xi_m\widehat{f}_m}{i\xi^2} - \frac{\lambda(\xi_1U_3 - \xi_3U_1)}{i\xi^2}\Big). \quad (35)$$

We are interested in the behavior of U_1, U_2, U_3, P with $\widehat{f} \in S(\mathbf{R}^N)$.

We define

$$L = \xi^8 + \lambda^2\xi_1^2\xi_3^2 + \lambda^2(\xi_1^2 + \xi_2^2)(\xi_2^2 + \xi_3^2),$$

$$\phi_{11} = \frac{(\xi^4 + \lambda\xi_1\xi_3)(\xi_2^2 + \xi_3^2)}{L}, \quad (36)$$

$$\phi_{12} = \frac{-(\xi^4 + \lambda\xi_1\xi_3)\xi_1\xi_2}{L} \quad (37)$$
$$+\frac{\lambda(\xi_2^2 + \xi_3^2)\xi_3\xi_2}{L}, \quad (38)$$

$$\phi_{13} = -\lambda\frac{(\xi_2^2 + \xi_3^2)\xi^2}{L} + \quad (39)$$
$$-\lambda\frac{(\xi^4 + \lambda\xi_1\xi_3)\xi_1\xi_3}{L}, \quad (40)$$

$$\phi_{31} = \lambda\frac{(\xi_2^2 + \xi_3^2)(\xi_1^2 + \xi_2^2)}{L} - \quad (41)$$
$$-\lambda\frac{(\xi^4 + \lambda\xi_1\xi_3)\xi_3\xi_1}{L}, \quad (42)$$

$$\phi_{32} = \frac{-\lambda\xi_1\xi_2(\xi_1^2 + \xi_2^2)}{L} \quad (43)$$
$$+\lambda\frac{(\xi^4 + \lambda\xi_1\xi_3)\xi_3\xi_2}{L}, \quad (44)$$

$$\phi_{33} = \frac{\lambda(\xi^2 - \xi_1\xi_3)(\xi_1^2 + \xi_2^2)}{L} \quad (45)$$
$$+\lambda\frac{(\xi^4 + \lambda\xi_1\xi_3)(\xi_1^2 + \xi_2^2)}{L}, \quad (46)$$

$$\phi_{21} = -\frac{i\xi_2}{\xi^2}\Big(\frac{\xi_1}{i\xi^2} - \frac{\lambda(\xi_1\phi_{31} - \xi_3\phi_{11})}{i\xi^2}\Big), \quad (47)$$

$$\phi_{22} = \frac{1}{\xi^2} - \frac{i\xi_2}{\xi^2}\Big(\frac{\xi_2}{i\xi^2} - \frac{\lambda(\xi_1\phi_{32} - \xi_3\phi_{12})}{i\xi^2}\Big), \quad (48)$$

$$\phi_{23} = -\frac{i\xi_2}{\xi^2}\Big(\frac{\xi_3}{i\xi^2} - \frac{\lambda(\xi_1\phi_{33} - \xi_3\phi_{13})}{i\xi^2}\Big), \quad (49)$$

$$\varphi_k = \frac{\xi_k}{i\xi^2} - \frac{\lambda(\xi_1\phi_{3k} - \xi_3\phi_{1k})}{i\xi^2}. \quad (50)$$

Lemma 2 *Let $\phi_{1k}, \phi_{2k}, \phi_{3k}, \varphi_k$ be given by (36)-(50), then the assumptions of Lemma 1 are satisfied*

(a) $\phi_{1k}, \phi_{3k} \quad \beta = 0;$
(b) $\xi_l\phi_{1k}, \xi_l\phi_{3k} \quad \beta = 0;$
(c) $\xi_l\xi_m\phi_{1k}, \xi_l\xi_k\phi_{3k},\ \xi_l\xi_m\phi_{2k} \quad \beta = 0;$
(d) $\xi_l\varphi_k \quad \beta = 0;$
(e) $\phi_{2k} \quad \beta = \frac{2}{3};$
(f) $\xi_l\phi_{2k} \quad \beta = \frac{1}{3}.$

