Proceedings of the 3rd IASME/WSEAS Int. Conf. on HEAT TRANSFER, THERMAL ENGINEERING AND ENVIRONMENT, Corfu, Greece, August 20-22, 2005 (pp1-5)

Hydraulic analysis of vertical two-pipe central heating networks

DR. LÁSZLÓ GARBAI, DR. LAJOS BARNA, DR. ZOLTÁN SZÁNTHÓ
Department of Building Service Engineering
Budapest University of Technology and Economics
1111 Budapest, Műegyetem rkp. 9-11.
HUNGARY
http://www.host.epgep.bme.hu

Abstract: - The study presents calculation methods for the determination of pressure and mass flow distribution, in consideration of the gravitational effect in vertical central heating networks. In the case of the prescribed flows, the pressure and mass flow distributions as well as the pump operating point are calculated. The forming pressure and mass flow distribution as well as the heating medium mass flow that reaches the radiators are determined with regard to prescribed or known pump operating points. The calculation method is based on node and loop equations and the ensuing non-linear equation system.

Keywords: - Hydraulic analysis, Central heating, Heating network, Gravitational effect, Pump characteristics

1 Introduction

The modernization of heating systems of high buildings started in Hungary in the past years. In the course of this process, the technological equipment of DH substations and the pump are generally replaced, but pipes can also be changed, new pipe sections can be added, and thermostatic valves can be installed. Prior to these works it is essential to conduct a hydraulic analysis of the central heating network (Garbai, L. [2], [8], Krope, J. and Goricanec, D. [3]).

In order to determine the characteristics of the fluid flow, exact calculations need to be done during the reconstruction of the heating systems. It is important to know, if the designed network is suitable to supply the demands of the consumers and to transport the planned hot water flows. We need, furthermore, to know what pump operating points will be obtained on the impact of the customer's interference.

It is important to consider the buoyancy effect in the computational model, because can be significant in the case of tall buildings and its magnitude with respect to the lift of the pump can not be disregarded. Practice shows that the buoyancy effect can cause unpredicted and unexplained flow phenomena in heating systems of tall buildings.

In our study we extend the method of hydraulic analysis to central heating networks.

The studied networks are two pipe and naturally looped systems. The described calculation method can be used to determine the flow characteristics for both symmetrical and asymmetrical network layouts.

It is necessary to point out that our investigation for central heating networks has not been mentioned previously in the literature and therefore counts as new scientific accomplishment.

In this paper – due to page restrictions – only the calculation method is described. Reporting specific calculation results was not possible. The results can differ significantly depending on the size of the network and the height of the building.

There are two groups of problems that can emerge in practice:

– a basic problem: the examination of the evolving flow pattern and consumption in the case of given input characteristics (pump character and characteristics) and network characteristics (coefficients of flow resistance), the examination of consumption possibilities, and the classification of the network.
– the determination of the optimal input characteristics (pump characteristics, operating point), which would produce minimal operating costs in the case of prescribed consumption or consumption possibilities and given network characteristics.

The solution of the problem was applied to a network containing four radiators (Fig. 1). According to this example the method can be applied to an optional number of networks.

2 Hydraulic analysis of vertical central heating networks

The gravitational effects caused by a density difference between the supply and return hot water density are to be considered in the hydraulic analysis. See network arrangement in Fig.1.

– **The basic challenge is to determine the evolving flow and pressure pattern in the case of a known pump operating point.**
  The geometric characteristics, with which the hydraulic resistance coefficients of the sections can be calculated, are given. The lift of the pump is known (Δp_{pump}).

The procedure is presented for the so-called nominal condition: the supply water temperature is 90°C and the return water temperature is 70°C. The medium temperature of the radiators is taken to be 80°C.

A system of node and loop equations is needed for the solution of the problem.

- **Node equations**

For node (3)

$$\dot m_{53} - \dot m_1 - \dot m_2 = 0 \text{, or } \dot V_{53} - \dot V_1 - \dot V_2 = 0 \,. \quad (1)$$

For node (5)

$$\dot m_{75} - \dot m_3 - \dot m_{53} = 0 \text{, or } \dot V_{75} - \dot V_3 - \dot V_{53} = 0 \,. \quad (2)$$

For node (7)

$$\dot m_{97} - \dot m_4 - \dot m_{75} = 0 \text{, or } \dot V_{97} - \dot V_4 - \dot V_{75} = 0 \,. \quad (3)$$

