

Determination by Proper Orthogonal Decomposition of the convection velocity of an annular jet and of a round jet.

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Abstract: - This article discusses the application of Proper Orthogonal Decomposition to the P.I.V. velocity fields of an annular jet and on a statistic of time-resolved tomographic images of the initial zone of the annular jet. The annular jet is an example of complex shear flow situations. Two axisymmetric shear layers, originating at the jet exit, one at the nozzle lip and the other at the centre body, eventually meet downstream or interact with each other. This article proposes especially a determination of the convection velocity of the primary vortices of an annular jet and compare with the convection velocity of a round jet.

Key-Words: - Particle Image Velocimetry, Proper Orthogonal Decomposition, Annular jet, round jet, convection velocity, Turbulence.

1 Introduction

Annular jet is used in the industrial domain, in combustion (burners, bluff bodies...) or in industrial treatment processes. The geometry of the annular jet is determined by the ratio D_i/D_o , where D_i represents the internal diameter and D_o the external diameter. The flow characteristics of the initial region of an annular jet discharging into stationary air have been investigated previously. Chigier [1], in 1964, studied them as a limiting case of coaxial jets. Because the configuration of the annular jet often adopted by authors was without afterbody or bullet at the nozzle exit, an internal recirculating region is formed downstream of the interface. This recirculation zone is the result of the lack of any air supply in the center and the entrainment of air from the main stream of the annular jet. The interests of the flow here are the interaction between the jet and the recirculation zone near the nozzle of the annular jet. Many authors, like Ko [2][3], work on small diameter ratio annular jets, with a ratio D_i/D_o smaller than 0.7. Only the fully-developed merging zone of a diameter ratio larger than 0.7 have been studied by Aly [4] in 1991.

The scientific interest in the study of turbulence has led to the development of a P.I.V. post-processing, which is able to bring out the inner driving mechanism of the flow [5]. In other words, the structure responsible for the development or the maintenance of flow instability has to be reached. So, in this context, Lumley [6][7] has proposed a method for identifying coherent and

instantaneous structures in turbulent flow. The method is the Proper Orthogonal Decomposition (P.O.D.). This method provides a base for the modal decomposition of a set of functions, such as data obtained in the course of the experiments. The most striking property of this decomposition is optimality: it provides the most efficient way of capturing the dominant components of an infinite-dimensional process with only a few functions. That is why the P.O.D. process has been applied in different turbulent flows to analyze experimental P.I.V. data with a view to extracting dominant features and trends: coherent structures[6][7][8][9]. P.O.D. provides an optimal set of basis functions for an ensemble of data. It is optimal in the sense that it is the most efficient way of extracting the most energetic component of an infinite dimensional process with only a few modes [5]. The present study relies on an experimental investigation of the initial zone of a large diameter ratio annular air jet by the use of Particle Image Velocimetry (P.I.V.), fast tomography imagery. Then, measurements of convection velocity will be determined by the use of a Proper Orthogonal Decomposition (P.O.D.) mode applied on tomographic images of the annular jet and of a round jet.

2 Experimental set-up

2.1 Annular jet

A diagram of annular jet is shown in fig 1. The annular jet is characterized by the outer diameter D_o , equal to 53.88 mm, and the inner diameter D_i , equal to 48.75 mm. The thickness of the jet e is thus equal to 2.565 mm.

In our setup, D_i/D_o is equal to 0.90. The exit velocities U_o are equal to 8 m/s, 15 and 30m/s which respectively correspond to the values of the Reynolds number Re , based on the thickness e , of 2048, 3840 and 7680. At a distance from the nozzle of 50 μ m, the cross sectional profile of the longitudinal, measured with hot wire anemometry, shows a “top hat” distribution.

