Proceedings of the 3rd IASME/WSEAS Int. Conf. on HEAT TRANSFER, THERMAL ENGINEERING AND ENVIRONMENT, Corfu, Greece, August 20-22, 2005 (pp6-11)

Modelling of non-steady-state conditions in a gas boiler heated room

DR. LÁSZLÓ GARBAI DR. LAJOS BARNA
Department of Building Service Engineering
Budapest University of Technology and Economics
1111 Budapest, Műegyetem rkp. 9.
HUNGARY
http://www.host.epgep.bme.hu

Abstract: In Hungary, the use of gas is of a significant proportion as compared to other energy resources. In the case of the most generally used „B" type gas boilers, the coordinated operation of the gas appliance and the chimney, as well as the supply of combustion air is an important question. Besides the construction of the chimney, when building envelops meet modern energetic requirements, this coordinated operation also depends on the introduction of the necessary air quantity.
For the appropriate adjustment of the gas boiler to the conditions of the building and its heating system, computer generated calculation results have to be obtained that can help the positioning of the appliance and the elaboration of the heating system. For the examination of the coordinated operation of the gas boiler and the heated space, a mathematical model needs to be developed to describe the non-steady-state conditions of the heating system and the building, and to help examine the non-steady-state condition of the room's air supply – appliance – chimney.

Key words: heating system, gas appliances, chimney, mathematical modelling, air supply

1 Introduction

In Hungary, the use of gas is of a significant proportion as compared to other energy resources. The heating of about 80% of family houses and blocks of flats is provided for by gas boiler operated central heating system. The prevalently used "B" type* gas boilers do not operate continuously but automatically turn on and off depending on the changes of indoor temperature. The temperature of the hot-water heating system connected to the boiler changes in harmony with the operation of the boiler. In the chimney, the extent of the draught depends on the operation of the boiler, and the temperature of the flue gas and the heating water.

For the appropriate adjustment of the gas boiler to the conditions of the building and its heating and ventilation system, computer generated calculation results have to be obtained that can help in choosing the appliance and in the planning of the chimney and the air supply of the room. Our goal is the development of a physical-mathematical model that can be used for examining the non-steady-state condition of the room and the gas boiler, the chimney and the air supply of the room.

* Type "B" appliances are connected to a flue through which combustion products can leave and the combustion air is drawn directly from the room. (CEN report CR 1749)

2 The physical model

The developed physical model is presented in Fig. 1. A "B" type boiler – supplied with an open combustion chamber – supplies a hot-water heating system. The heated space is connected to its environment in part through the heat directed toward the outdoors and in part through the ventilation that develops due to the draught of the chimney. See the main characteristics of the energetic connection below.

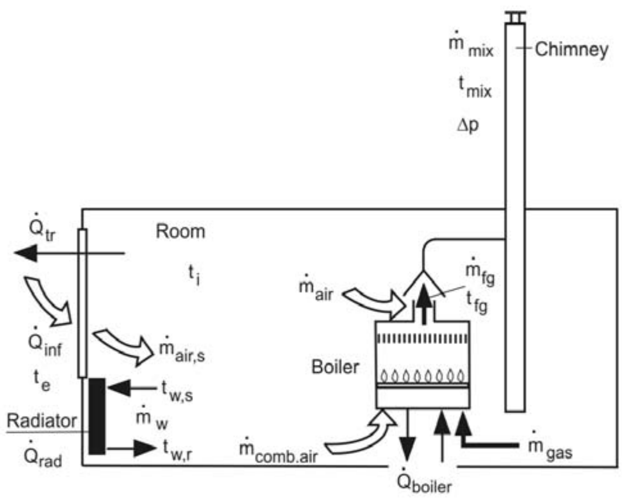

Fig. 1 The physical model of a flat heated by an individual gas boiler

On the basis of the physical model, an energetic block diagram of the flat heated with the gas boiler was developed, which in turn led to the elaboration of the mathematic model. The model can be divided into two main parts: the boiler – heating system – room – walls model and the boiler – chimney – combustion-air supply model (Fig.2). The details of the sub-models are as follows.