- **Loop equations**

For loop (1)

$$p_4 = p_3 - l_{13}\,\rho_{90}\,g + h\,\rho_{80}\,g + l_{24}\,\rho_{70}\,g - \\ - \Delta p_{1,3} - \Delta p_{1,1'} - \Delta p_{1\,rad} - \Delta p_{2',2} - \Delta p_{2,4}\,, \quad (4)$$

$$p_4 = p_3 + h\,\rho_{80}\,g + \Delta p_{3,3'} - \Delta p_{2\,rad} - \Delta p_{4',4}\,. \quad (5)$$

The two equations after subtraction

$$h\,\rho_{80}\,\mathrm{g} - \Delta \mathrm{p}_{3,3'} - \Delta p_{2\,\mathrm{rad}} - \Delta p_{4',4} + l_{13}\,\rho_{90}\,g - \\ - h\,\rho_{80}\,\mathrm{g} - l_{24}\,\rho_{70}\,\mathrm{g} + \Delta p_{13} + \Delta p_{1,1'} + \Delta p_{1\,\mathrm{rad}} + \\ + \Delta p_{2',2} + \Delta p_{2,4} = 0\,. \quad (6)$$

Following reduction, assuming that $l_{13} \approx l_{2,4}$

$$- \Delta p_{3,3'} - \Delta p_{2\,\mathrm{rad}} - \Delta p_{4',4} + l_{13}\,(\rho_{90} - \rho_{70}) \cdot g + \\ + \Delta p_{13} + \Delta p_{1,1'} + \Delta p_{1\,\mathrm{rad}} + \Delta p_{2',2} + \Delta p_{2,4} = 0\,. \quad (7)$$

For loop (2)

$$p_6 = p_5 - l_{35}\,\rho_{90}\,g + l_{46}\,\rho_{70}\,g + h\,\rho_{80}\,g - \\ - \Delta p_{3,5} - \Delta p_{3,3'} - \Delta p_{2\,rad} - \Delta p_{4',4} - \Delta p_{4,6} \quad (8)$$

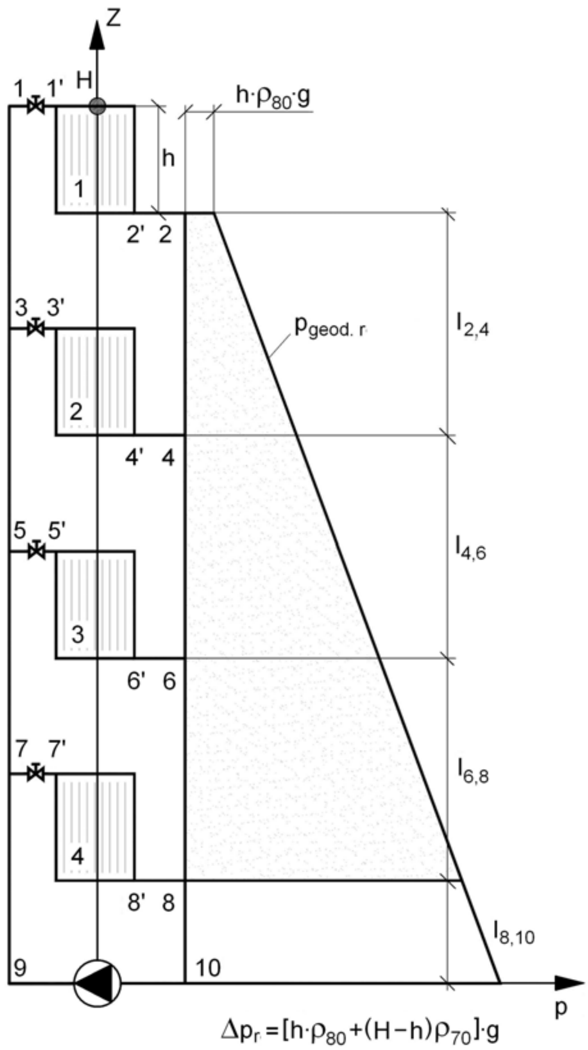

Fig.1

$$p_6 = p_5 + h\,\rho_{80}\,g - \Delta p_{5,5'} - \Delta p_{3\,rad} - \Delta p_{6',6}\,. \quad (9)$$

The two equations after subtraction and reduction:

$$- \Delta p_{5,5'} - \Delta p_{3,\mathrm{rad}} - \Delta p_{6',6} + l_{3,5}\,(\rho_{90} - \rho_{70}) \cdot g + \\ + \Delta p_{3,5} + \Delta p_{3,3'} + \Delta p_{2,\mathrm{rad}} + \Delta p_{4',4} + \Delta p_{4,6} = 0\,. \quad (10)$$