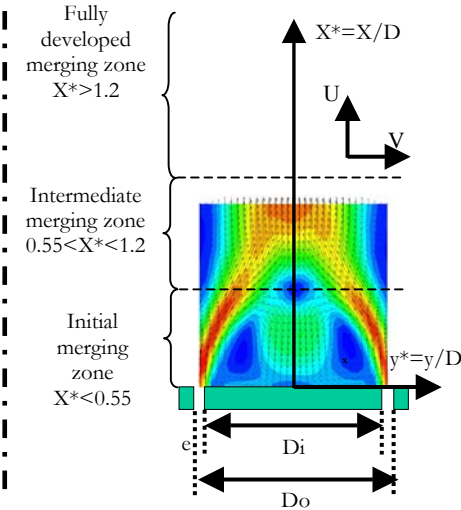


Fig 1: annular jet

2.2 Round jet

The burner consists of a vertical cylindrical tube with a 62-mm-diameter core and a 6-mm-diameter nozzle, d_o . The nozzle lip thickness is less than 0.2mm. The nozzle contraction is profiled to obtain a well-organized jet. Honeycombs and grids provide a homogenous flow. The exit velocity U_o is equal to 13.5 m/s.

2.3 cross correlation PIV set-up

The experimental setup of the P.I.V. measurements is presented in fig 2. A double pulsed Nd-Yag laser is used to set up the light sheet. The output energy is nearly 30 mJ for each laser pulse. The wavelength is 532 nm. The laser beams are focused onto a sheet across the median plane of the annular jet by one cylindrical lens ($f=0.02$ m) and one spherical lens ($f=0.5$ m). The time delay between the two pulses, which depends on the exit velocity U_o , is 8 μ s. The observation field is 2.8×2.5 cm².

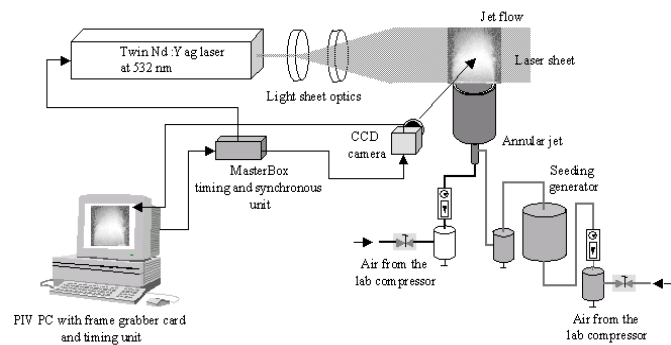


Fig 2 : experimental PIV set-up

In this study, the video images are recorded by a LAVISION Flow Master 3S camera. The frame grabber, using a pixel clock, digitizes the analogue video signal to an accuracy of 12 bits. In the frame grabber, each field is digitized in 1280*1024 pixels with grey levels. The acquisition frequency is 4 Hz. Interrogation of the recorded images is performed by two-dimensional digital cross correlation analysis using “Davis 6.2.2.” For all velocity fields, the sampling window has a size of 16 by 16 pixels (0.377 by 0.377 mm) and there is a 50 % overlap with the next window. 400 P.I.V. images have been recorded. We observed less than 1 % of false vectors calculated in the flow.

2.4 Fast tomographic imaging set-up

The laser sheet is obtained by a 4W, 515nm Argon-Ion continuous laser. The tomography system is composed of a high-speed digital camera Kodak Ektapro 8 bits. For the annular jet we used 256x256 pixels at 4500 images/s. For the round jet, we used 128x256 pixels, 0.13mm/pixels at 9000 images/s.

2.5 Flow seeding.

Flow seeding is one of the most important aspects of P.I.V. measurements. The intake air is seeded with 2-3 μ m diameter olive oil droplets. These were generated in an atomizer by passing air through a bath of olive oil. The air pressure varied from 1.5 to 2.5 bars (fig 2). The particle seeding density was controlled by the flow rate of air through the atomizer.

3 Proper Orthogonal Decomposition.

Coherent structures are present in turbulent flow and P.I.V. is able to highlight those on the largest scale at any given moment. The scientific interest in the study of turbulence has led to the development of P.I.V. post-processing, which is able to bring out the inner driving mechanism of the flow [5].

Lumley [6][7] has proposed a method for identifying coherent and instantaneous structures in turbulent flow. The method is Proper Orthogonal Decomposition (P.O.D.). P.O.D. provides an optimal set of basis functions for a set of data, Delville [12], Graftieux [13]. It is optimal in the sense that it is the most efficient way of extracting the most energetic components of an infinite dimensional process with only a few modes. Each velocity field can be decomposed as a linear combination of proper mode ϕ such that:

$$u_i(x,y) = \sum_{k=0}^{N-1} a_{i,k} \phi_k(x,y) \text{ and } \delta_{kk'} \lambda_k = \langle a_k a_k^* \rangle \quad (1)$$

where λ_k is the energy contained in the mode k . The computation has been done with 400 velocity fields. This number is sufficient for a good representation of the flow.