Proof:

$$|\phi_{3k}| \le \Big(\frac{\lambda|\xi|^4 + |\xi|^6}{\lambda^2|\xi|^4 + |\xi|^8}\Big), \quad (51)$$

$$|\xi_l|\Big|\frac{\partial\phi_{3k}}{\partial\xi_l}\Big| \le \frac{|\xi|(\lambda|\xi|^3+|\xi|^5)}{\lambda^2|\xi|^4+|\xi|^8} + \frac{|\xi|(\lambda^2|\xi|^7+|\xi|^{11})}{\lambda^4|\xi|^8+|\xi|^{16}},$$

$$|\xi_1\xi_2|\Big|\frac{\partial^2\phi_{3k}}{\partial\xi_1\partial\xi_2}\Big| \le \frac{|\xi|^2(\lambda|\xi|^2+|\xi|^4)}{\lambda^2|\xi|^4+|\xi|^8} + \frac{|\xi|^2(\lambda^2|\xi|^6+|\xi|^{10})}{\lambda^4|\xi||^8+|\xi|^{16}} + \frac{|\xi|^2(\lambda^3|\xi|^{10}+|\xi|^{16})}{\lambda^6|\xi|^{12}+|\xi|^{24}},$$

$$|\xi_1\xi_3|\Big|\frac{\partial^2\phi_{3k}}{\partial\xi_1\partial\xi_3}\Big| \le \frac{|\xi|^2(\lambda|\xi|^2+|\xi|^4)}{\lambda^2|\xi|^4+|\xi|^8} + \frac{|\xi|^2(\lambda^2|\xi|^6+|\xi|^{10})}{\lambda^4|\xi||^8+|\xi|^{16}} + \frac{|\xi|^2(\lambda^3|\xi|^{10}+|\xi|^{16})}{\lambda^6|\xi|^{12}+|\xi|^{24}},$$

$$|\xi_2\xi_3|\left|\frac{\partial^2\phi_{3k}}{\partial\xi_2\partial\xi_3}\right| \leq \frac{|\xi|^2(\lambda|\xi|^2+|\xi|^4)}{\lambda^2|\xi|^4+|\xi|^8} + \frac{|\xi|^2(\lambda^2|\xi|^6+|\xi|^{10})}{\lambda^4|\xi||^8+|\xi|^{16}} + \frac{|\xi|^2(\lambda^3|\xi|^{10}+|\xi|^{16})}{\lambda^6|\xi|^{12}+|\xi|^{24}},$$

$$|\xi_1\xi_2\xi_3|\left|\frac{\partial^3 U_3}{\partial\xi_1\xi_2\xi_3}\right| \leq \frac{\lambda^4|\xi|^{16}+|\xi|^{25}}{\lambda^8|\xi|^{16}+|\xi|^{32}} + \frac{|\xi|^3(\lambda^2|\xi|^9+|\xi|^{13})}{\lambda^6|\xi|^{12}+|\xi|^{24}} + \frac{|\xi|^3(\lambda^2|\xi|^5+|\xi|^9)}{\lambda^4|\xi|^8+|\xi|^{16}} + \frac{|\xi|^3(\lambda|\xi|+|\xi|^3}{\lambda^2|\xi|^4+|\xi|^8},$$

From formulas above (a)-(f) are satisfied.

Lemma 2 implies that

$$\left\|\frac{\partial u_i}{\partial x_2}\right\|_q \leq c\|f\|_q, \qquad i=1,3, \tag{52}$$

$$|u_i|_{2,q} \leq c\|f\|_q, \qquad i=1,2,3, \tag{53}$$

$$|p|_{1,q} \leq c\|f\|_q, \tag{54}$$

$$\left\|\frac{\partial u}{\partial x_2}\right\|_{\frac{3q}{3-q}} \leq c\|f\|_q, \tag{55}$$

$$|u_i|_{1,q} \leq \|f\|_q, \qquad i=1,3, \tag{56}$$

$$|u_2|_{1,\frac{3q}{3-q}} \leq c\|f\|_q, \tag{57}$$

$$\|u_i\|_q \leq \|f\|_q, \qquad i=1,3, \tag{58}$$

$$\|u_2\|_{\frac{3q}{3-2q}} \leq c\|f\|_q, \tag{59}$$

where c is a constant which depends on n, q. This give us the proof of Theorem 1.