For loop (3)

$$p_8 = p_7 - l_{5,7}\,\rho_{90}\,g + l_{6,8}\,\rho_{70}\,g + h\,\rho_{80}\,g - \\ - \Delta p_{5,7} - \Delta p_{5,5'} - \Delta p_{3,\,rad} - \Delta p_{6,6'} - \Delta p_{6,8}\,, \quad (11)$$

$$p_8 = p_7 - \Delta p_{7',7} - \Delta p_{4,\,rad} - \Delta p_{8,8'} + h\,\rho_{80}\,g\,. \quad (12)$$

The two equations after subtraction and reduction:

$$- \Delta \mathrm{p}_{7',7} - \Delta p_{4,\,\mathrm{rad}} - \Delta p_{8,8'} + l_{5,7}\,(\rho_{90} - \rho_{70}) \cdot g + \\ + \Delta \mathrm{p}_{5,7} + \Delta p' + \Delta p_{3,\,\mathrm{rad}} + \Delta p' + \Delta p_{6,8} = 0\,. \quad (13)$$

For loop (4)

$$p_{10} = p_9 - l_{7,9}\,\rho_{90}\,g - \Delta p_{7,9} - \Delta p_{7',7} - \Delta p_{rad,4} - \\ - \Delta p_{8,8'} - \Delta p_{8,10} + h\,\rho_{80}\,g + l_{8,10}\,\rho_{70}\,g\,, \tag{14}$$

$$p_{10} = p_9 - \Delta p_{pump}\,. \tag{15}$$

The two equations after subtraction and reduction:

$$-\Delta p_{pump} + (l_{7,9}\,\rho_{90} - l_{8,10}\,\rho_{70})\cdot g + h\,\rho_{80}\,g + \\ + \Delta p_{7,9} + \Delta p_{8,10} + \Delta p_{4,rad} + \Delta p_{7',7} + \Delta p_{8,8'} = 0\,. \tag{16}$$

Unknowns in the equations: $\dot V_1$, $\dot V_2$, $\dot V_3$, $\dot V_4$.

$\dot V_{9,7}$ equals to the sum of the demand of radiators:

$$\dot V_{9,7} = \dot V_1 + \dot V_2 + \dot V_3 + \dot V_4\,. \tag{17}$$

The unknowns can be determined with loop equations in a way that every Δp is to be substituted with the expression:

$$\Delta p = R\cdot \dot V^2 \tag{18}$$

where $R = 8\cdot\left(\lambda\frac{l}{d} + \Sigma\varsigma\right)\cdot\rho\frac{1}{d^4\pi^2}$,

because $\Delta p = \left(\lambda\frac{l}{d} + \Sigma\varsigma\right)\cdot\frac{\rho}{2}w^2$

and $\dot V = \frac{d^2\pi}{4}w.$

The equation system to be solved is thus the following:

$$-R_{3,3'}\dot V_2^2 - R_{2,rad}\,\dot V_2^2 - R_{4',4}\,\dot V_2^2 + \\ + l_{1,3}(\rho_{90} - \rho_{70})\cdot g + \\ + R_{1,3}\,\dot V_1^2 + R_{1',1}\,\dot V_1^2 + R_{1,rad}\,\dot V_1^2 + \\ + R_{2',2}\,\dot V_1^2 + R_{2,4}\,\dot V_1^2 = 0\,, \tag{19}$$

$$-R_{5,5'}\dot V_3^2 - R_{3,rad}\,\dot V_3^2 - R_{6',6}\,\dot V_3^2 + \\ + l_{5,3}(\rho_{90} - \rho_{70})\cdot g + R_{5,3}\,(\dot V_1 + \dot V_2)^2 + \\ + R_{3,3'}\,\dot V_2^2 + R_{2,rad}\,\dot V_2^2 + R_{4,4}\,\dot V_2^2 + \\ + R_{4,6}\,(\dot V_1 + \dot V_2)^2 = 0\,, \tag{20}$$

$$-R_{7',7}\,\dot V_4^2 - R_{4,rad}\,\dot V_4^2 - R_{8,8'}\,\dot V_4^2 + \\ + l_{5,7}(\rho_{90} - \rho_{70})\cdot g + R_{7,5}\,(\dot V_1 + \dot V_2 + \dot V_3)^2 + \\ + R_{5,5'}\,\dot V_3^2 + R_{3,rad}\,\dot V_3^2 + R_{6,6'}\,\dot V_3^2 + \\ + R_{6,8}\,(\dot V_1 + \dot V_2 + \dot V_3)^2 = 0\,, \tag{21}$$