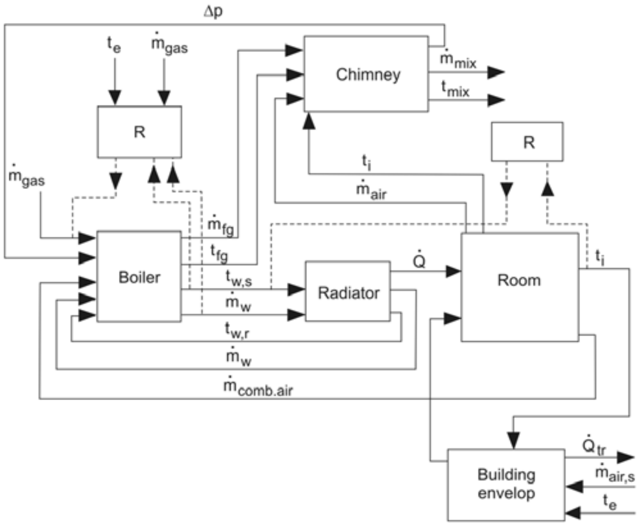

Fig.2. Energetic block diagram of a room heated by a gas boiler

3 The calculation model

3.1. The model of the heating system

The components of the model: the gas boiler, the hot-water heating system, the room and its walls. The heat produced by the gas boiler enters the room through the heating system and the radiator. Depending on the supply and return temperatures and in function of the temperature of the room, the heat transmitted by the radiator and, therefore, the temperature of the air of the room both change.

The initial equations were the following:

– the heat balance of the heat circuit:

$$\frac{d}{d\tau}\left(M_w \cdot \frac{t_{w,s}+t_{w,r}}{2}\right) = \dot{Q}_{boiler} - \dot{Q}_{rad} \qquad (1)$$

– the heat balance of the room:

$$\frac{d}{d\tau}(M_{air} \cdot t_i) = \dot{Q}_{rad} - \dot{Q}_{tr} - \dot{m}_{air} \cdot c_{p,air} \cdot (t_i - t_e)\,, \qquad (2)$$

where

$$\dot{Q}_{rad} = k_{rad} \cdot A_{rad} \cdot \left(\frac{t_{w,s}+t_{w,r}}{2} - t_i\right), \qquad (3)$$

is the heat flow transmitted by the radiator and

$$\dot{Q}_{tr} = k_{wall} \cdot A_{wall} \cdot (t_i - t_e)\,, \qquad (4)$$

is the transmission heat loss of the walls.

With the introduction of a tag for the medium temperature of the heating medium ($t_{w,av}$) and its substitution into equations (1) and (2), the result after the rearrangement of the equations is:

$$\frac{d}{d\tau}t_{w,av} = \frac{\dot{Q}_{boiler}}{M_w} - \frac{k_{rad} \cdot A_{rad}}{M_w} \cdot t_{w,av} + \frac{k_{rad} \cdot A_{rad}}{M_w} \cdot t_i \qquad (5)$$

and

$$\frac{d}{d\tau}t_i = \frac{k_{rad} \cdot A_{rad}}{M_{air}} \cdot (t_{w,av} - t_i) - \frac{k_{wall} \cdot A_{wall}}{M_{air}} \cdot (t_i - t_e) - \frac{\dot{m}_{air} \cdot c_{air}}{M_{air}} \cdot (t_i - t_e) = \frac{k_{rad} \cdot A_{rad}}{M_{air}} \cdot t_{w,av} + \left(-\frac{k_{rad} \cdot A_{rad}}{M_{air}} - \frac{k_{wall} \cdot A_{wall}}{M_{air}} - \frac{\dot{m}_{air} \cdot c_{air}}{M_{air}}\right) \cdot t_i + \left(\frac{k_{wall} \cdot A_{wall}}{M_{air}} \cdot t_e + \frac{\dot{m}_{air} \cdot c_{air}}{M_{air}} \cdot t_e\right) \qquad (6)$$

With the constants combined, the equations can also look as follows:

$$\frac{d}{d\tau}t_{w,av} = a_{11} \cdot t_{w,av} + a_{12} \cdot t_i + a_{13}\,, \text{ and} \qquad (7)$$

$$\frac{d}{d\tau}t_i = a_{21} \cdot t_{w,av} + a_{22} \cdot t_i + a_{23} \qquad (8)$$