4 Oseen problem in the whole space

We will consider the problem

$$\begin{cases} \mathcal{R}\tilde{v}\cdot\nabla u + \lambda g\times u - \mu\Delta u + \nabla p = \mathcal{R}f, \\ \nabla\cdot u = 0. \end{cases} \tag{60}$$

Let $\tilde{v} = e_2$, $g = e_2$. We investigate the following problem

$$\begin{cases} \tilde{v}\cdot\nabla u + \lambda g\times u - \mu\Delta u + \nabla p = f, \\ \nabla\cdot u = 0. \end{cases} \tag{61}$$

In establishing estimates for (60), it is important to single out the dependence of the constants entering the estimates on the dimensionless parametr $\mathcal{R}$. The estimates for (60) will be obtained if we make replacements

$$f \to f/\mathcal{R},$$
$$p \to p/\mathcal{R},$$
$$x_i \to \mathcal{R}x_i.$$

Theorem 2. *Given $f \in L^q(\mathbf{R}^n)$, $1 < q < \infty$, there exists a pair of functions (u,p) with $u \in D^{2,q}$, $\nabla p \in L^q$, $u_1, u_3 \in L^q$, $\frac{\partial u_1}{\partial x_2}$, $\frac{\partial u_2}{\partial x_2}$, $\frac{\partial u_3}{\partial x_2} \in L^q$, satisfying (60) and moreover*

$$\mathcal{R}\left\|\frac{\partial u}{\partial x_2}\right\|_q + \mathcal{R}\left\|\frac{\partial u_1}{\partial x_l}\right\|_q + \mathcal{R}\left\|\frac{\partial u_3}{\partial x_l}\right\|_q + \|u_1\|_q + \|u_3\|_q + |p|_{1,q} + |u|_{2,q} \leq c_1\mathcal{R}\|f\|_q. \tag{62}$$

Moreover, if $1 < q < 4$, $i = 1,3$ then

$$\mathcal{R}\left\|\frac{\partial u}{\partial x_2}\right\|_q + \mathcal{R}\left\|\frac{\partial u_i}{\partial x_l}\right\|_q + \|u_i\|_q + \mathcal{R}^{1/4}|u_2|_{1,\frac{4q}{4-q}} + |p|_{1,q} + |u|_{2,q} \leq c_2\mathcal{R}\|f\|_q. \tag{63}$$

Also, if $1 < q < 2$, $i = 1,3$ then

$$\mathcal{R}\left\|\frac{\partial u}{\partial x_2}\right\|_q + \|u_i\|_q + \mathcal{R}^{1/2}\|u_2\|_{\frac{2q}{2-q}} + |u_i|_{1,q} + \mathcal{R}^{1/4}|u_2|_{1,\frac{4q}{4-q}} + |p|_{1,q} + |u|_{2,q} \leq c_3\mathcal{R}\|f\|_q, \tag{64}$$

where c_j, $j = 1,2,3$ depend on n, q.

Proof: We shall look for a solution to (60) corresponding to $f \in C_0^\infty(\mathbf{R}^n)$ of the form (22), (23).

Replacing (22), (23) into (60) furnishes to the following algebraic system for U and P

$$\left.\begin{array}{l}(\xi^2+i\xi_2)U_1(\xi)+i\xi_1P(\xi)+\lambda U_3(\xi)=\widehat{f}_1(\xi),\\ (\xi^2+i\xi_2)U_2(\xi)+i\xi_2P(\xi)=\widehat{f}_2(\xi),\\ (\xi^2+i\xi_2)U_3(\xi)+i\xi_3P(\xi)-\lambda U_1(\xi)=\widehat{f}_3(\xi),\\ i\xi_mU_m=0,\quad m=1,2,3.\end{array}\right\} \tag{65}$$

Solving (65) for U and P delivers

$$\begin{array}{l}U_1=\frac{(\xi^2(\xi^2+i\xi_2)-\lambda\xi_1\xi_3)(\xi^2\widehat{f}_1-\xi_1\xi_m\widehat{f}_m)}{L_1},\\ -\frac{\lambda(\xi_2^2+\xi_3^2)(\widehat{\xi}^2\widehat{f}_3-\xi_3\xi_m\widehat{f}_m)}{L_1}\\ L_1=\lambda^2(\xi_2^2\xi_1^2+\xi_2^4+\xi_3^2\xi_1^2+\xi_3^2\xi_2^2)+\\ +(\xi^2+i\xi_2)^2\xi^4-\lambda\xi_1^2\xi_3^2,\end{array} \tag{66}$$