4. Velocity characteristics of annular jet.

4.1 Aerodynamic characteristics of the initial zone of annular jet.

For the annular jet, it appears that the stagnation point is put through important radial fluctuations and, axially, the maximal fluctuations are localized on the external-mixing layer. This was also observed by Ko and Chan [3], but they used a hot wire and this cannot measure null velocity. Therefore, for a spatial quantification, the P.I.V. technique has been used [9]. Fig 3 shows an example of the mean velocity field with the corresponding Reynolds decomposition fluctuation fields. The initial merging zone extends from the jet exit to the tip of the potential core. This zone contains a recirculating zone. The stagnation point which marks the end of the recirculating region is located at $x/Do=0.5$.

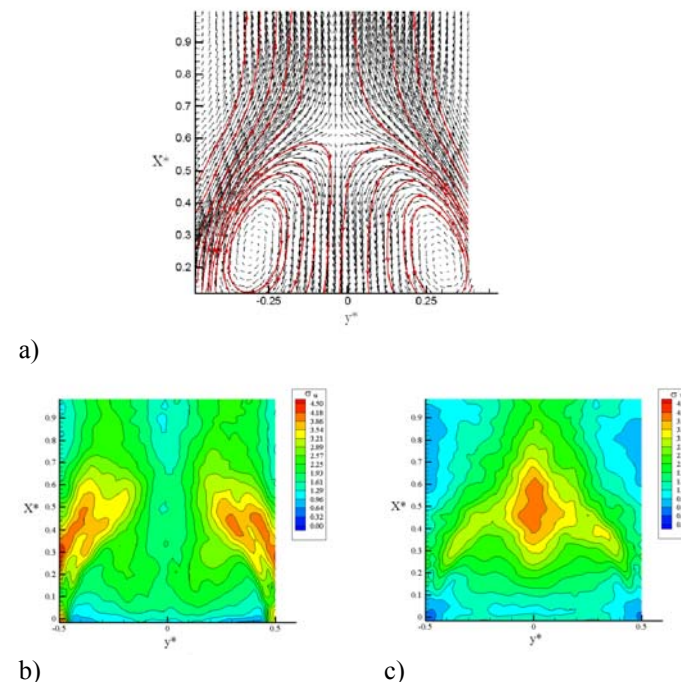


Fig 3: Aerodynamic characteristics of the annular jet: $Re_c = 7680$. a) Average velocity field calculated with 400 P.I.V. fields. b) Reynolds decomposition radial velocities c) Reynolds decomposition axial velocities.

4.2 Results of Proper Orthogonal Decomposition application

In this study the P.O.D. is applied on P.I.V. velocity fields of the recirculation zone of this annular jet.

Reynolds decomposition velocity fluctuations show two effects: the importance of oscillations for the stagnation point and the air entrainment in the annular jet. Thus, to evaluate the importance and the influence of each mode on an instantaneous velocity field, each instantaneous field reconstructed by choosing P.O.D. modes, and then the Reynolds decomposition radial fluctuations are calculated. A reconstruction with the first mode and the k^{th} mode, using the projection value of the instantaneous field on the modes, automatically shows what instability is represented by this k^{th} mode. Fig 4 shows the reconstruction of all the instantaneous fields with the first two modes and without mode 1. So, it is clear that with modes 0+1 the position and the intensity of the Reynolds decomposition radial fluctuations are the same as those calculated for all modes, contrary to the reconstruction without mode 1. So mode 1 is responsible for the radial fluctuations of the stagnation point.