Initial conditions in the moment $\tau = 0$:

$$t_{w,av} = t_{w,av0} = \frac{t_{e0} + t_{v0}}{2}\,; \text{ and}$$

$$t_i = t_{i0}\,; \; t_e = t_{e0} = const\,.$$

3.2 The model of the boiler

The energy flow entering the appliance in the moment when the boiler starts operating:

$$\dot{Q} = \dot{m}_{gas} \cdot H_s + \dot{m}_{gas} \cdot c_{p\,gas} \cdot t_i + \dot{m}_{comb.air} \cdot c_{p\,air} \cdot t_i \qquad (9)$$

After the burner:

$$\dot{Q} = (\dot{m}_{gas} + \dot{m}_{comb.air}) \cdot t_{comb} \cdot c_{p\,fg} = \dot{m}_{fg} \cdot t_{comb} \cdot c_{p\,fg} \qquad (10)$$

Flue gas temperature after the burner calculated from equation (1) and (2):

$$t_{comb} = \frac{\dot{m}_{gas} \cdot H_s}{\dot{m}_{fg} \cdot c_{p\,fg}} + t_i \qquad (11)$$

The energy flow leaving the appliance is

$$\dot{Q} = \dot{m}_{fg} \cdot c_{fg} \cdot t_{fg} + \dot{m}_w \cdot c_w \cdot (t_{w,s} - t_{w,r}) + \dot{Q}_{loss} \qquad (12)$$

where

$\dot{Q}_{loss}$ is the heat loss of the appliances caused by convection and radiation.

Neglecting the appliance's convection and radiation losses, the heat flow transmitted by the hot flue gas and the heated water within the heat exchanger is:

$$\dot{m}_{fg} \cdot c_{p,fg} \cdot (t_{comb} - t_{fg}) = \dot{m}_w \cdot c_w \cdot (t_{w,s} - t_{w,r}) = = A_{he} \cdot k_{he} \cdot \Delta t_{av} \tag{13}$$

After reductions and the introduction of constants (A_0, A_1, A_2, A_3), the form of equation (13) is:

$$\dot{m}_{fg} \cdot c_{p,fg} \cdot (t_{comb} - t_{fg}) = = A_0 \cdot A_1 \cdot t_{comb} + A_0 \cdot A_1 \cdot t_{fg} - A_0 \cdot A_2 - A_3 \tag{14}$$

From the equations, the temperature of flue gas leaving the appliance is:

$$t_{fg} = \frac{A_0 \cdot A_2 + A_3}{A_0 \cdot A_1 + \dot{m}_{fg} \cdot c_{p,fg}} \tag{15}$$

3.3 The model of the chimney and the air-inlet

Draft, induced by the density difference at H long, vertical section of the chimney covers: flow resistance of the air inlet, pressure difference necessary to increase the velocity of room air entering the draft hood, the flow resistance of chimney and the wind pressure. Using the tags from Fig. 1:

$$\Delta p = g \cdot H \cdot (\rho_e - \rho_{mix.av}) = \Delta p_{air,in} + \frac{\rho}{2} w_{air}^2 + + \Delta p_{draft\,hood} + \Delta p_{chimney} + \Delta p_{wind} . \tag{16}$$

Assuming that the characteristic of the air inlet is:

$$\Delta p = C_1 \cdot \dot{m}_{air,s}^2$$

where C_1 = const.,

equation (16) will become:

$$g \cdot H \cdot (\rho_e - \rho_{mix.av}) = C_1 \cdot \dot{m}_{air,s}^2 + + \left(\frac{\dot{m}_{air}}{A_{draft\,hood} \cdot \rho_i}\right)^2 \frac{\rho_i}{2} + + \zeta_{draft\,hood} \left(\frac{\dot{m}_{air}}{A_{draft\,hood} \cdot \rho_i}\right)^2 \frac{\rho_i}{2} + + \left(\lambda \frac{H}{D} + \zeta\right) \cdot \left(\frac{\dot{m}_{mix}}{A \cdot \rho_{mix,av}}\right)^2 \frac{\rho_{mix,av}}{2} + C_2 , \tag{17}$$

where $C_2 = \Delta p_{wind}$ = const.