$$\begin{array}{l}U_3=\frac{(\xi^2+i\xi_2)\xi^2(\widehat{f}_3\xi^2+\widehat{f}_m\xi_3\xi_m)}{G_1}\\ +\ \frac{(\xi_1^2+\xi_2^2)(\lambda\xi^2\widehat{f}_1-\lambda\xi_1\xi_m\widehat{f}_m)}{G_1}\\ +\frac{\lambda\xi_1\xi_3(\xi^2\widehat{f}_3-\lambda\xi_m\xi_3\widehat{f}_m)}{G_1},\\ G_1=\lambda^2(\xi_2^2\xi_1^2+\xi_2^4+\xi_3^2\xi_1^2+\xi_3^2\xi_2^2)+\\ +(\xi^2+i\xi_2)^2\xi^4-\lambda^2\xi_1^2\xi_3^2,\end{array} \tag{67}$$

$$P(\xi)=\frac{\xi_m\widehat{f}_m-\lambda(\xi_1U_3-\xi_3U_1)}{i\xi^2}, \tag{68}$$

$$U_2(\xi)=\frac{\xi^2\widehat{f}_2-\xi_3\xi_m\widehat{f}_m}{\xi^2(\xi^2+i\xi_2)}+\frac{\lambda(\xi_2(\xi_1U_3-\xi_3U_1))}{\xi^2(\xi^2+i\xi_2)}. \tag{69}$$

We are dealing with the behavior of U_1, U_2, U_3, P with $\widehat{f}\in S(R^N)$.

We define

$$\begin{array}{rcl}\phi_{11}&=&\frac{(\xi^2(\xi^2+i\xi_2)-\lambda\xi_1\xi_3)\xi^2}{H_1}\\ &&+\frac{\lambda(\xi_2^2+\xi_3^2)\xi_3\xi_1}{H_1}\end{array} \tag{70}$$

$$\begin{array}{l}H_1=\lambda^2(\xi_2^2\xi_1^2+\xi_2^4+\xi_3^2\xi_1^2+\xi_3^2\xi_2^2)+\\ +(\xi^2+i\xi_2)^2\xi^4-\lambda\xi_1^2\xi_3^2,\end{array}$$

$$\begin{array}{rcl}\phi_{12}&=&\frac{(\xi^2(\xi^2+i\xi_2)-\lambda\xi_1\xi_3)\xi_1\xi_2}{H_1}+\\ &&+\frac{\lambda(\xi_2^2+\xi_3^2)\xi_3\xi_1}{H_1},\end{array} \tag{71}$$

$$\begin{array}{rcl}\phi_{13}&=&\frac{(\xi^2(\xi^2+i\xi_2)-\lambda\xi_1\xi_3)\xi_1\xi_3}{H_1}+\\ &&+\frac{\lambda(\xi_2^2+\xi_3^2)\xi_3\xi_2}{H_1},\end{array} \tag{72}$$

$$\begin{array}{rcl}\phi_{31}&=&\frac{(\xi^2+i\xi_2)\xi^2\xi_3\xi_1)}{H_1}+\\ &&+\frac{(\xi_1^2+\xi_2^2)(\lambda\xi^2+\lambda\xi_1\xi_3-\lambda\xi_1\xi_3}{H_1},\end{array} \tag{73}$$

$$\begin{array}{rcl}\phi_{32}&=&\frac{(\xi^2+i\xi_2)\xi^2(\xi_3\xi_2)+(\xi_1^2+\xi_2^2)(\lambda\xi_1\xi_2)}{H_1}\\ &&+\frac{\lambda\xi_1\xi_3-\lambda\xi_2\xi_3)}{H_1},\end{array} \tag{74}$$

$$\begin{array}{l}\phi_{33}=\frac{(\xi^2+i\xi_2)\xi^2(\xi^2+\xi_3^2)}{H_1}+\\ +\frac{(\xi_1^2+\xi_2^2)(-\lambda\xi_1\xi_3)+\lambda\xi_1\xi_3(\xi^2-\lambda\xi_3^2)}{H_1},\end{array} \tag{75}$$

$$\phi_{21}=\frac{\xi_3\xi_1}{\xi^2(\xi^2+i\xi_2)}+\frac{\lambda(\xi_2(\xi_1\phi_{31}-\xi_3\phi_{11}))}{\xi^2(\xi^2+i\xi_2)}, \tag{76}$$

$$\phi_{22}=\frac{\xi^2-\xi_3\xi_2}{\xi^2(\xi^2+i\xi_2)}+\frac{\lambda(\xi_2(\xi_1\phi_{32}-\xi_3\phi_{12}))}{\xi^2(\xi^2+i\xi_2)}, \tag{77}$$

$$\phi_{23}=\frac{-\xi_3^2}{\xi^2(\xi^2+i\xi_2)}+\frac{\lambda(\xi_2(\xi_1\phi_{33}-\xi_3\phi_{13}))}{\xi^2(\xi^2+i\xi_2)}, \tag{78}$$

$$\pi_k=\frac{\xi_k-\lambda(\xi_1\phi_{3k}-\xi_3\phi_{1k})}{i\xi^2}. \tag{79}$$

Lemma 3 *Let $\phi_{1k}, \phi_{2k}, \phi_{3k}, \pi_k$ be given by (70)-(79) then the assumptions of Lemma 1 are satisfied*