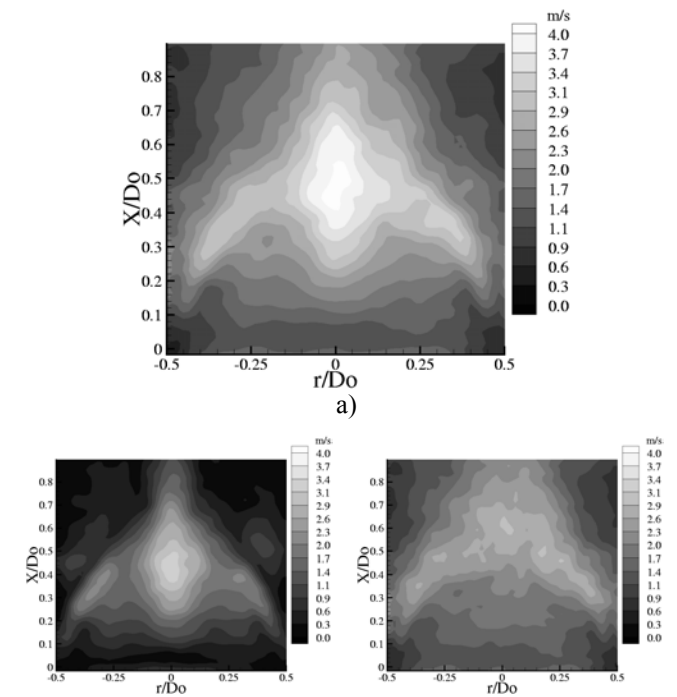


Fig 4 Influence of mode 1 on Reynolds decomposition radial velocity fluctuations: a) Radial velocity fluctuations b) Radial velocity fluctuations reconstructed with mode 0 and 1. c) Radial velocity fluctuations reconstructed without mode 1.

5 Determination of the convection velocity U_c .

5.1 Velocity convection of the primary vortices of the annular jet.

Two different methods have been applied to determine the convection velocity U_c of the primary vortices.

5.1.1 Classical Method.

The first one consists in determining U_c by calculating the ratio between a distance d and a time. The distance is measured between two primary vortices, and the time is determined from the frequency of appearance f of the vortices.

An algorithm for the detection of the core of a vortex has been used to measure the probability density function of the distance between two primary vortices. The algorithm was developed by Graftieux [11]. S is the area of a mesh, centered on a point P . U_M is the velocity at a point M included in S . Γ is thus a scalar function and its value lies between -1 and $+1$, according to the direction of rotation of the axisymmetric vortices.

$$\Gamma(P) = \frac{1}{S} \int_S \frac{\overrightarrow{PM} \wedge \overrightarrow{U_M}}{\|\overrightarrow{PM}\| \|\overrightarrow{U_M}\|} dS \quad (2)$$

This algorithm has been applied to all the velocity fields for all U_o velocities. Thus, we have calculated the pdf of inter-vortex distances d in the external (Fig 5) and internal (Fig 6) mixing layers. There are significant real differences between the two layers.

In the external layer, the more probable distance d_m is about 0.06. There is also, for $U_o = 8$ m/s, a second peak localized for $d/D_o = 0.12$. This represents the distance between paired vortices, double of the distance d_m .

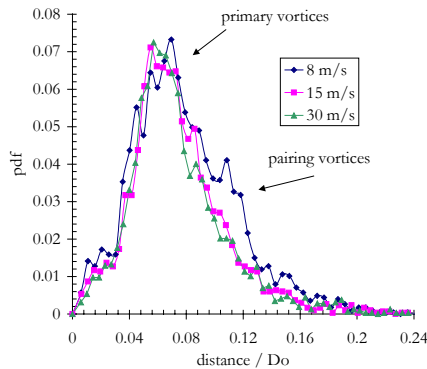


Fig 5 Probability density function of the distance d_m for $U_o=8, 15$ and 30 m/s. External layer of annular jet.

In the internal mixing layer, the distance d_m depends on the velocity U_o . At the beginning of the internal shear layer, a small recirculation zone defined in the previous part influences the convection velocity of the vortices. The frequency, measured by hot wire anemometry, is the same as the frequency of the external mixing layer. Because the distance d_m is different according to the velocity, U_c^* is not a constant.

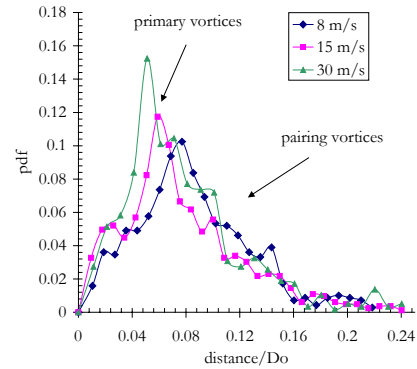


Fig 6 Probability density function of the distance d_m for $U_o=8, 15$ and 30 m/s. Internal mixing layer of annular jet.