After simplifying and reducing equation (17), the result is:

$$g \cdot H \cdot \frac{p_e}{R_{air} \cdot T_e}\left(1 - \frac{T_e}{T_{mix,av}}\right) = C_1 \cdot \dot{m}_{air,s}^2 + + (1 + \zeta_{draft\,hood}) \cdot \left(\frac{1}{2 \cdot A_{draft\,hood}^2}\right) \frac{R_{air} \cdot T_i}{p_i} \cdot \dot{m}_{air}^2 + + \left(\lambda \frac{H}{D} + \zeta\right) \cdot \left(\frac{1}{2 \cdot A^2}\right) \frac{R_{air} \cdot T_{mix,av}}{p_{mix,av}} \cdot \dot{m}_{mix}^2 + C_2 . \tag{18}$$

In order to receive an equation easier to handle, the following notations are introduced:

$$g \cdot H \cdot \frac{p_e}{R_{air} \cdot T_e} = B_0 ,$$

$$(1 + \zeta_{draft\,hood}) \cdot \left(\frac{1}{2 \cdot A_{drafthood}^2}\right) \frac{R_{air} \cdot T_i}{p_i} = B_1 ,$$

$$\left(\lambda \frac{H}{D} + \zeta\right) \cdot \left(\frac{1}{2 \cdot A^2}\right) \frac{R_{air}}{p_{mix,av}} = B_2 ,$$

Thus, the new form of the equation (14) is:

$$1 - \frac{T_e}{T_{mix,av}} = \frac{C_1}{B_0} \cdot \dot{m}_{air,s}^2 + \frac{B_1}{B_0} \cdot \dot{m}_{air}^2 + + \frac{B_2}{B_0} \cdot T_{mix,av} \cdot \dot{m}_{mix}^2 + \frac{C_2}{B_0} . \tag{19}$$

As

$\dot{m}_{air,s} = \dot{m}_{comb.air} + \dot{m}_{air}$ és $\dot{m}_{mix} = \dot{m}_{fg} + \dot{m}_{air}$

equation (19) expressed in a different way is:

$$1 - \frac{T_e}{T_{mix,av}} = \frac{C_1}{B_0} \cdot (\dot{m}_{comb.air} + \dot{m}_{air})^2 + \frac{B_1}{B_0} \dot{m}_{air}^2 + + \left(\frac{B_2}{B_0} \cdot T_{mix,av}\right) \cdot (\dot{m}_{fg} + \dot{m}_{air})^2 + \frac{C_2}{B_0} . \tag{20}$$

After raising the equation to second power and rearranging it:

$$\frac{T_e}{T_{mix,\,av}} = 1 - \left(\frac{C_1}{B_0} + \frac{B_1}{B_0} + \frac{B_2}{B_0} \cdot T_{mix,\,av}\right) \cdot \dot{m}_{air}^2 - - \left(\frac{2C_1}{B_0} \dot{m}_{comb.air} + \frac{2B_2}{B_0} \cdot \dot{m}_{fg} \cdot T_{mix,0} + + \frac{2B_3}{B_0} \cdot \dot{m}_{fg} \cdot T_{mix,\,av}\right) \cdot \dot{m}_{av} - - \left(\frac{C_1}{B_0} \dot{m}_{comb.air}^2 + \frac{B_3}{B_0} \cdot \dot{m}_{fg}^2 \cdot T_{mix,\,av}\right) - \frac{C_2}{B_0} . \tag{21}$$

As the next step, the constants in equation (21) are substituted with the constants shown below:

$$a_0 = 1 - \frac{C_2}{B_0}\ ,\ a_1 = \frac{C_1}{B_0} + \frac{B_1}{B_0}\ ,\ a_2 = \frac{B_2}{B_0}\ ,\ a_3 = \frac{B_3}{B_0}\ ,$$

$$a_4 = \frac{2C_1}{B_0} \cdot \dot{m}_{\text{comb.air}},\ a_5 = \frac{2B_2}{B_0} \cdot \dot{m}_{\text{fg}}\ ,\ a_6 = \frac{2B_3}{B_0} \cdot \dot{m}_{\text{fg}}\ ,$$

$$a_7 = \frac{C_1}{B_0} \cdot \dot{m}^2_{\text{comb.air}},\ a_8 = \frac{B_2}{B_0} \cdot \dot{m}^2_{\text{fg}}\ ,\ a_9 = \frac{B_3}{B_0} \cdot \dot{m}^2_{\text{fg}}\ ,$$