(a) *by* $\phi_{1k}\phi_{3k}$, $\beta=0$,
(b) $\xi_l\phi_{1k}, \xi_l\phi_{3k}$, $\beta=0$,
(c) $\xi_l\xi_k\phi_{1k}, \xi_l\xi_k\phi_{3k}, \xi_l\xi_k\phi_{2k}$, $\beta=0$,
(d) $\xi_l\pi_k$, $\beta=0$,
(e) ϕ_{2k}, $\beta=1/2$,
(f) $\xi_2\phi_{2k}$, $\beta=0$,
(g) $\xi_l\phi_{2k}$, $\beta=1/4$.

Proof:

$$|\phi_{3k}(\xi)|\le\frac{\lambda|\xi|^4+|\xi|^6+|\xi|^5}{\lambda^2|\xi|^4+|\xi|^8+|\xi|^7+|\xi|^6}, \tag{80}$$

$$|\xi_l|\left|\frac{\partial\phi_{3k}}{\partial\xi_l}\right|\le\frac{|\xi_l|[\lambda|\xi|^3+|\xi|^5]}{\lambda^2|\xi|^4+|\xi|^8}+\frac{|\xi_l|(\lambda^3|\xi|^7+|\xi|^{13})}{\lambda^4|\xi|^8+|\xi|^{16}}, \tag{81}$$

$$\begin{array}{l}|\xi_1\xi_3|\left|\frac{\partial^2\phi_{3k}}{\partial\xi_1\xi_3}\right|\le\frac{|\xi_1|\xi_3(\lambda|\xi|^2+|\xi|^4+|\xi|^3)}{\lambda^2|\xi|^4+|\xi|^8}\\ +\frac{|\xi_1|^2(\lambda|\xi|^3+|\xi|^5+|\xi|^4)(|\lambda|^2|\xi|^3+|\xi|^7+|\xi|^6)}{\lambda^4|\xi|^8+|\xi|^{16}}\\ +\frac{|\xi_1||\xi_3|(\lambda|\xi|^4+|\xi|^6)(\lambda^2|\xi|^2+|\xi|^6+|\xi|^5)}{\lambda^4|\xi|^8+|\xi|^{16}}\\ +\frac{|\xi_1||\xi_3|(\lambda|\xi|^4+|\xi|^6+|\xi|^5)(\lambda^2|\xi|^3+|\xi|^7)^2}{\lambda^6|\xi|^{12}+|\xi|^{24}}.\end{array} \tag{82}$$

Similarly for $\frac{\partial^3\phi_{3k}}{\partial\xi_2\xi_1\xi_3},\ldots\frac{\partial^3\phi_{3k}}{\partial\xi_3\xi_2\xi_1}, \frac{\partial^2\phi_{3k}}{\partial\xi_1\partial\xi_2}, \frac{\partial^2\phi_{3k}}{\partial\xi_1\partial\xi_3}$, etc. applying (80)-(82) we get (a)-(c).

$$|\phi_{2k}|\le\frac{|\xi_1|^\beta|\xi_2|^\beta|\xi_3|^\beta}{|\xi|^2+|\xi_2|},\qquad\beta=1/2, \tag{83}$$

$$|\xi_lU_2|\le\frac{|\xi_l||\xi_1|^\beta|\xi_2|^\beta|\xi_3|^\beta}{|\xi|^2+i|\xi_2|},\beta=1/4, \tag{84}$$

$$|\xi_2U_2|\le\frac{|\xi_2||\xi_1|^\beta|\xi_2|^\beta|\xi_3|^\beta}{|\xi|^2+|\xi_2|},\qquad\beta=0, \tag{85}$$

$$\begin{array}{l}|\xi_lP|\le\frac{(|\xi_1|^\beta|\xi_2|^\beta|\xi_3|^\beta)(\xi_l\xi_m-\lambda\xi_l(\xi_1\phi_{3k}-\xi_3\phi_{1k}))}{|\xi^2|},\\ \beta=0.\end{array} \tag{86}$$