$$-\Delta p_{pump} + (l_{7,9}\,\rho_{90} - l_{8,10}\,\rho_{70})\cdot g - \\ - h\,\rho_{80}\,g + R_{7,9}\,(\dot V_1 + \dot V_2 + \dot V_3 + \dot V_4)^2 + \\ + R_{8,10}\,(\dot V_1 + \dot V_2 + \dot V_3 + \dot V_4)^2 + \\ + R_{7,7'}\,\dot V_4^2 + R_{4,rad}\,\dot V_4^2 + R_{8,8'}\,\dot V_4^2 = 0 \tag{22}$$

Following further reductions:

$$-(R_{3,3'} + R_{2,rad} + R_{4,4'})\,\dot V_2^2 + \\ + (R_{1,3} + R_{1',1} + R_{1,rad} + R_{2',2} + R_{2,4})\,\dot V_1^2 + \\ + l_{1,3}(\rho_{90} - \rho_{70})\cdot g = 0\,, \tag{23}$$

$$-(R_{5,5'} + R_{3,rad} + R_{6,6'})\,\dot V_3^2 + \\ + (R_{5,3} + R_{4,6})\,(\dot V_1 + \dot V_2)_1^2 + \\ + (R_{3,3'} + R_{2,rad} + R_{4',4})\,\dot V_2^2 + \\ + l_{5,3}(\rho_{90} - \rho_{70})\cdot g = 0\,, \tag{24}$$

$$-(R_{7',7} + R_{4,rad} + R_{8',8})\,\dot V_4^2 + \\ + (R_{7,5} + R_{6,8})\,(\dot V_1 + \dot V_2 + \dot V_3)^2 + \\ + (R_{5',5} + R_{3,rad} + R_{6',6})\,\dot V_3^2 + \\ + l_{5,7}(\rho_{90} - \rho_{70})\cdot g = 0\,, \tag{25}$$

$$-\Delta p_{pump} + \\ + (R_{7,9} + R_{8,10})\cdot(\dot V_1 + \dot V_2 + \dot V_3 + \dot V_4^2)^2 + \\ + (R_{7',7} + R_{4,rad} + R_{8',8})\,\dot V_4^2 + \\ + (l_{7,9}\,\rho_{90} - l_{8,10}\,\rho_{70})\cdot g - h\,\rho_{80}\,g = 0\,. \tag{26}$$

The received non-linear equation system can be solved, for example, with serial approximation. We establish the value of $\dot V_1$, following which we will be able to determine the value of $\dot V_2$, $\dot V_3$, and $\dot V_4$ with equations (23), (24), and (25). These values are then substituted in equation (26), and then it can be verified whether the equation is fulfilled. Should it not be fulfilled, the procedure is to be repeated with a different $\dot V_1$ value.

Another problem can also appear where the lift of the pump is not specified but instead the characteristic of the pump is given. It is practical to describe this problem with a parabolic approximation seen below:

Proceedings of the 3rd IASME/WSEAS Int. Conf. on HEAT TRANSFER, THERMAL ENGINEERING AND ENVIRONMENT, Corfu, Greece, August 20-22, 2005 (pp1-5)

$$\Delta p_{\text{pump}} = A\left(\dot{V}_1 + \dot{V}_2 + \dot{V}_3 + \dot{V}_4\right)^2 + $$
$$+ B\left(\dot{V}_1 + \dot{V}_2 + \dot{V}_3 + \dot{V}_4\right) + C$$

Then in equation (26) Δp needs to be replaced with the above formula. The obtained equation system can be solved with the following $\dot{V}_1, \dot{V}_2, \dot{V}_3$, and $\dot{V}_4{}^2$ unknowns.