The new form of the equation is:

$$\frac{T_\text{e}}{T_{\text{mix,av}}} = a_0 - (a_1 + a_2 \cdot T_{\text{mix,0}} + a_3 \cdot T_{\text{mix,av}}) \cdot \dot{m}^2_{\text{air}} - \\ - (a_4 + a_5 \cdot T_{\text{mix,0}} + a_6 \cdot T_{\text{mix,av}}) \cdot \dot{m}_{\text{air}} - \\ - (a_7 + a_8 \cdot T_{\text{mix,0}} + a_9 \cdot T_{\text{mix,av}})\ . \tag{22}$$

The unknowns in equation (22) are:

$$\dot{m}_{\text{air}}\ ,\ T_{\text{mix,0}}\ \text{és}\ T_{\text{mix,av}}\ ,$$

while the other factors can be determined from the combustial equations and from the flow and geometric characteristics of the combustion gas outlet.

Further equations are:

– Initial temperature of flue gas after mixing combustion air and room air:

$$T_{\text{mix}}\big|_{z=0} = T_{\text{mix,0}} = \frac{\dot{m}_{\text{fg}} \cdot T_{\text{fg}} + \dot{m}_{\text{air}} \cdot T_\text{i}}{\dot{m}_{\text{fg}} + \dot{m}_{\text{air}}}\ . \tag{23}$$

– The medium temperature of the mixture flowing through the chimney can also be described on the basis of the temperature of the mixture that enters and then leaves the chimney (if we conceive the chimney as a heat exchanger):

$$T_{\text{mix, av}} = \frac{T_{\text{mix,0}} + T_{\text{mix}}(H)}{2} = T_{\text{mix,0}} - \\ - \frac{1}{2}(T_{\text{mix,0}} - T_\text{o})e^{-\frac{k \cdot U}{(\dot{m}_{\text{fg}} + \dot{m}_{\text{air}}) \cdot c_{\text{p,mix}}} H} \tag{24}$$

Equations (22), (23) and (24) form an equation system. It can be used for the examination and sizing of chimneys as regards thermal and fluid dynamics but can also be utilised for the determination of the chimney's operating point when the heating system connected to the appliance is in a non-steady-state.

4 Steps of calculation

The steps of modelling were the following:

• Initially, the gas boiler is not in operation, a temperature evolves in the room, and a given air current develops through the gas appliance and the chimney.

• At the sign of the room thermostat, at a $\Delta\tau = 0$ moment, the gas appliance starts its operation and transmits ΔQ heat quantity into its surroundings during a $\Delta\tau$ period. As a result, the supply water temperature increases. At the flue gas outlet of the appliance, a flue gas mass flow of t_{fg} temperature appears. The air of the room mixes to this in the draft hood and an initial mixture temperature develops together with an increasing flow through the chimney.

• The operating point of the chimney can be calculated with iteration cycles by repeating the following steps:

1. From the heat load of the gas appliance, as given by the manufacturer, it is possible to calculate the mass flow of the flue gas which leaves the heat exchanger.

2. The temperature of flue gas can be calculated from equation (15) according to the actual return temperature of the heating circuit.

3. By assuming an initial mass flow value for the combustion air entering the draft hood, $t_{\text{mix,0}}$ and $t_{\text{mix,av}}$ can be calculated from (23) and (24), respectively.

4. Room air mass flow can be determined with equation (22) and can be compared with the assumed value.

5. Should they differ, air mass flow has to be modified and the above described iteration process has to be repeated. Iteration is to be continued until desired accuracy is met. At which point, the data for the chimney's operating point is available for the given condition of the heating circuit.

6. The mass flow of combustion air has to be recalculated and checked by taking into account the pressure difference between what is calculated in the draft hood and what is measured in the room, as well as the pressure difference necessary to accelerate the combustion air and the flow resistance of the gas appliance.

To calculate the next chimney operating point for the given heating circuit, the above described method has to be repeated.