The results of the determination of the convection velocity U_c^* of primary vortices are:

U_o (m/s)	external mixing layer			internal mixing layer		
	8	15	30	8	15	30
U_c^*	0.31 ± 0.03	0.27 ± 0.02	0.28 ± 0.02	0.39 ± 0.03	0.27 ± 0.02	0.24 ± 0.02

Table 1: determination of the convection velocity

5.1.2 Determination by P.O.D.

The second method is an application of an inhomogeneous filter called Proper Orthogonal Decomposition (P.O.D.) on a statistic of time-resolved tomographic images of the initial zone of the annular jet. It can be applied on scalar fields, such as images (Sirovich [12][13]), or P.I.V. fields (Patte-Rouland [14]). P.O.D. is optimal in the sense that it is the most efficient way of extracting the most energetic component of an infinite dimensional process with only a few modes. As relation (1), each image can be decomposed as a linear combination of proper mode ϕ such that:

$$Im_i(x,y) = \sum_{k=0}^{N-1} a_{i,k} \phi_k(x,y) \quad (3)$$

The computation has been done with 400 images for $U_o=8$ m/s. The first five different modes are displayed on the Fig 7. The first one has the topology of the mean image of the flow. The other modes are characteristic of a convection flow. Each presents an alternation of positive and negative zones in the external mixing layer. The negative values of a mode are represented by a black pixel and positive values by a white one.

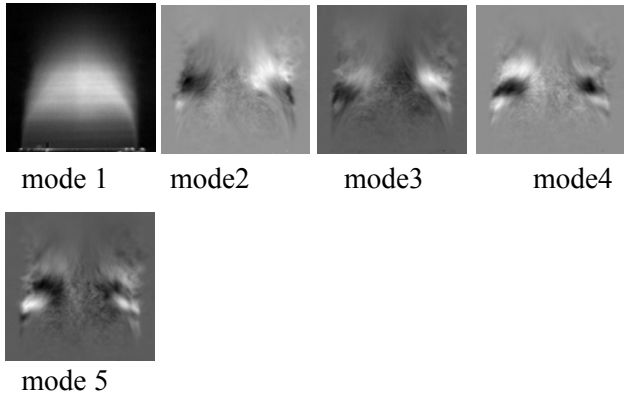
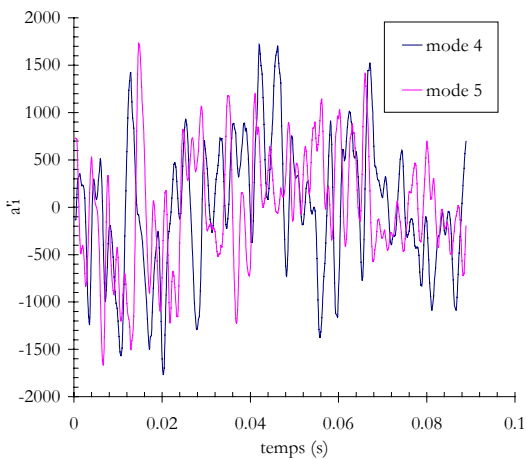
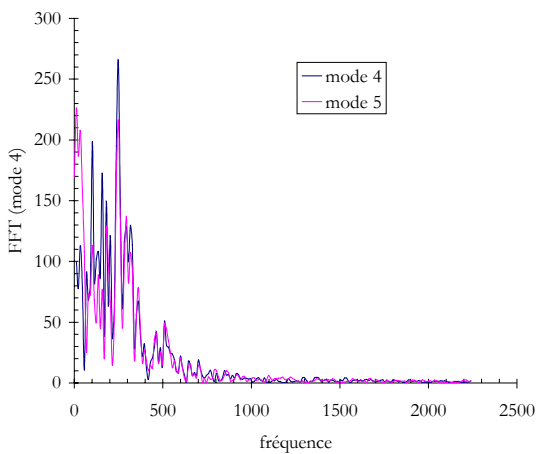


Fig 7: The first 5 modes calculated from P.O.D., for $U_o=8$ m/s.