– **The inverse problem: it is a task of balancing, to determine the pressure pattern, the necessary chokes, and the pump operating point for the prescribed consumptions**

If $\dot{V}_1$, $\dot{V}_2$, $\dot{V}_3$ and $\dot{V}_4$ are given or prescribed, then the pump lift needed for circulation, the pressure pattern of the network, and the necessary chokes can be determined:

Equations (23) … (26) are to be summed up. Following reduction, these equations are received:

$$\Delta p_{pump} = (l_{7,9} + l_{5,7} + l_{3,5} + l_{1,3}) \cdot \rho_{90} \cdot g +$$
$$+ R_{7,9}(\dot{V}_1 + \dot{V}_2 + \dot{V}_3 + \dot{V}_4)^2 + R_{5,7}(\dot{V}_1 + \dot{V}_2 + \dot{V}_3)^2 +$$
$$+ R_{3,5}(\dot{V}_1 + \dot{V}_2)^2 + R_{1,3}\dot{V}_1^2 -$$
$$- (l_{2,4} + l_{4,6} + l_{6,8} + l_{8,10}) \cdot \rho_{70} \cdot g - h\,\rho_{80}\,\mathrm{g}\,. \tag{27}$$

As the next step, we shall return to the system of equations (23) … (26), from which $R_{7,7}$, $R_{5,5}$, $R_{3,3}$, and $R_{1,1}$ choke factors are calculated, and on the basis of which the pressure drop can be determined for the sections and the pressure pattern can be established.

– **Analysis of gravitational effects and lift**

In a circulation realized exclusively on the basis of gravitational effects, circulation may stop in the radiator on the lowest floor. Circulation is unstable. Sufficiently stable heating cannot be realized in multi-storied buildings on the basis of the gravitational principle.

– **The influence of gravitational effects and lift in pump heating**

The effects of gravitation 'de-balance' the system, which then needs to be re-balanced. The pressure pattern can become divergent and greater choke is to be applied on the higher floors than on the lower ones. Gravitational effects can become significant, which should be compensated with chokes in order to avoid change in the prescribed heating water distribution. If we want to take the gravitational effect into account when selecting the pump, then its smallest value is to be considered, that is, by taking the supply and return water temperature present in the early phase of the heating period. Otherwise this could cause troubles in terms of supply.

3 Conclusion

It is necessary to elaborate a calculation method for vertical central heating networks in order to be able to determine the pressure and mass flow distribution, taking into account the gravitational effect.

Such a calculation method has recently been developed recently. It is based on node and loop equations and on ensuing non-linear equation system.

Calculations show that gravitational effects can become significant, which need to be compensated with balancing valves in order to avoid change in the prescribed heating water distribution. If we want to take the gravitational effect into account when selecting the pump, then its smallest value is to be considered, that is, by taking the supply and return water temperature present in the early phase of the heating period, or else this could cause troubles in terms of supply.

Symbols:

d – pipe diameter
g – acceleration due to gravity
h – heights
l – pipe length
$\dot{m}$ – mass flow rate
p – pressure
Δp – pressure difference
R – coefficient of flow resistance
t – temperature
$\dot{V}$ – volume flow rate
w – velocity

Greek symbols

ρ – density
λ – coefficient of flow resistance due to friction
ζ – drag coefficient

Indices:

rad – radiator
r – return
90, 80, 70 – water temperature in the pipe, °C

References:

[1] Jeppson, W. R.: *Analysis of flow in pipe networks.* Ann Arbor Science Publishers, 1976, Ann Arbor, Michigen, 48106

[2] Garbai, L., Dezső, Gy.: *Áramlás energetikai cső-vezeték rendszerekben* (Flow in Energetic Pipe-lines), Műszaki Könyvkiadó, Budapest, 1986

[3] Krope, J., Goricanec, D.: Analysis of pipe networks including pumps *Energy build.* [Print ed.], 1991, 17, pp. 141-145.

[4] Lajos, T., *Az áramlástan alapjai* (Basis of Fluid Mechanics), Műegyetemi Kiadó, Budapest, 2000

[5] Krope, A., Goricanec, D., Krope, J., Nacrtovanje drevesne poti in dimenzioniranje prenosnega cevnega omrezja na podlagi najmanjsih stroskov (Tree path design and dimensioning of transport pipe networks on the basis of minimized costs). *Stroj. Vestn*., 2000, Vol. 46, pp. 573-57

[6] Roos, Hans: *Hydraulik der Wasserheizung*, 5. korr. Aufl. Oldenburg Verlag, 2002

[7] Halász, G., Kristóf, G., Kullmann, L., *Áramlás csőhálózatokban* (Flow in Pipe Networks) Műegyetemi Kiadó, Budapest, 2002

[8] Garbai L.: Távhőellátás (District Heating) *Lecture notes on Budapest University of Technology and Economics*, 2004

[9] Garbai L., Szánthó Z. (2004): Hydraulic Analysis of Vertical two-pipe Central Heating Networks, *Proceedings of the fourth Conference on Mechanical Engineering*, Budapest University of Technology and Economics (Hungary), May 27-28, 2004, ISBN 963 214 748 0, Vol. 1. pp. 269-274