5 Calculation results

Based on the model, a calculation program has been created which can calculate the mass-flow of the room air entering the draft hood, though not yet the heat balance of the heating system and the room. The program was used in order to determine the chimney point and the mass-flow of the room air entering the draft hood when a gas appliance is turned on and the return water temperature is still cold. Afterwards the above described factors were examined for the case when the return water temperature is already hot.

Fig.3 reveals an example of the results of the calculation. The mean flue gas temperature, the draft caused by the density difference in the vertical section of the chimney and therefore the mass flow of the room air all steadily increase.

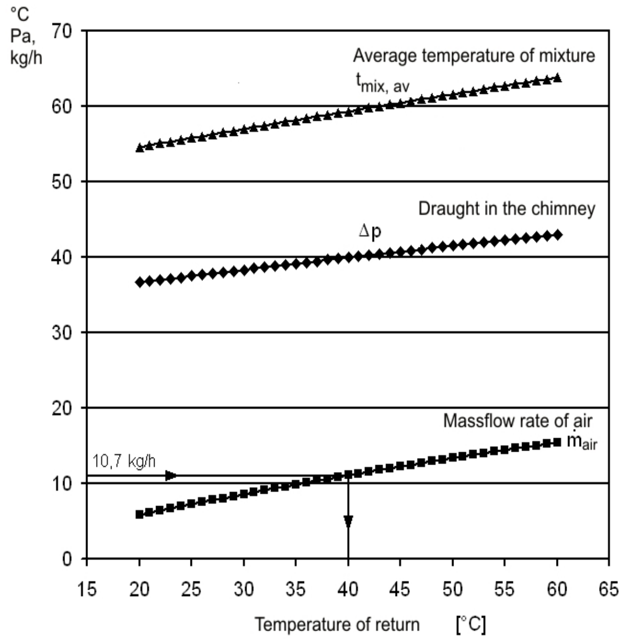

Fig.3 Calculation results

6 Examination of the non-steady-state process through measurement

Establishing the initial data for modelling requires measurement data which describe the operating parameters of the gas appliances, with special regard to non-steady-state operating conditions. Therefore, measurements have been conducted to describe the operating conditions of the boiler and the chimney.

The following temperature measurement points were established on the equipment in the university department's gas technical laboratory:

- the temperature of the flue gas leaving the gas appliance,
- the temperature of the flue gas–air mixture close to the point it entered the flue pipe,
- the inner wall temperature of the flue liner once the mixture got into the funnel,
- close to the chimney outlet, along the diameter, the temperature of the gas–air mixture and the inner wall temperature of the flue liner.

As follows, the changes of the characteristic temperature data of a Vaillant VU18 type wall mounted boiler and the adjoining chimney are presented. In the initial state, the boiler is cold. In the course of the measurement, the boiler and the chimney heat up. Once the operating balance is reached, the boiler shuts down followed by a period of falling temperature.

The measured environmental, gas- and water-side characteristics:

t_{room}	=	21,1	°C
t_{gas}	=	18,7	°C
p_{gas}	=	3040	Pa
$t_{w,supply}$	=	51,1	°C
$t_{w,return}$	=	28,3	°C
m_w	=	520	Litre/h

Figures 4, and 5 indicate a few measurement results.

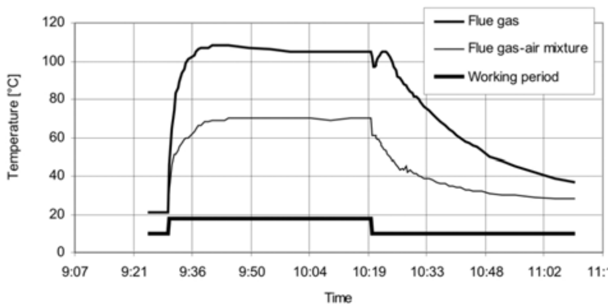

Fig.4 Temperature change around the appliance

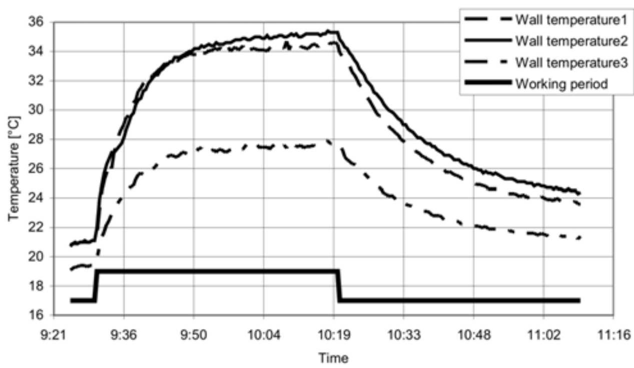

Fig.5 Temperature change at the chimney entrance

7 Conclusions

In Hungary, in the case of the generally used „B" type gas boilers, the coordinated operation of the gas appliance and the chimney, as well as the supply of combustion air is an important question.