The mode 4 and mode 5 shows an alternation of black and white zones in the external mixing layer of the annular jet. The temporal evolution of a_4 and a_5 show a frequency equal to 249 Hz for the two modes. We can determine a characteristic distance d_p , as the length of a white, or black zone.



a)



b)

Fig 8 a) Temporal evolution of the reconstruction coefficient of the mode 4 and 5, b) FFT of a).

In Fig 8 a, is plotted the evolution of the reconstruction coefficient of modes 4 and 5. By applying a Direct Fourier Transform, we find a characteristic frequency equal to 249 Hz, for the two modes. The characteristic spatial distance d_p is equal to 0.208 D_o for mode 4 and 0.212 D_o for mode 5. Then, U_c^* is so equal to 0.34. It shows that the two methods give the same order of U_c^* .

5.2 Velocity convection of the round jet

We applied Proper Orthogonal Decomposition (P.O.D.) on a statistic of time-resolved tomographic images of the round jet. The computation has been done with 400 images for $U_o=13.5$ m/s. The first four different modes are displayed in the Fig 9.

The first one has the topology of the mean image of the flow. The other modes are characteristic of a convection flow. The negative values of a mode are represented by a black pixel and positive values by a white one.

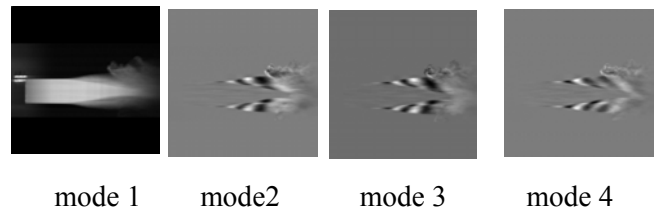


Fig 9 The first 4 modes calculated from P.O.D., for a round jet $U_o=13.5$ m/s.

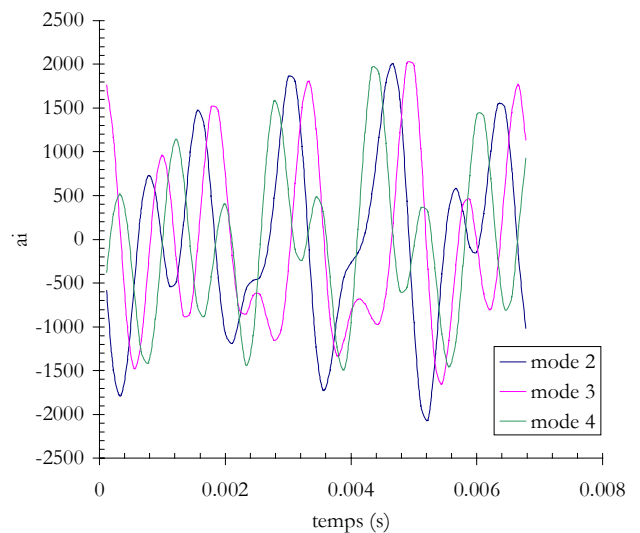


Fig 10 Temporal evolution of the reconstruction coefficient of the mode 2, 3 and 4.

The temporal evolution of a_2 , a_3 and a_4 show a frequency equal to $f=(620\pm 40)$ Hz. We can determine a characteristic distance d_p , as the length of a white, or black zone. For the mode 2, this length is $\Delta x=(21.72 \pm$

0.29) mm. So, $U_c^* = U_c/U_0 = 0.50 \pm 0.05$. This result is characteristic of a round jet (Gutmark [16]). We can note that the value of U_c^* for annular jet is smaller than U_c^* for round jets.

5 Conclusions

Measurements of convection velocities of primary vortices have been carried out, and a new method, from the temporal evolution of a P.O.D. mode applied on images has been used. The results give the same value of U_c^* in the external mixing layer. Whatever U_c^* , the value is smaller than U_c^* for round jets where it equal to 0.5. In our experimental setup, the core potential is shorter than the one in the round jet. So, the local velocity is smaller. Then, the jet converges toward the axis of the jet. This could explain the smaller value of U_c^* than round jet.

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