From Lemma 3 it follows

$$\left\| \frac{\partial u}{\partial x_2} \right\|_q \leq C\|f\|_q, \tag{87}$$

$$\left\| \frac{\partial u_i}{\partial x_l} \right\|_q \leq c\|f\|_q, \qquad i = 1, 3, \tag{88}$$

$$\|u_i\|_q \leq c\|f\|_q, \qquad i = 1, 3, \tag{89}$$

$$|p|_{1,q} \leq c\|f\|_q, \tag{90}$$

$$|u|_{2,q} \leq c\|f\|_q, \tag{91}$$

$$|u_2|_{1, \frac{4q}{4-q}} \leq c\|f\|_q, \tag{92}$$

$$\|u_2\|_{\frac{2q}{2-q}} \leq c\|f\|_q. \tag{93}$$

This give us the proof of Theorem 2.

Remark: The extension to the exterior problem we can find in [N1], and the Navier-Stokes equations with the Coriolis force was studied in [N2].

Acknowlegment: *The research was supported by the Academy of Sciences of the Czech Republic, Institutional Research Plan no. AV0Z10190503 and by the Grant Agency of the Academy of Sciences No.IAA10019505.*

5 References

[A] R. A. Adams, *Sobolev Spaces*, (1975) Academic Press, New York

[B] H. Brenner, *The Stokes Resistance of an Arbitrary Particle II* ,Chem. Engng. Sci. **19**, (1959) 599-624

[FHM] R. Farwig, T. Hishida, D. Müller, *L^q Theory of a singular operator arising from fluid dynamics*, (2003) Prepint Nr.2259, University of Darmstadt

[G1] G. P. Galdi, *An introduction to the mathematical theory of the Navier-Stokes equations:Linearised steady problems*, Springer Tracts in Natural Philosophy, Vol.**38**, (1998) 2nd edition.

[G2] G. P. Galdi, *On the motion of a rigid body in a viscous luquid: a mathematical analysis with applications* , Handbook of Mathematical Fluid Dynamics, Volume 1, (2002) Edited by Friedlander, D.Serre, Elsevier

[G3] G. P. Galdi, *Steady flow of a Navier-Stokes fluid around a rotating obstacle*, Preprint, (2002)

[GHT] B. Guenther , R. T. Hudspeth, E. A. Thomann, *Hydrodynamic forces on submerged rigid bodies-steady flow*, J. Math. Fluid Mech. **2**, (2001), 219-266.

[K] G. Kirchoff, G.,*Uber die Bewegung eines Rotationskorpers in einer Flussigkeit, Crelle*, **71**, (1869) 237-281

[L] P. I. Lizorkin, *(L^p, L^q) Multipliers of Fourier Integrals*, Dokl. Akad. Nauk SSSR, **152**, (1963) 808-811; English Translation: Soviet Math. Dokl., **4**, 1963, 1420-1424

[Le] L. G. Leal:*The slow motion of slender rod-like particles in a second-order fluid* Jornal of Fluid Mech. **69**, (1975), 305-337

[N] J. Nečas, *Les Methodes Directes en Theorie des equations Elliptiques*, Masson et C^{ie} (1967)

[N1] Š. Nečasová, *Stokes and Oseen flow with the Coriolis force in an exterior domain*, Preprint (2005)

[N2] Š. Nečasová, *Asymptotic properties of the steady fall of a body in a viscous fluid*, Mathem. Models and Appl. Sciences **27**, (2002), 1969-1995

[PC] E. S. Pettyjohn, E.B. Christiansen *Effect of particle shape on free-settling rates of isometric particles*, Chem. Eng. Prog. **44**, (1948), 157-172

[S] D. Serre, *Chute Libre d'un Solide dans un Fluide Visqueux Incompressible.* Existence. Jap. J. Appl. Math., 4,**(1)**,(1987) 99-110

[T] W. Thomson, (Lord Kelvin), *Mathematical and Physical Papers*, Vol. **4**, Cambridge University Press, (1882)

[W1] H. F. Weinberger, *On the steady fall of a body in a Navier-Stokes fluid*, Proc. Symp. Pure Mathematics, **23**, (1973), 421-440

[W2] H. F. Weinberger, *Variational Principles for a Body Falling in Steady Stokes Flow*, J. Fluid Mech., **52**, (1972), 321-344