For the examination of the coordinated operation of the gas boiler and the heated space, a mathematical model needs to be developed to help describe the non-steady-state conditions of the heating system and the building, and can also be used also for the examination of the non-steady-state condition of the room's air supply – appliance – chimney.

Based on the model, a calculation program has been created which can calculate the mass-flow of the room air entering the draft hood, though not yet the heat balance of the heating system and the room.

Establishing the initial data for modelling requires measurement data. Therefore, measurements have been conducted to describe the operating conditions of the boiler and the chimney.

The measurements conducted so far clearly demonstrate the non-steady character of the early stage of the process, namely that the diluted flue gas temperature and the inner wall temperature constantly change in the case of intermittently operating boilers.

Further steps of modelling include the comparison of calculated and measured parameters and improvements on the model.

Nomenclature:

A flow cross-section, m^2;
c_p specific heat, J/kg, K;
D the characteristic size of the chimney (diameter), m;
g gravity acceleration, m/s^2;
H average height of the chimney, m;
H_s combustion heat of the natural gas, J/m^3;
k heat transmission coefficient, J/m^2,K;
$\dot{m}$ mass flow rate, kg/s;
M heat capacity
p pressure, Pa;
$\dot{Q}$ energy flow rate, heat flow rate, W;
R gas constant, J/kg,K;
t temperature, °C;
T absolute temperature, K;
U specific surface of the wall of the chimney (surface/length), m;
V volume, m^3;
V_{gas} natural gas mass flow entering the gas appliance, m^3/s;
w speed, m/s;

Greek symbols

v specific volume, m^3/kg;
λ friction factor (-);
ρ density, kg/m^3;

Subscripts

av average
comb combustion
e outside
fg flue gas
i inside
mix mixture
r return
rad radiator
s supply
w water

References:

[1] Barna, L., Poroszlai, I., Légbevezető elemek alkalmazása gázkészülékek helyiségeinek szellő-zőlevegő ellátására (Application of Air Inlets for Ventilating-Air Supply of Rooms with Gas Appliances), in: *Magyar Épületgépészet*, Vol. L 2001/8, p. 7-10.

[2] Barna, L., Gázkészülékek égési levegő ellátásá-nak méretezése 1. rész: „A" típusú gázkészülékek (Sizing of Air Supply in the Room of Gas Appliances, 1st Part: „A" Type Gas Appliances), in: *Magyar Épületgépészet*, Vol. LI 2002/3, p. 27-30.

[3] Barna, L., Gázkészülékek égési levegő ellátásá-nak méretezése 2. rész: „B" típusú gázkészülékek (Sizing of Air Supply in the Room of Gas Appliances, 2nd Part: „B" Type Gas Appliances), in: *Magyar Épületgépészet*, Vol. LI 2002/8, p. 25-28.

[4] Homonnay, G. (ed.), *Épületgépészet 2000 Vol. I, Fűtéstechnika* (HVAC 2000 Vol. I, Heating Technic, Épületgépészet Kiadó, Budapest, 2001.

[5] Krope, J., Goricanec, D., Analysis of pipe networks including pumps. *Energy build.*. [Print ed.], 1991, Vol. 17, pp. 141-145.

[6] Garbai, L. – Barna, L. – Vigh, G., Modelling of Instacioner Conditions in a Space Heated by an Individual Gas Boiler, *Gépészet 2004 Conference*, Budapest University of Technology and Economics, 2004, Proceedings pp. 264-268.

[7] Garbai, L. – Barna, L., Air Supply Modelling in a Room Heated by an Individual Gas Boiler, *Energy for Buildings 6th International Conference*, Vilnius, Lithuania, 2004, Proceedings pp. 428-435.